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[p.394]

VI - THE HEDONICAL CALCULUS

PROBLEM - To find (α) the distribution of means and (β) of labour, the (γ) quality and (δ) number of population, so that there may be the greatest possible happiness.

DEFINITIONS -

(**1.**) *Pleasure* is used for "preferable feeling" in general (in deference to high authority, though the general term does not appear to call up with equal facility all the particulars which are meant to be included under it, but rather the grosser[1] feelings than for instance the "joy and felicity" of devotion). The term includes absence of pain. *Greatest possible happiness* is the greatest possible, integral of the differential 'Number of enjoyers × duration of enjoyment × degree thereof ' (*cf*. axiom below).[2]

(**2.**) *Means* are the distributable proximate means of pleasure, chiefly wealth as destined for consumption and (what is conceivable if not usual in civilisation) the unpurchased command of unproductive labour. **[p.395]**

(**3.**) An individual has a *greater capacity for happiness* than another, when for the same amount whatsoever of means he obtains a greater amount of pleasure; *and also* for the

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[1] **[p.394]** Compare the base associations of "Utilitarianism". Surely, as Mr. Arnold says, a pedant invented the term.

[2] **[p.394]** The greatest possible value of $\iiint dp\,dt\,dn$ (where *dp* corresponds a just perceivable increment of pleasure, *dt* to an instant of time, *dn* to a sentient individual).

same increment (to the same, amount) whatsoever of means a greater increment of pleasure.

This "definition of a thing" is doubtless (like Euclid's), imperfectly realised. One imperfection is that some individuals may enjoy the advantages not for *any* amount of means, but only for values above a certain amount. This may be the case with the higher orders of evolution. Again one individual may have the advantages in respect of one kind of means, another of another. But, if one individual has the advantages in respect of most and the greatest pleasures, he may be treated as having more capacity for pleasure in general. Thirdly, the two advantages may not go together. If "the higher pleasures, such as those of affection and virtue, can hardly be said to come from pleasure-stuff at all" (as Mr Barratt says in his able Note in MIND X. often cited below), it is possible (though not probable?), that the enjoyers of the higher pleasures should derive from the zero, or rather a certain minimum, of means (and *à fortiori* for all superior values) an amount of pleasure greater than another class of enjoyers, say the sensual, can obtain for any amount whatsoever of means; *while at the same time* the sensual obtain greater increments, of pleasure for the same increments of means (above the minimum). In such a case the problem would be complicated, but the solution not compromised. Roughly speaking, the first advantage would dominate the theory of population; the second the distribution of means. A fourth imperfection in the statement of the definition is that the units whose capacities are compared are often *groups* of individuals, as families. With these reservations the reality of the definition may be allowed.

But it may be objected that differences of capacity, though real, are first not precisely ascertainable, and secondly artificial being due to education. But, first, even at present we can roughly discriminate capacity for happiness. If the higher pleasures are on the whole most pleasurable -- a fact of which the most scientific statement appears to have been given by Mr. Sully (*Pessimism,* Note to chap. 11) -- then those who are most apt to enjoy those pleasures tend to be most capable of happiness. And, as Mr Barratt says, it "seems (speaking generally) to be the fact that, the higher a being in the scale of evolution, the higher its capacity for pleasure"; while greater precision might be attainable by improved examinations and hedonimetry. Further it will be seen that some of the applications of the problem turn upon *supposed*, rather than ascertained, differences of capacity. The second objection, William Thompson's, would hardly now be maintained in face of what is known about heredity. But it is worth observing that his conclusion, equality of distribution, follows from his premiss only in so far as a proposition like our first postulate (below) is true of wealth and labour applied to *education*, in so far as it is true that improvement is not proportionately increased. by the increase of the means of education.

(**4.**) An individual has more *capacity for work* than another, when **[p.396]** for the same amount whatsoever of work done he incurs a less amount of fatigue, *and also* for the same increment (to the same amount) whatsoever of work done a less increment of fatigue.

This fourth definition may present the same imperfections as the third. Indeed the fourth definition is but a case of the third; both stating relation between means and pleasure. The third definition becomes the fourth, if you *change the signs* of means and pleasure, put means produced for means consumed and the pains of production for the pleasures of consumption. Or not even the latter change, in so far as labour is sweet (which is very far according to Fourier). It is submitted that this identification confirms the reality of the third definition, since the reality of the fourth is undisputed. Of course, if we identify the definitions, we must bear in mind that they are liable to be separated in virtue of the second imperfection above noticed.

AXIOM. -- Pleasure is measurable, and all pleasures are commensurable; so much of one sort of pleasure felt by one sentient being equateable to so much of other sorts of pleasure felt by other sentients.

Professor Bain has shown (*Emotions and Will*, Third Ed.) how one may correct one's estimate of one's own pleasures upon much the same principle as the observations made with one's senses; how one may correctly estimate the pleasures of others upon the principle "Accept identical objective marks as showing identical subjective states," notwithstanding personal differences, as of activity or demonstrativeness. This "moral arithmetic" is perhaps to be supplemented by a moral differential calculus, the Fechnerian method applied to pleasures in general. For Wundt has shown that sensuous pleasures may thereby be measured, and, as utilitarians hold, all pleasures are commensurable. The first principle of this method might be : Just-perceivable increments of pleasure, of all pleasures for all persons, are equateable (cf. Wundt, *Phys*. *Psych*., p. 295). Implicated with this principle and Bain's is the following : Equimultiples of equal pleasures are equateable; where the multiple of a pleasure signifies exactly similar pleasure (integral or differential) enjoyed by a multiple number of persons, or through a multiple time, or (time and persons being constant) a pleasure whose degree is a multiple of the degree of the given pleasure. The last expression is open to question (though see Delbœuf *Étude pscyhophysique*, vii. and elsewhere), and is not here insisted upon. It suffices to postulate the practical proposition that when (agreeably to Fechnerian conceptions) it requires *n* times more just-perceivable increments to get up to one pleasure from zero than to get up to another, then the former pleasure enjoyed by a given number of persons during a given time is to be sought as much as the latter pleasure enjoyed by *n* times the given number of persons during the given time, or by the given number during the multiple time. Just so one cannot reject the practical conclusions of Probabilities, though one may object with Mr. Venn to speaking of *belief* being numerically measured. Indeed these principles of μετρητικη` are put forward not as proof against metaphysical subtleties, but **[p.397]** as practical; self-evident, *à priori*, or by whatever 'εθαγωγη` or 'εθισμο`ς is the method of practical axioms.

Let us now approach the Problem, attacking its inquiries, separately and combined, with the aid of appropriate **POSTULATES**.

(α) The *first postulate* appropriate to the first inquiry is : The rate of increase of pleasure decreases as its means increase. The postulate asserts that the second differential of

pleasure with regard to means is continually negative. It does not assert that the first differential is continually positive. It is supposable (though not probable) that means increased beyond a certain point increase only pain. It is also supposable that "the higher pleasures" do not "come from pleasure-stuff at all," and do not increase with it. Of course there are portions of the utilitarian whole unaffected by our adjustments; at any rate the happiness of the stellar populations. But this does not invalidate the postulate, does not prevent our managing our " small peculiar " for the best, or asserting that in respect thereof there tends to be the greatest possible happiness. The proposition thus stated is evidenced by every-day experience; experience well focused by Buffon in his *Moral Arithmetic,* Laplace in his *Essay on Probabilities,* William Thompson in his *Inquiry into the Distribution of Wealth,* and Mr. Sidgwick in the *Methods of Ethics*.

This empirical generalisation may be confirmed by "ratiocination" from simpler inductions, partly common to the followers of Fechner, and partly peculiar to Professor Delbœuf. All the formulae suggested for the relation between quantity of stimulus and intensity of sensation agree in possessing the property under consideration; which is true then of what Prof. Bain would describe, as pleasures of mere intensity; coarse pleasures indeed but the objects of" much expenditure. Thus pleasure is not proportionately increased by increased glitter of furniture, nor generally by increased scale of establishment; whether in the general case by analogy from the Fechnerian experiments on the senses (*cf*. Fechner *Psychophysik*, ix. 6), or by a more *à priori* "law of relation" in the sense of Wundt.

But not only is the function connecting means and pleasure such that the increase of means does not produce a proportionate increase of pleasure; but this effect is heightened by the function itself so varying (on repetition of the conditions of pleasure) that the same means produce less pleasure. The very parameter in virtue of which such functional variation occurs is exhibited by Prof. Delbœuf in the case of eye-sensations *(Étude pscyhophysique,* &c.) ; that a similar variation holds good of pleasures in general is Bain's Law of Accommodation. Increase of means then, affording proportionately increased repetition of the conditions of pleasure, does not afford proportionately increased pleasure. Doubtless there are compensations for this loss; echoes of past pleasures, active habits growing up in the decay of passive impressions. Indeed the difference of individuals in respect to these compensations constitutes a large part of the difference of capacity for pleasure.

It may now be objected: increased means do not operate solely by **[p.398]** repeating old pleasures, but also by introducing to new (*e.g.* travel); also the "compensations" may *more than counterbalance* the accommodations. It is generally replied : In so far as a *part* only of happiness increases *only proportionately* to its means, the second differentia1 of happiness with regard to means does not cease to be negative. That second differential cannot be *continually* negative. Its being negative for a space *may* not affect the reasoning. If it does affect the reasoning, one conclusion, the inequality of distribution, would probably (if the pleasure-curve is not very complicated) become *à fortiori*. Not only would the less capable receive then still less means, but even the equally capable might then not all receive equal means.

This being postulated, let us mark off the degrees of capacity for happiness on an abscissa (supposing that capacity is indicated by the values of a *single* variable; if by the values of a *function of several* variables, the proof differs only in complexity). At each degree, erect an ordinate representing the number of individuals of that degree of capacity. On the rectangle corresponding to each individual it is required to construct a parallelopiped representing his means. Let us proceed to impart the distribuend means -- in the first inquiry a given distribuend to given distributees doing each a given amount of labour -- by way of small increments. Let us start with the assumption that each individual has and shall retain that minimum of means just sufficient to bring him up to the zero-point of happiness (a conception facilitated by, though not quite identical with the economical "natural minimum of wages"). Thereafter who shall have the first increment of means? By definition an individual of the highest capacity (at least supposing the *minimum* to be the same in all capacities). Who shall have the next increment of means? *Another* individual of the highest capacity, in preference to *the same* individual by the postulate. Thus a first, dividend will be assigned to the first section (all the individuals of the highest capacity) exclusively. But they will not continue sole assignees. Their means only, being continually increased, must by the postulate reach a point such that an increment of means can be more felicifically assigned to an individual of the second *section* (the next highest capacity) than to one of the first. The second section will then be taken into distribution. Thus *the distribution of means as between the equally capable of pleasure is equality; and generally is such that the more capable of pleasure shall have more means.*

The law of unequal distribution is given by a plane curve, in the plane of the capacities and means, say a *megisthedone*. To different distribuends correspond megisthedones differing only by a *constant.* For it is educible from the postulate that there is *only one family* of megisthedones. We may have any number of *maxima* by *tacking* between different members of the family. But the *greatest possible value* is afforded by the *continuous solution.*

If we now remove the condition that each individual shall retain his minimum, what happens? Simply that the megisthedones may now dip below the minimum line. But it is improbable that they should dip very low under the minimum at the lower end while they rise very high above **[p.399]** the minimum at the higher end; since excessive physical privations cannot be counter-balanced by any superfluity of refined pleasures. In fact, if we assume that the zero of means corresponds to *infinite pain* of privation, (*cf*. Wundt's curve of pleasure and pain) then by investigating the radius of curvature it is shown that, as the distribuend diminishes, the megisthedone tends to become a horizontal line. In famine the distribution even between unequals is equality -- abstracted ulterior considerations, as of posterity.

These conclusions may be affected by the imperfections of the third definition. By the first imperfection, if the " minimum " line were not horizontal. Secondly, suppose that the individuals who have less capacity for pleasures in general have a special capacity for particular pleasures. The bulk of means will be distributed as before. But there will be a

residue distributed according to a *second megisthedone*. The second megisthedone superimposed upon the first will more or less deform it. Lastly, the unit distributee is often a *group (e.g.*, a married couple, in respect of their common *ménage*). The conclusions may be affected, in so far as the most capable groups are made up of individuals not most capable *as individuals*.

(β) The distribution of labour (to which attention has been called by Mr. Barratt) is deduced by a parity of reason from the parallel *second axiom* : that the rate of increase of fatigue increases as the work done increases ; which is proved by common experience and (for muscular work) by the experiments of Prof. Delbœuf (*Étude pscyhophysique)*. It appears indeed from Prof. Delbœuf's formulæ the first and second postulates are to a certain extent implicated (whereby the first postulate gains strength). Let us now arrange our individuals according to their *capacity for work* and proceed as before. Who shall do the first increment of work? Of course one of the most capable of work. And so on. *The distribution of labour as between the equally capable of work is equality; and generally is such that the most capable of work shall do more work* -- so much more work, as to suffer more fatigue.

The inquiry presents the same declensions as the first. In particular co-operatives are to be compared *not inter se*, but with the *similar* operatives in similar co-operative associations: except indeed so far as the work done is a *symmetric* function of the effort of fellow-workers. It is deducible that the rowers of a νηο`ς 'ε′ισης shall have equal fatigue; but the fatigue of the pilot is not to be equated to that of the oarsman. All the while it is to be recollected that the fatigue or *pain of work* under consideration may be negative.

(αβ) To combine the first and second inquiries, determine by the Differential Calculus the constants of a *megisthedone* and a *brachistopone* such that the means distributed by the former may be equal to the work distributed by the latter *and* that the (algebraical) sum of the pleasures of consumption and the pains of production may be the greatest possible. Or, *ab initio*, by the Calculus of Variations, we may determine the *means* and *fatigue* as *independent variable functions* satisfying those two conditions.

$$\text{Let} \quad \mathrm{V} = \int_{x_0}^{x_1} n[\mathrm{F}(xy) - p - c\{y - f(xp)\}]dx$$

where x is degree of *either* capacity, or more elegantly a third variable in terms of which both capacities may be expressed; x_1 and x_0 are the given limits of integration (the number and quality of the distributees being not **[p. 400]** in the present inquiry variable); n is the number of each section; F(xy) is a unit's pleasure of consumption, being a function of x his quality (capacity for pleasure) and the *independent variable y* his means; p is the unit's pain of work, another independent variable function; c is the constant incidental to problems of *relative* maximum; $f(xp)$ is the work done by the unit, being a function of his quality (capacity for work) and fatigue (effort).

Greatest possible happiness = greatest possible value of

$$\int_{x_0}^{x_1} n[\mathrm{F}(xy) - p]dx =$$

greatest possible value of V, *c* being taken so that

$$\int_{x_0}^{x_1} n[y - f(xp)]dx = 0\,.$$

The second term of the variation of V,

$$n\left[\delta y^2 \frac{d_2\mathrm{F}}{dy^2} + \delta p^2 \frac{d_2 f}{dp^2}\right]$$

is continually negative by the postulates. Therefore the greatest possible value of V is when its first term of variation vanishes. The first term of variation,

$$n\delta y\left[\left(\frac{d\mathrm{F}}{dy}\right) - c\right] + n\delta p\left[c\left(\frac{df}{dp}\right) - 1\right]$$

vanishes only when both

$$\left(\frac{d\mathrm{F}}{dy}\right) = c \text{ and } \left(\frac{df}{dp}\right) = \frac{1}{c}\,.$$

If these equations hold, the two rules (α and β) hold. Q.E.D. The combined solution takes for granted that the means of pleasure and the pain of work *are independent* variables. And to a certain extent this may fail to be the case. An individual may want *strength* or *time* to *both* enjoy the means and do the work which the double rule assigns to him. In that case there will be a compromise between the two rules.

(**γ**) The *third postulate* simplifying the third inquiry is that capacity for pleasure and capacity for work generally speaking go together; that they both rise with evolution.[1] The *quality of population should be the highest possible* evolution -- provided[2] that the first imperfection of the third definition does not give us pause. To advance the whole population by any the same degree of evolution is then desirable; but it is probably not the most desirable application of a given quantity of *means of education*. For it is probable that the highest in the order of evolution are most *capable of education* and improvement. In the general advance the most advanced should advance most.

[1] **[p.400]** See *New and Old Methods of Ethics* (by the present writer), p.72
[2] **[p.400]** *Ibid*., p.77.

(δ) The *fourth postulate* essential to the fourth inquiry is that, as population increases, means (the distribuend) increase at a decreasing rate. This is given by the Malthusian theory with regard to the products of extractive labour. And this is sufficient. For the second **[p.401]** differential of the whole means with regard to population is still negative, even though a *part* of means increase *proportionately* to the number of population; for instance, unproductive labour requiring little or no materials (*e.g.*, ballet-dancers), or those manufactured articles of which the cost is not appreciably affected by the cost of the raw material. From this Malthusian premiss it is deduced that *population should be limited*; but the hedonical conclusion is not necessarily of the same extent as the Malthusian (*cf*. below αβγ and βδ). A simple inquiry under this head is the following. Assuming that all the sections (degrees of capacity or orders of evolution) multiply equally, and that each section reproduces exactly his kind, to find the rate of increase?

(γδ) A more important inquiry is: *not* assuming that all sections multiply equally, to find the average issue for each section, so that the happiness of the next generation may be the greatest possible.

First let us introduce a conception more appropriate than was possible under the preceding head; namely, that each section does not reproduce exactly its kind, but that the issue of each (supposed endogamous) section ranges on either side of the parental capacity, as thus -- $\nu = \beta\varepsilon^{\frac{-(\xi-x)^2}{b^2}} \times \frac{n}{2}$; where ξ is the capacity of the parental section, *n* its number (= something like $\mathrm{A}\varepsilon^{-\frac{\xi^2}{a^2}}$, since the parental generation is to be conceived as ranging under a curve of possibility; *cf*. Galton, Quetelet, &c.), ν is the number of issue of capacity *x*. Perhaps *b* is constant for all the curves of issue; the variation of β alone determines the natural maximum, or artificial limit, of the average issue. But neither the symmetry of the curves of possibility, nor the particulars of this conception, are postulated.

The *fifth postulate* appropriate to this case is that to substitute in one generation for any number of parents an equal number each superior in capacity (evolution) is beneficial for the next generation. This being granted, either analytically with the aid of Mr. Todhunter's *Researches* (chap. II.), or by unaided reason, it is deduced that the average issue shall be as large as possible for all sections above a determinate degree of capacity, but zero for all sections below that degree.

But can we be certain that this method of *total selection* as it might be termed holds good when we provide not only for the next generation, but for the indefinite future? In the continuous series of generations, wave propagating wave onward through all time, it is required to determine what wavelet each section of each wave shall contribute to the proximate propagated wave, so that the whole sum of light of joy which glows in the long line of waves shall be the greatest possible. If in the distant future, agreeably to the views of Herbert Spencer, population tends inartificially to become nearly stationary; if to the contemplator of all time generations fade into differentials; we may conceive formed a

differential equation connecting the population of one generation with the population of its successor and involving an *independent variable function,* the average issue for each section. By the Calculus of Variations (if the calculator is not at sea) it is educed that the average issue shall be as large as possible **[p.402]** for all sections above a (for each time) determinate degree of capacity, but zero for all sections below that degree. But a further postulate is required for so long as the movement of population is not amenable to infinitesimal calculus; while the present initial irregular disturbances are far from the tranquil waves of the "stationary" state. This *sixth postulate* might be: To substitute in one generation for any number of parents an equal number each superior in capacity (evolution) is beneficial for all time. This postulate being granted, *if possible* let the most beneficial selection be not *total.* Then a total selection can be arranged more beneficial!

If only we have swum through the waves to a *terra firma*, our position need not appear outlandish. For, first, these rules are very general, founded on very abstract tendencies and requiring to be modified in practice. Thus our principle of selection might be modified, in so far as endogamy should not be the rule, if the higher orders of evolution have a greater tendency to reversion (in violation of the fifth and sixth postulates), and so forth. Again, since to exclude some sections from a share of domestic pleasures interferes with the principle of (α), it could not be expedient to sacrifice the present to the future, without the highest scientific certainty and political security. Again to indicate an ideal, though it can only be approached 'ανθρωπι′νως, may be useful. What approach is useful in such cases is to be determined by Mr. Todhunter's principle (*Researches*, chap. II.). Again mitigations might be provided for the classes not selected. (*Cf.* Galton "The weak could find a welcome and refuge in celibate monasteries, &c."; also Sully, *Pessimism*, p. 392). In particular they might have the benefit of rule (β) now almost cut away by the struggle of competition. Again *emigration* might supplement total selection; emigration from Utopia to Atopia -- some unprogressive country where the prospect of happiness might be comparatively zero.

(αβδ) In the preceding analysis (γδ) the distribution of means (and labour) was supposed given. But the reasoning is unaffected, if the distribution of means is supposed variable; provided that the later postulates are not affected by that distribution. And this they might be on Mr. Doubleday's hypothesis. But in Herbert Spencer's more probable view of the relation of affluence to populousness, the first rule (α) will become *à fortiori*.

Under this head may be considered the question: *What is the fortune of the least favoured class in the Utilitarian community?* Let us consider first the case of *emigration* for the benefit of the present generation. Let us start with the supposition, however inappropriate, that the distribuend does not vary with population; as in an isolated island where the bounty of nature could not be affected by human exertion. The happiness of the present generation may be symbolised

$$\int_{x_0}^{x_1} n[\mathrm{F}(xy) - cy]dx + c\mathrm{D}$$

where D is the given distribuend and the rest of the notation is as above (αβ). By the third postulate, x_1 is given as the highest existing degree of capacity. What remains variable is x_0, the abscissa of emigration. At the **[p. 403]** limit $F(x_0 y_0) - cy_0 = 0$. Now c is positive, for it equals $\left(\frac{d\mathrm{F}}{dy}\right)$, the first differential of pleasure with regard to means, which (presupposed a utilitarian intelligence) is probably never negative (above, Postulate I.). But this is not postulated. Only, if $\left(\frac{d\mathrm{F}}{dy}\right)$ is negative, we are dealing with the *external case* of the inquiry; determining what sections shall *immigrate* (from Atopia). For if the Utopians have such a plethora of means that their happiness would be increased by a diminution of their means, then immigration will set in until the point of satiety be at least repassed. Then c is positive; and y is essentially positive. Therefore $F(x_0 y_0)$ is positive. It cannot be zero, the zero-point of pleasure corresponding to a positive minimum of means.

In this case the *condition of the least favoured class is positive happiness***.** This conception assists us to conceive that a similar answer would be obtained, if the increase of the distribuend with increasing population were *small***.**

Small in relation to the megisthedonic share of the least favoured class. Write the distribuend $\int_{x_0}^{x_1} nf(xp\mathrm{N})dx$; where p is the effort of each unit worker, so far supposed given as a function of x; N is the number of population $= \int_{x_0}^{x_1} n\,dx$. Differentiate the distribuend with regard to x_0. Substitute x for x_0 and call the curve so presented the *Malthusian*. Then *the condition of the least favoured class is positive, zero, or negative happiness,* according as at the limit the ordinate of the Malthusian is less than, equal to, or greater than that of the megisthedone.

Our uncertainty as to the condition of the lowest class increases, when we consider the case of *selection* for the benefit of the next generation.

Let $n = \phi(x)$ be the curve of possibility for the present generation. Let $v = \mathrm{B}\varepsilon^{\frac{-(x-\xi)^2}{b^2}} \times \frac{n}{2}$ be the curve of issue for capacity ξ; where B is the natural maximum of issue. Then n^1 the line of possibility for the next generation, is $\int_{x_0}^{x_1} \frac{1}{2}\mathrm{B}_{x+z}\varepsilon^{\frac{-z^2}{b^2}}\phi(x+z)dz$, where by the fifth postulate x_1 is given as the highest existing degree of capacity ; what is variable is x_0, the abscissa of total selection. The happiness of the next generation $\mathrm{H}^1 = \int_{-\infty}^{+\infty}[n^1(\mathrm{F}(xy) - cy)]dx + c\mathrm{D}$, where ∝ is a convenient designation for the utmost extent of *variation* -- variation in the Darwinian sense. x_0 is given by the equation

$\frac{d\mathrm{H}^1}{dx_0} = 0$; from which it is by no means clear that the condition of the least favoured in the second generation is above zero.

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In fact the happiness of some of the lower classes may be sacrificed to that of the higher classes. And again the happiness of part of the second generation may be sacrificed to that of the succeeding generations. Moreover (it is convenient, though out of order, here to add) our uncertainty increases when we suppose the laboriousness also of population variable. *Nothing indeed appears to be certain from a quite abstract point* of view, except that the required limit is above the starving-point; both because in the neighbourhood of that point there would be no work done, and -- before that consideration should come into force and above it -- because the pleasures of the most favoured could not weigh much against the privations of the least favoured. (*Cf.* Wundt's pleasure-curve.)

It may be admitted however that a limit below the zero of happiness, even if abstractedly desirable, would not be humanly attainable; whether because discomfort in the lower classes produces political instability (Aristotle, &c.), or because only through the comfort of the lower classes can population be checked from sinking to the starving-point (Mill, &c.). Let Politics and Political Economy fix some such limit above zero. If now Hedonics indicate a limit still superior (in point of comfort) -- well. But if abstract Hedonics point to a limit *below* that hard and fast line which the consideration of human infirmity imposes, what occurs? Simply that population shall press up against that line without pressing it back.

(βδ) When labour, as well as number of population, is variable, in order that the vanishing of the first term of variation may correspond to a maximum, there is needed in addition to the second and fourth postulates a further condition between the portions of the second term of (the distribuend's) variations which are under the integral sign. A *seventh postulate,* more than sufficient for the purpose, is that the surface W = *f*(*p*, N), representing the work of a unit in terms of his effort and the number of population, should (for each capacity) have no parabolical or hyperbolical points. This is probable, in so far as it is probable that the functions with which we have to deal are simple. But if this condition fails there fails not the second rule (β); there *might* fail the proof which Hedonics might (as just shown) give that there is a limit of population required for the *well-being,* superior to and quite distinct from the limit which is known to be required for the *being* of society. In short the effect of this last consideration is slightly to diminish the probability (previously even?) that, there is such a distinct hedonical limit.

(βγδ) Under this head should be considered whether rule (β) does not interfere with rule (γδ). And this upon Mr. Herbert Spencer's theory of population it would do. (Contrast, however, Champagny, *Les Antonins***,** III., p. 277.) The present then may have to be sacrificed to the future; though in general how much of the present it is expedient to sacrifice to the future must be as nice a question in political, as in personal prudence.

(αβγδ) Contemplating the combined movements we seem to see the vast composite flexible organism, the play and the work of whose members are continually readjusted, by degrees advancing up the line of evolution; the parts about the front advancing most, the members of the other extremity more slowly moving on and largely dying **[p.405]** off. The final shape of the great organism, whether its bounding line of possibility shall be ultimately perpendicular, whether the graduation of (in a Greek sense) *aristocracy* or the level of modern revolution, is the ideal of the future, is still perhaps a subject more for prejudice than judgment. Utilitarianism, indifferent about the means, with eye undistorted by prepossessions, looks only to the supreme end.

COROLLARIES. The application of these inquiries is (I.) to first principles (II.) to subordinate rules of conduct.

I. The end of conduct is argued to be Utilitarianism, as exactly defined in the *Methods of Ethics***,** by deducing from that general principle maxims of common sense; perhaps as the constitution of matter is proved by deducing from the theory experimental laws. What inferior accuracy in the moral universe indeed! But before that inferiority should prejudice, let it be settled what degree of accuracy was here to be expected. No one would listen to Prof. Clerk Maxwell πιθανολογου^ντος about the atoms without a mathematical correspondence of his theory and the facts. But we have a large experience of the progress of Physics; it is well seen how she goes; but is the movement of Morals so familiar that the true science should be manifest by her method! Whatever the method -- for Universal Eudæmonism prescribes no dogma about the origin of her supremacy; affiliated as readily to practical reason as pure passion, the "Faith" of a Green or "Ideals" of a Grote -- whatever our faith, when we descend from faith to works, requiring a criterion for alternative actions, it may be divined that we shall not far err in following, however distantly, the procedure of the *Methods of Ethics*.[1]

Consider first then Equality, the right of equals to equal advantages and burdens, that large section of distributive justice, that deep principle which continually upheaves the crust of convention.

πολλα′ων πολι′ων κατε′λνσε κα′ρηνα
᾽ηδ᾽ ᾽′ετι και᾽ λν′σει· τον^ γα᾽ρ κρα′τος ᾽εστι με′γιστον

All this mighty moral force is deducible from the practical principle of exact Utilitarianism combined with the simple laws of sentience (α and β).

But Equality is not the whole of distributive justice. There may be needed an ᾽αξι′α for unequal distribution. Now inequalities of fortune -- abstracted the cases of governor and general and every species of trustee for the advantage of others -- are generally explained by utilitarians as the consequence of conventions clear and fixed and preventing

[1] **[p.405]** Pp. 90, 346, 392, &c., 2d edn. *Cf*. Buffon, *Moral Arithmetic*: "Le sentiment n'est en géneral qu' un raisonnement implicite mains clair, mais souvent plus fin et toujours plus sûr que le produit direct de la raison." (He is proving our first postulate.)

confusion and encouraging production, but not otherwise desirable, or rather of which the necessity is regretted. Yet in the minds of many good men among the moderns and the wisest of the ancients, there appears a deeper sentiment in favour of aristocratical privilege -- the privilege of man above brute, of civilised above savage, **[p.406]** of birth, of talent, and of the male sex. This sentiment of right has a ground of utilitarianism in supposed differences of *capacity*. Capacity for pleasure is a property of evolution, an essential attribute of civilisation (α). The grace of life, the charm of courtesy and courage, which once at least distinguished rank, rank not unreasonably received the means to enjoy and to transmit (α). To lower classes was assigned the work of which they seemed most capable; the work of the higher classes being different in kind was not to be equated in severity (β, *cf*. Livy II., 32). If we suppose that capacity for pleasure is an attribute of skill and talent (α); if we consider that production is an *unsymmetrical function* of manual and scientific labour (β); we may see a reason deeper than Economics affords for the larger pay, though often more agreeable work, of the aristocracy of skill and talent. The aristocracy of sex is similarly grounded upon the supposed superior capacity of the man for happiness, for the ενεργει^αι of action and contemplation; upon the sentiment --

> "Woman is the lesser man, and her passions unto mine
> Are as moonlight unto sunlight and as water unto wine."

Her supposed general incapacity is supposed to be compensated by a special capacity for particular emotions, certain kinds of beauty and refinement. Agreeably to such finer sense of beauty the modern lady has received a larger share of certain *means,* certain luxuries and attentions (Def. 2 ; a *sub finem*). But gallantry, that "mixed sentiment which took its rise in the ancient chivalry," has many other elements. It is explained by the polite Hume as attention to the weak *(Essay*, 14), and by the passionate Rousseau φνσικωτε′ρως *(Émile*, 4). Now attention to the weaker sex, and woman's right not only to certain attentions in polite society but to some exemption from the harder work of life, are agreeable to the utilitarian theory: that the stronger should not only do more work, but do so much more work as to suffer more fatigue where fatigue must be suffered (β). It may be objected: consideration should equally be due from the stronger to the weaker members of the same sex. But in the latter case there is wanting a natural instinct predisposing to the duties of benevolence; there has been wanting also a fixed criterion of strength to fix the associations of duty ; and lastly competition has interfered, while competition between man and woman has been much less open (and much less obviously useful to the race). Altogether, account being taken of existing, whether true or false, opinions about the nature of woman, there appears a nice consilience between the deductions from the utilitarian principle and the disabilities and privileges which hedge round modern womanhood.

Utilitarian also is the custom of family life, among other reasons, in so far as (contrasted with communistic education) it secures for the better-born better educational influences (γ); in particular a larger share of good society in early life. The universal principle of the struggle for life, as Mr Barratt may suggest, conduces to Utilitarian selection. This being borne in mind, there appears a general correspondence between the population-theory

above deduced (γδ) and the **[p.407]** current ethics of marriage, which impose[1] only a precedent condition, success, hereditary or personal, in the struggle for life. Concerning the classification of future society common-sense anticipates no utopia of equality. Physical privations are pitied; the existence of a subordinate and less fortunate class does not seem to accuse the bounty of Providence. (*Cf*. Burke on the "labouring poor" in *Regicide Peace*, 3.) With the silence of common-sense accords the uncertain sound of exact Utilitarianism (αγδ).

But, if egoist or intuitionist are not to be altogether converted by the deductive process of Mr Sidgwick, at least the dealing with his exact definition may tend to mark out and reclaim from the indefinite one large common field of conduct, one of the virtues of the intuitionist, one of the gratifications of the egoist -- rational benevolence. For can there be a rational wish to please without a willingness to estimate the duration of the pleasure, the susceptibility, as well as the number, of the pleased?

Exact Utilitarianism may also, as Mr. Barratt thinks plausible, present the end of Politics; of Politics as based upon self-interest. A political "contract" for the adjustment of conflicting interests should have two qualities. It should be clear and fixed, universally interpretable in the same sense. It should be such that the naturally more powerful class, those who though fewer outweigh the more numerous by strength ability and capacity to co-operate, should not have reason to think that they would fare better under some other contract. Two contracts present these qualities; the rough and ready *iso*cratical, the exact possibly *aristo*cratical, Utilitarianism. The first contract excels in the first quality; the second in the second.

II. That the same reasonings should lead up to a general principle and down again to its applications -- that the theory should be tolerably certain, the practice indefinitely remote -- is not more paradoxical than that the demonstrator of the atom-theory should foresee the remote possibility of its application, no less a possibility than to triumph over the second law of Thermodynamics (Clerk Maxwell, *Theory of Heat*, p. 308). The triumphs of Hedonics, if equally conceivable, are equally remote; but they do not so certainly become more conceivable when considered more remote; for what if in the course of evolution the subtlety of science should never overtake the subtlety of feeling! Faint and vague and abstracting many things which ought not to be abstracted, the Hedonical Calculus supplies less a definite direction than a general bias, here briefly and diffidently indicated.

The end of action being defined as above, the Jacobin ideal 'All equal and rude,' J. S. Mill's ideal 'All equal and cultivated,' are not necessarily desirable, not paramount ends to be sought by revolution or the more tedious method of depopulation. Pending a scientific hedonimetry, the principle 'Every man, and every woman, to count for one' should be very cautiously applied. In communistic association (if such should be) the distribution of produce should be rather upon the principle of Fourier than of Owen.

[1] **[p.407]** In respect to population.

Universal equal suffrage **[p.408]** is less likely to be approved than plural votes conferred not only (as Mill thought) upon sagacity, but also upon capacity for happiness.

The play of the struggle for life is to be encouraged, in the present state of society, within limits, without prejudice to the supremacy of the supreme principle. Mr. Barratt indeed from the same premisses, the utility of competition, infers a different conclusion: that Utilitarianism should resign in favour of Egoism. But surely the inference is, not that the Utilitarian should change his destination from Universal to Egoistic Hedonism (points *toto cælo* apart, as the chart of Sidgwick shows); but that, while constant to his life's star, he should *tack* (in the present state of storm at least) more considerably than the inexperienced voyager might advise. No one can misunderstand this "self-limitation " of Utilitarianism -- for it has been explained by Mr. Sidgwick; least of all the Egoist -- for a similar delegation, without abdication, of the supreme command is much more necessary in the case of the supremacy of self-love (Butler, &c.).

Lastly, while we calculate the utility of pre-utilitarian institutions, we are impressed with a view of Nature, not, as in the picture left by Mill, all bad, but a first approximation to the best. We are biassed to a more conservative caution in reform. And we may have here not only a direction, but a motive, to our end. For, as Nature is judged more good, so more potent than the great utilitarian has allowed are the motives to morality which religion finds in the attributes of God.

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