curves, especially at low pressures, must be considered rather as a convenient way of showing which dots in the figure correspond to any given pressure than as an attempt at interpolation.

IX. "The Asymmetrical Probability Curve." By F. Y. EDGEWORTH, M.A., D.C.L. Communicated by Sir G. G. STOKES, F.R.S. Received June 14, 1894.

(Abstract.)

The asymmetrical probability curve is the second approximation—the symmetrical probability curve being the first approximation—to the law of frequency which governs the set of values assumed by a function of numerous independently fluctuating small quantities. The curve may be written

$$y = \frac{1}{\sqrt{\pi}c} e^{\frac{-x^2}{c^2}} \left[- \frac{2j}{c^3}\left(\frac{x}{c} - \frac{2}{3}\frac{x^3}{c^3}\right)\right];$$

where $y\Delta x$ is the number of errors occurring between x and $x+\Delta x$, $c^2/2$ is the mean square of errors, and j is the mean cube of errors—errors measured from the centre of gravity. This form is obtained by completing the analysis which Todhunter, after Poisson, has indicated ('History of Probabilities,' Art. 1002); and independently by obtaining a general form for the asymmetric probability curve, and deducing therefrom the Poissonian formula in the case when the asymmetry is slight—the only case to which that formula is applicable.

Among the peculiarities of the asymmetric probability curve are the want of coincidence between the arithmetic mean and the position of the greatest ordinate, and the descent of the curve at one extremity below the abscissa—the ordinate appearing to denote *negative* probability.

An important case of the general curve is afforded by the *Binomial*, for which each of the independent elements admits of only *two* values. The approximate form of the Binomial, obtained directly by Laplace (Todhunter, 'History,' Art. 993), is deducible from the general theory. The general, or multinomial, probability curve can always be represented by a binomial.

The principle of the asymmetric probability curve affords an extension of the theory of *correlation* investigated by Messrs. Galton and Hamilton Dickson ('Roy. Soc. Proc.,' 1886, p. 63). The symmetrical probability surface

$$z = \frac{1}{\sqrt{\pi}\sqrt{1-r^2}} e^{\frac{-(x^2-2rxy+y^2)}{1-r^2}}$$

becomes now slightly distorted, so that the locus of the most probable y deviation, corresponding to an assigned x deviation is no longer a straight line, but a parabola.

X. "The Differential Covariants of Twisted Curves, with some Illustrations of the Application to Quartic Curves." By R. F. Gwyther, M.A., Fielden Lecturer in Mathematics, Owens College, Manchester. Communicated by Professor Horace Lamb, F.R.S. Received May 31, 1894.

(Abstract.)

The object of the earlier parts of this paper is to obtain relations connecting what Halphen* calls the canonical invariants of the curve, without the intervention of what are called by him the fundamental invariants. In any geometrical investigation it is the canonical invariants which present themselves, and the relations between the consecutive canonical invariants, and the values of their differential coefficients must, if the method of investigation is to be used, be expressed, in terms of canonical invariants only, without the intervention of the other series of invariants which Halphen treats as fundamental.

In the paper, the notation of Halphen's paper cited above is generally followed, but the mode of initial investigation, as in a previous paper on covariants of plane curves,† is made to depend upon a homographic transformation with infinitesimal arguments.

Writing ξ, η, ζ for the coordinates of a current point, x, y, z for the coordinates of a point on a standard curve, and y_n and z_n for $d^ny/dx^n, n!$ and $d^nz/dx^n . n!$, the arguments of a covariant function are shown to be

$$f = \xi - x,$$

$$g = z_3\{\eta - y - y_1(\xi - x)\} - y_3\{\zeta - z - z_1(\xi - x)\}/y_2z_3 - y_3z_2,$$

$$h = y_2\{\zeta - z - z_1(\xi - x)\} - z_2\{\eta - y - y_1(\xi - x)\}/y_2z_3 - y_3z_2,$$

with

$$a_n = y_2z_n - y_nz_2/y_2z_3 - y_3z_2,$$

$$b_n = y_nz_3 - y_3z_n/y_2z_3 - y_3z_2,$$

while $y_2z_3 - y_3z_2$ is an invariant which will not appear independently.

The other conditions that a function $\phi(f, g, h, a_n, b_n \ldots)$ may be a covariant function are (1) that it shall be isobaric, counting $f, g, h, a_n,$

* "Sur les Invariants Différentiels des Courbes Gauches," 'Journal de l'École Polytechnique,' cahier 47, vol. 28, 1880.

† 'Phil. Trans.,' vol. 184 (A), 1893, p. 1171.