SECTION III
MONEY

(H)

MEASUREMENT OF CHANGE IN VALUE OF MONEY

[This article consists of two papers, the first and third of three Memoranda which were presented to the British Association for the Advancement of Science in the years 1887, 1888, 1889 respectively. The three were prepared by the present writer acting as Secretary of a Committee appointed for the purpose of investigating the best methods of ascertaining and measuring Variations in the Value of the Monetary Standard. The second Memorandum dealing with a special aspect of the subject is reprinted as a separate Article, the second of the present section. The first and third Memoranda being *in pari materia* are here put together. The collocation of an originally somewhat diffuse disquisition with afterthoughts which occurred two years later does not form a model of order and unity. The composition is like the " scene of man," according to Pope, " a mighty maze "; but that it is " not without a plan " may appear from the following *résumé.*

The " natural method of calculating a measure of change," in the phrase of Mr. Flux (*Journal of the Statistical Society*, 1921, p. 175), is to determine the change in the money value of the articles consumed by the population under consideration. This standard is considered in the first section of the first Memorandum. The simplest form of this standard is " the comparative money-cost of a fixed schedule of articles " (Flux, *loc. cit. Cp.* **N**, below, p. 396). The most refined form of this standard compares the amounts of money required to procure the same *satisfaction* at different epochs (*Cp.* Sidgwick, *Political Economy*, Book I., ch. ii. § 3; Bowley, *Journal of the Statistical Society*, 1921, p. 351). Over against this Consumption-Standard is the Production- or Labour-Standard, which compares the amounts of money procured by the same " Real Cost " in the sense of effort-and-sacrifice. This standard is propounded in the last section of the third Memorandum. Quantity of labour may not be a very distinct idea; as Adam Smith says, " the greater part of people understand better what is meant by a quantity of a particular commodity." There is, however, a very distinct difference between this and the first standard. Thus, if gold prices fall through increased production of commodities, as perhaps in the eighties of last century, according to the first standard there may appear a serious appreciation demanding correction; according to the second standard there may be no change in the " real value " (Marshall and Ricardo) of money, gold is behaving very well. These two standards may be regarded as species of a genus which may be described as subjective, or personal, in contrast with objective standards which are less directly adapted to definite human purposes. Such is the character of the " Indefinite Standard " consisting of a mere average, as propounded in Section VIII. of the first Memorandum, where it will be observed that some hypothesis involving sporadic dispersion of prices—before (with respect to standards of the first genus) repeatedly stated not to be implied—is now for the first time introduced. This may be considered as the first species of the objective genus. A second species is presented when we consider not only the fact of an average change in prices, but also as its cause the change in the relative quantity (and velocity) of circulation. This species may be identified with the second method of the third Memorandum; connected in 1889 with the name of Foxwell, and subsequently elaborated by Professor Irving Fisher. Within this species there is a variety which not only connects the change in prices with change in the quantity of circulation, but also considers as the cause of that cause change in the quantity of gold, or other primary money on which the circulation (cheques and notes) is based. The indefinite standard may also be divided into species of which one is irrespective of the quantities of commodities, the other takes account of quantities. The second species includes the variety just now mentioned (second part of Section IX., First Memorandum), and more generally cases in which the conditions of a perfect market are not realised in such wise as to render the first species appropriate (first part of Section IX.). A cross-division of the indefinite standard is formed by the use of different averages; in particular the Arithmetic Mean, the Median, the Geometric Mean and the Mode, the familiar four to which the character of objective and not directly adapted to special purposes principally appertains.

The genus index-number of prices may be defined so as to include the measurement of differences between different *places* in the value of money (p. 290).

Another cross-division of the genus is formed by the limitation of the population or district to which the computations refer. The index-numbers examined on Sections III.–V. of the third Memorandum may perhaps be considered as thus referring to a particular interest, foreign trade.

Or these, like several of the remaining sections, may be considered as dealing with imperfect measures, symptoms and indications of one or other of the index-numbers above defined. Section III. of the first Memorandum is thus related to the Consumption-Standard of Section II. Section IV. and V. of the first Memorandum may be subordinated to the Production-Standard of Section VII., third Memorandum. Section I. of the second Memorandum bears a similar relation to Section VI. Section I. of the second Memorandum might also be treated as an imperfect substitute for Section II. of that Memorandum (an abridgment in which account is not taken of the pull upon currency exercised by the repeated sale of a commodity). It is a question whether the Capital Standard proposed by Professor Nicholson should be treated as subsidiary or as a new substantive index-number.

There is finally an index-number subordinated not to one or other of the ends defined, but to several of them; adapted to secure the maximum of utility, regard being had to the different kinds of advantage described in the different sections. Such is the purport of the "mixed modes" in Section X. of the first Memorandum corresponding to Professor Wesley Mitchell's "general purpose" index-number (see **N**, below, p. 387).

It may be asked, How does the indefinite standard enter into a compound of this sort? The answer that it contributes the important attribute of a "true mean" (Section VIII., First Memorandum, p. 233), a "good average" (*Journal of the Statistical Society*, 1923, p. 572). Such is the purport of that compromise between the principles of the Consumption-Standard and the more objective species which the British Association Committee seemed to sanction (Section X. of First Memorandum). The weighted Arithmetic Mean prescribed by the Committee would be proper in the absence of any hypothesis as to the dispersion of the data. But the formula may be used with more confidence when the hypothesis of sporadic distribution is known to be realised.

For further elucidations of the Memoranda the reader is referred to two papers dealing with hostile criticism, viz. *The doctrine of Index-numbers according to Mr. Correa Walsh*, ECONOMIC JOURNAL, 1923, and the *Calculation of Index-numbers by Mr. Correa Walsh*, "Journal of the Statistical Society," 1923. Mr. Walsh makes a rejoinder in *The Quarterly Journal of Economics*, May, 1924.]

FIRST MEMORANDUM

CONTENTS.

Introductory Synopsis.

The object of this paper is to define the meaning, and measure the magnitude, of variations in the value of money. It is supposed that the prices of commodities (including services), and also the quantities purchased, at two epochs are given. It is required to combine these data into a formula representing the appreciation or depreciation of money.

It will appear that beneath the apparent unity of a single question there is discoverable upon a close view a plurality of distinct problems. Many different branches have been traced, and the number might be largely increased if every bifurcation were followed out to its logical end. But it is not to be supposed that the innumerable ramifications which a formal logic might be able to distinguish would all repay cultivation. The most rigorous analysis may be content with a dozen distinct cases; and for the purpose of an introductory summary these may be reduced under a still smaller number of headings.

To one taking a general view of the subject there stand out four main types, four modes of measurement distinct in idea and definition, though occasionally coincident in practice. The *first* sort of measure is based upon the change in the prices of finished products, the object being to find, or rather show how to find at any future time, a ratio or *Unit* such that the creditor in the future receiving as many Units as he at present receives pounds may derive as much advantage in the way of consumption then as now. The *second* sort of measure is based upon all the articles which trade deals with, the object being to find a Unit such that the debtor in the future, paying as many Units as at present pounds, may not be more hampered in his business then than now. There is *thirdly* the measure of that appreciation which it is the object of bimetallism and similar projects to correct? The *fourth* sort of measure is required not so much for any particular practical object as for the more general purposes of Monetary science, to interpret the past and forecast the future.

Let us add a few words on each of these methods separately, to explain more clearly either the means adopted or the end proposed, or how far those means are conducive to that end.

(1) The general principle of the first method may be embodied in slightly different rules, of which the following two may claim to be the best. (α) In order to ascertain the change in the value of money between two epochs, find the national [1] expenditure per head upon finished products or articles of consumption (including unproductive services) at each epoch. The ratio of the new to the old expenditure is the required measure of depreciation, or Unit. Otherwise (β) thus (the general principle being interpreted somewhat differently): Find the quantities of each article consumed at the two epochs, and take the mean of each couple. Multiply each of these mean items by the old price of the corresponding article and add together these amounts. Proceed similarly with the new prices. The ratio of the latter sum to the former is the required Unit. There are other formulæ, in all more than half a dozen. But there is not much to choose, among them. And the exercise of a choice may exceed the powers and province of the writer.[2]

The advantages of rendering money a steady measure of value-in-use would be considerable wherever there may be violent fluctuations of general retail prices.[3] Such oscillation in the purchasing power of money intensifies the ups and downs of Fortune—so trying both to the sentient and the moral nature of man. The disturbance superadded by a bad currency might be annulled by a corrected standard. The honest labourer would not be cheated of his reward by miscalculations of the value

[1] See below p. 223; and p. 213.

[2] To choose between the first of the rules just given and the second is beyond the scope of this paper.

[3] The advantages of a "Tabular Standard of Value" have been pointed out by many writers. See Jevons, *Currency and Finance*, p. 122, and the references given in the note.

of currency. Those who had laid out their lives upon the faith of a fixed income would not be disappointed of their just hopes. The provision for the widow and the orphan would be more secure. The endowments of learning would preserve that constancy of competence which is favourable to the cultivation of the liberal arts.

These great advantages seem capable of being largely realised. For it is shown by statistics, such as those of Engel [1] and the Massachusetts Labour Reports,[2] that there is considerable constancy in the budgets of family expenditure. Thus in Massachusetts in 1885 the average workman spent out of 100 dollars 29·5 upon groceries, 19·7 upon provisions, 4·3 upon fuel, and so on. Suppose a Unit or corrected dollar continually equivalent to the amounts of groceries, provisions, fuel, etc., which in 1885 were respectively purchased for ·295, ·197, and ·043. There is reason to believe that such a Unit would afford a tolerably constant sum of satisfaction to the Massachusetts working family. But we cannot expect an equally perfect measure, when we construct a Unit, not for a class, but a nation.[3]

(2) The desirability of prescribing separately for different interests is even more strongly brought before us when we consider the second of the methods above defined. It purports to be a *sliding scale* for general use, adapted to all trades. But what fits all indiscriminately cannot fit many exactly. We may say of such a project what Steuart says of a certain "ideal standard," that it is "acting like the tyrant who adjusted every man's length to that of his own bed, cutting from the length of those who were taller than himself, and racking and stretching the limbs of such as he found to be of a lower stature." It would not be unreasonable, however, to construct beds of different sizes, adapted to the average height of markedly different classes of persons, say little boys and men. Similarly, when average prices have largely varied, a scale sliding with the average variation, however imperfectly fitted to particular trades, may be suitable to industry as a whole. The illustration shows the spirit in which our calculation should be performed. What should

[1] *Volkswirtschaftliche Zeitfrage*, Heft 24. *Inst. Natl. de Statistique*, N. 5.

[2] For 1885. See also Young, *Labor in Europe and America*.

[3] Professor Foxwell writes: "I think it would also for many purposes be extremely convenient to have an index-number, or numbers, indicating the altered purchasing power of selected amounts of consumers' incomes, estimated in the corrected standard. I mean that having first determined, by our principal standard, the corrected value of £1 for the given year, we should then find the alteration in the purchasing power of the new standard £1 for different incomes: *e. g.* for incomes of £50, £100, £200, £500, £1,000, and £10,000."

we think of an upholsterer who, having to construct different types of bed, should invoke the aid of the British Association Anthropometric Committee nicely to determine *l'homme moyen* for different ages? The labours of that committee would not be more misspent than ours, if we attempted in framing a universal sliding scale to determine the ideally best *weight* for each item entering into the combination. Almost any combination of the more important articles of trade is likely to be equally imperfect and equally serviceable (see p. 228).

The advantages aimed at by this method may be presented under two aspects. That steady secular decline of prices which, according to many eminent writers, is a cause of the depression of trade, might be corrected. The advantages offered by bimetallists would be attained. There might be also another benefit which not even bimetallists venture to promise. The sudden violent oscillations in general prices, occasioned by the derangement of credit, would be arrested. For, as the supply of money to meet debts became deficient, the demand for money to meet debts would proportionately dwindle; the amount of debts in "standard" currency inversely varying with the value of metallic money.[1] The hunger for gold would be less felt just as the means of satisfying it were less abundant. Heretofore a contraction of currency has acted like an atmospheric depression in the physical world. The drain and rush of the medium has produced a storm. But in the new commercial Cosmos, equilibrium between debts and currency being continually preserved, the stormy winds of Panic will have ceased to blow. Hitherto the relation between liabilities and currency has been that of a continent to the ever-changing level of the sea. Each ampler tidal wave has rendered harbours unserviceable, and dislocated trade, and strewn the shore with wrecks. But the latest invention of science is a sort of *floating dock*, which shall rise with the flood and sink with the ebb, so that the argosies of commerce may be safely landed, whatever the level of the transporting medium.

These are fascinating images, ideal possibilities, which the sober thinker may entertain while he is conscious how remote and uncertain is the realisation; how numerous the difficulties and objections. Perhaps the new organisation of the money market would develop new varieties of roguery. Certainly complications would arise between liabilities to the foreigner

[1] This action is well exemplified in the plan proposed by William Cross, that the standard should vary *per saltum*; a correction being made as often, say, as money was appreciated (or depreciated) by 3 per cent.

expressed in gold, and engagements with the home trader expressed in the adjusted currency. It is alleged, too, that the business of banking would be impeded. In fine, the common sense of business men appears opposed to the scheme; and, on the question what is at present practicable and what not, the opinion of practical men, even unsupported by reasons, is conclusive.

(3) The third inquiry is, What is the appreciation (or depreciation) which it is the object of bimetallism and similar projects to correct? What is that mean (or function) of prices which the bimetallist would desire to keep constant? Of course, if prices varied all in much the same ratio, like the lengths of shadows with the advancing day, the answer would be very simple. That ratio is the required measure. But suppose that one large category of prices is pretty uniformly elevated, while another is *en bloc* depressed; we desiderate a measure which, like the two preceding, may be independent of the particular hypothesis that there has been a uniform average price-variation all over the field of industry.*

It is to be observed that the Unit required for this purpose cannot be restricted to a particular geographical or industrial area. Rather the averaging must be extended over the whole system of countries in monetary communication—that is, over the greater part of the civilised and uncivilised world.

(4) When we consider the next type, the fourth definition of our problem, there once more is pressed upon us the expediency of limiting the area of markets over which our measurement is to extend. It may be doubted whether a standard based upon the variation of all prices indiscriminately would—abstracted from some definite particular purpose such as those contemplated in the preceding paragraphs—be of much scientific use. It would be like taking the mean barometric pressure over a large continent. It is more useful to observe the variation of pressure at particular stations, in order to predict what changes will be propagated to neighbouring regions, what storms are coming. Suppose, for the sake of illustration, that at any station the reading of a single barometer was not sufficient to give the true pressure; that each instrument was liable to a proper disturbance over and above the general atmospheric change. The heat or cold, for example, of different situations might cause a misleading

* I have omitted two sentences identifying the desiderated Mean with one of the two preceding or some cross between them. Rather, the formula which the writer is here feeling after is to be identified with the Currency Standard defined in Section 2 of the Third Memorandum (below p. 261, *et seq.*).

expansion or contraction of the mercury. On such a supposition it might be proper, in order to measure the pressure at any station, to take a mean between the readings of several barometers. Upon well-known hydrostatical principles, no particular importance, other things being equal, would attach to the reading of the barometer which contained a particularly large mass of mercury.

These conceptions appear appropriate to our problem. We should demarcate a certain region of industry, and estimate in terms of that special group of articles an index-number indicative of changes which are likely to become general. The zone of observation most suitable to our purpose would probably be as it were the coast-line of trade, those articles of world-commerce which are most sensitive to changes propagated from abroad. In taking such a mean of observations the " weights " are not necessarily proportioned to the masses of commodity. *Primâ facie* and in the abstract pepper may afford as good an index as cotton (see p. 243 and context). The writer has given rules for taking the mean of these observations. But he is aware how difficult it is to define the proper zones; how hardly susceptible of perfection is the science of monetary meteorology.

Contemplating all these types we discern a property common to most of them, the desirability of treating separately selected interests, rather than operating upon all commodities indiscriminately. To construct such partial measures does not seem to be the business of this Committee, or at least this Memorandum. We may, however, hope that our theoretical diagnosis of different purposes may be of use to those who undertake the more practical task of prescribing for different interests.

SECTION I.

Description and Division of the Problem.

The business of this Committee is to measure a fact, not to speculate about its causes or consequences. Should a fall in the value of money have occurred we need not trace that phenomenon to its sources. Whether it takes its rise on the side of the precious metals or of commodities—whether, in Dr. Johnson's phrase, it is the pence that are few or the eggs that are many—it is not our part to determine. The consequences of the change are equally outside our province. It is open to us to hold with Hume that, when prices are rising owing to the influx of money, " everything takes a new face; labour and industry gain life." With General Walker we may predicate the converse attributes of falling prices. Or we may accept Professor Marshall's[1] qualified, or Mill's[2] negative, statement of those effects. We have to leave speculation and apply ourselves to measurement.

But, while we are not called upon to decide such controverted questions, we cannot be as indifferent to the decision as might at first sight have appeared. For it is only in the simpler kinds of measurement that the metretic art can be entirely divorced from theory about its subject-matter. To measure the height of a man we do not require a knowledge of anthropology. We may even ascertain the mean stature of a nation without much special knowledge. But difficulties arise when we have to do not with one attribute, such as *height*, but with two (or more) attributes: for instance, the masses and velocities of a system of bodies. Take the simple case of a number of heavy particles at rest, and suppose that different velocities are imparted to the different particles between two given epochs. It would not be very easy for one coming fresh to the study of mechanics so to define his confused general idea of the *change of motion* which had occurred as to be able to express it in terms of the data: namely, the masses, say M_1, M_2, etc., M_n, and the imparted velocities (which, in order to minimise difficulties, we will suppose all in the same direction) V_1, V_2, etc., V_n. It is plausible to say that the problem is purely statistical, that we seek a merely objective result. The difficulty is that any combination—at least, any symmetrical combination—of the data is in a sense objective. We must call in mechanical science to determine what combinations are worth forming and what are insignificant. Consider the two combinations $M_1 V_1^2 + M_2 V_2^2 + \text{etc.} + M_n V_n^2$ and $M_1^2 V_1 + M_2^2 V_2 + \text{etc.} + M_n^2 V_n$. *Prima facie*, these are both equally "objective," and they seem equally simple. But while the former (the expression of energy) constitutes a spell for opening all the secret chambers of Nature, the latter could only be significant on some very peculiar hypothesis, for some very out-of-the-way purpose.

Similarly, in the problem before us we have to combine two sets of data, the prices of different articles and the quantities thereof. Indeed, our problem is rather more complicated. We may have to take account of a third attribute, the *quality* or species of wares; to consider, for instance, whether the price and quantity of labour or of materials shall enter *pari passu* and symmetrically into that combination of our data which we desiderate.

In order to discover the principle on which this combination

[1] *Third Report on Industrial Depression*, Appendix C, Vol. II. p. 422.

[2] *Political Economy*, Book III. chap. xiii. § 4.

is to be effected, we may be led into the most perplexed regions of monetary science. We are brought against the question, What is the relation between the amount of money in a country and the general scale of prices?—the question which has been called by a distinguished authority [1] " one upon which the most contradictory opinions have been expressed by economists of reputation." And even where there are no fundamental differences of theory, yet practice may vary according to the practical end in view. Some may aim at the construction of a tabular standard, adapted only to contracts extending over a long period of time; others may desiderate a more flexible standard, which may mitigate the effects not only of the secular, but also of the more [2] transient variations in the value of money; others may seek only an index of the future course of prices—a sort of monetary barometer.

There are therefore many methods—not one method—of " measuring and ascertaining variations in the value of money." The path which we have to investigate has many bifurcations. To decide at each turn which is the right direction is either impossible, or at least presumptuous. It is impossible when both ways are right, directed to different but equally legitimate ends. And, even where there must be a right and wrong, it is not becoming here to pronounce upon points controverted by high authorities. The course adopted is to trace separately the alternative paths, indicating the difference without expressing a preference.

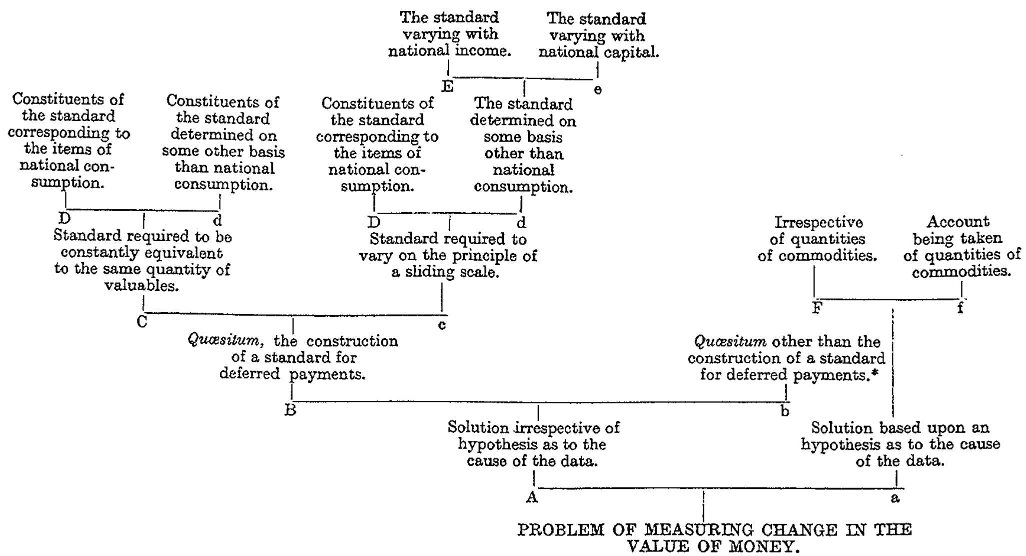
In this memorandum it is proposed to distinguish the various cases of the general problem, and to construct the formula appropriate to each case. The numerical determination of the quantities which enter into the formulæ—both the compilation of the proper figures from explicit statistics, and, where these are wanting, the more speculative arts of inferring unknown prices and amounts from imperfect data and indirect indications—these parts of the subject are not treated by the present writer. They may be considered in a future Report of the Committee and in separate memoranda contributed by other members.

The delicate subdivisions of the subject are exhibited in the annexed diagram by means of a regular *logical tree*.[3] In examining this tree of knowledge we shall give priority to the branches

[1] General Walker in his *Money*.

[2] As Professor Marshall hopes; *Contemporary Review*, March 1887.

[3] As logical and genealogical trees for the most part, like the trees in the poet Parnell's *Hermit*, " depending grow," it may be as well to point out to the reader that our tree, like those cultivated by some of the earlier logicians, is trained *upwards*.



* A variant definition of *b* is offered by the phrases used in the remarks now prefixed to this article—"not adapted to any special or definite purpose." So understood, the attribute *b* would often occur with, could hardly occur without, the attribute $\bar{a}$, implying the existence of a true mean or good average.

on the left. As soon as we have reached the definition of each ultimate species we shall add its properties—the treatment adapted to that particular case. We shall not only trace out the form of each branch, but also gather the fruit at its extremity, before we go on to the branch nearest on the right.

The whole subject is first divided according as the method adopted is (A) irrespective of any hypothesis as to the cause of the price movements or (a) is based on some such theory. Deferring the treatment of the latter case (a) we proceed to divide the former according as (A B) the practical purpose in view is to construct a standard or "Unit" for deferred payments, or (A b) some other purpose. Postponing the latter case, we may complete the definition of the former by explaining that the Unit (a term borrowed from Professor Marshall's recent article in the *Contemporary Review*), as used here in a general sense, means a sum of money estimated to be equivalent at present (or at some future time) to what a Unit of money, say a pound, was worth at some past time: in such wise that it may be just or expedient for debtors to pay, and creditors to receive, as many Units now (and from time to time) as they contracted to pay and receive pounds at the initial epoch. The general idea of a Unit may be specialised according as it is required that (A B C) the Unit should constantly be equivalent to the same quantity of valuables, or (A B c) that it should not represent a constant purchasing power, but one varying with the means of debtors, after the manner of a *sliding scale*. Lastly the kinds and quantities of the valuables entering into the Unit may either (A B C D) correspond to the items of national consumption, or (A B C d) may be selected on some other principle. In this arrangement priority does not import any preference.

Section II.

Determination of a Standard for Deferred Payments; based upon the items of national consumption; calculated to afford to the consumer a constant value-in-use; no hypothesis being made as to the causes of the change in prices. (A B C D.)

According to this arrangement, the first case for which we have to prescribe is where, apart from any hypothesis as to the cause of the movement of prices, we want to construct a Unit adapted to deferred payments, and where it is required that the Unit should be constantly equivalent to the same amount of valuables, the kinds and proportions of the valuables corresponding to the items of the national expenditure. Upon reflection it will be found that the last attribute involves, or is deduced from, some such condition as the following—that the *advantage* which an average person derives from the expenditure of a Unit should be constant.[1] *

From this condition, owing to the unequal consumption[2] of different individuals it follows that the precision of our calculation cannot be great. That is to say, we cannot be certain that between considerable limits some other ratio than the one which we have chosen would not be as good as the one which we have chosen.

It may be worth adding that even if we could suppose that all commodities were consumed in the same proportions by all

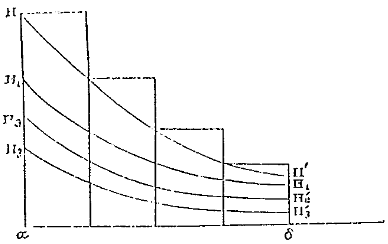


Fig. 1.

individuals, yet the mere difference in the size of fortunes and of debts would introduce an inaccuracy. To show this let us first suppose that all fortunes would be equal but for the payment of debts; and let us represent the average amounts of commodities consumed by the height of the columns in the annexed diagram, the divisions of the horizontal line being equal. Now suppose a person, from being a consumer of the average amount of each article, becomes debtor to the extent of a certain sum, expressed in *Units*[3] of tabular standard. Theoretically he would retrench something of his expenditure on each article, contracting as it were the *margin of final utility*. He might thus fall back upon

[1] *Cp.* Horton, *Silver and Gold*, chap. iv. "In the average annual consumption of provisions . . . we should have at least fixed a definite portion of utility. . . . By enlarging the sphere of consumption on which to base the average . . . we still more nearly attain a measure of the value of Money."

* *Cp.* references to Sidgwick and Bowley given above, p. 195.

[2] *Cp.* Professor Marshall, *Industrial Conference.*

[3] The term *Unit* is here employed in the sense proposed by Professor Marshall, *Contemporary Review*, March 1887.

the curve $H_1H'_1$ instead of the original boundary. And if his debt increased he might have to fall back upon an interior frontier, the next *isohedone*, as we might call this family of curves. Conversely, in the case of a creditor. Now, in order that our standard should be applicable to debts of various sizes it is virtually assumed that the ratio $HH_1 : H'H'_1$ is the same as $H_1H_2 : H'_1H'_2$, and so on for other columns and curves. But this assumption is without evidence, or rather contrary to evidence. Or, if it be held sufficient that the standard should represent the utility corresponding to the *average* debt, still even for this purpose our method of determining the proportions (by the totals consumed) is arbitrary. *A fortiori* when we admit all kinds of inequalities of fortune and other irregularities. Thus it may plausibly be contended in virtue of the analogies of Fechner's law that, where the total wealth of a people has increased, an equal quantity of utility is represented by a larger quantity of wealth.[1] In this

[1] The standard defined in this section, the Consumption Standard as it may be called, appears to be particularly appropriate to the case in which National Wealth is regarded as a constant quantity. Otherwise there is apt to arise a divergence between two attributes which we have hitherto assumed to be conjoined, namely, the condition that the Unit should be constantly equivalent to the same quantity of valuables, and that it should afford, on an average at least, the same quantity of value-in-use, the same "Final Utility." For, according to the *Law of Diminishing Utility* (expounded by Laplace, Jevons, and others), the same increment of means tends to afford a smaller increment of advantage when the fortune to which addition (or from which subtraction) is made is ampler. If, then, National Wealth increasing, the average fortune becomes larger, the Unit which is equivalent to the same quantity of things will no longer correspond to the same quantity of advantage. The average scale of living being higher, the same amount of goods will not appear of the same importance to the average consumer [see *Recent Writings on Index-numbers*, below, 3.] Accordingly, in such a case we must make a choice between the following two conditions for the definition of our Standard or Unit. The first condition is that the Unit should constantly be equivalent to the same quantity of valuables. Or since, agreeably to the views here adopted, quantity of valuables cannot be in general defined irrespective of subjective considerations, it might be more philosophical to lay down as the first condition that the Unit should constantly afford the same quantity of utility; *abstracting the change of National Wealth*, supposing that the fortune of the average consumer remained constant. The alternative condition is that the utility afforded should be constant, that circumstance *not* being abstracted. As a matter of nomenclature, it seems better to restrict the symbol C, the term *Consumption Standard*, to the former definition. The latter arrangement may be regarded as a variety of the genus *sliding-scale*, designated by c.

Dr. Julius Lehr, in the important contribution to our subject made in his *Beiträge zur Statistik der Reise* (Frankfort, 1885), seems to assume the proposition that the utility derived from wealth at both the compared epochs is the same; or at least that the final utility at each epoch is the same, or rather a quantity of the same order. For he takes as the measure of the importance of an article the number of *Genusseinheiten* afforded by its consumption. Now, Dr. Lehr's *Genusseinheit* and Jevons' *Final Utility* are quantities of the same dimension.

case Method A B c D (explained below) might be the legitimate deduction from the principle on which we here suppose method A B C D to depend.

It is important to realise how loose is the character of the calculation even under the most favourable conditions. To expend meticulous care in determining our weights when our balance is thus rough is nugatory. It is taking care of the pence and leaving the pounds to take care of themselves, a course dictated rather by proverbial than practical wisdom.

It follows from the condition above stated that the frequent resale of an article (such as cotton) forms no reason (with reference to the purpose of Section III) why it should be counted more than once. Nor should materials, as distinguished from finished products, be counted, or only as representative of finished products. Upon the same principle the price of stipendiary labour [1] (domestic wages and many professional payments) enters in as an independent item; but the price of industrial labour (ordinary wages) only as representative, and in the absence, of the finished products.

A hundredweight of diamonds, say, affords so many times more *Genusseinheiten* than a hundredweight of iron, as the Final Utility of the former is greater than the Final Utility of the latter. To determine the number of *Genusseinheiten* conferred by (the objective unit, *e. g.* hundredweight, of) each species of article, Dr. Lehr in effect takes the mean of the Final Utilities at each epoch. Now he who takes a mean assumes that the quantities of which he takes a mean are of the same order.

[1] The exclusion of "services" as distinguished from material commodities has been maintained on the ground that so-called "unproductive" labourers are paid out of the proceeds of productive industry. The money which we expend on singers and dancers finds its way to butchers and bakers. To include in the National Inventory the outlay on Singing and Dancing as well as the total expenditure on Bread and Meat is therefore to count the same portion of wealth twice over. And no doubt this remark is relevant, where the object is to measure the quantity of Wealth defined as something *material*. But for the present purpose would it not be theoretically as reasonable to omit Bread and Meat and base our standard exclusively upon the price of theatrical entertainments and such like, upon the ground that what we pay to the butcher and baker finds its way to the Music Halls which they frequent? "No," it may be replied, "for a good part of their income must be expended on material necessities." Well, but by parity a good part of the wages of "unproductive" labour may be expended on immaterial utilities. What is earned by teaching literature may be spent in tickets for the opera. Theoretically it is as arbitrary to exclude altogether immaterial utilities as it would be to include nothing but them. The difference between the two errors is only one of degree and practical importance. As a matter of fact in the existing world, of the two defective methods the less imperfect is that which includes material, and excludes immaterial, utilities. But the converse might be true in some happy island, where the material necessities of life were obtained almost for nothing, and the principal monetary transactions were constituted by the exchange of mutual services.

In constructing the formula for combining the quantities and prices thus defined, we may first distinguish the abstract and ideally simple case in which exactly the same quantity of each article is consumed at the two epochs. In this case the method of procedure is that indicated by Professor Sidgwick in his *Political Economy* (Book I. chap. ii. § 3): "Summing up the amounts of money paid for the things consumed [1] at the old and the new prices respectively," and [to find the value of the Unit at the later epoch] dividing the latter sum by the former.

A difficulty arises when we introduce the concrete circumstance that the quantities consumed at the two epochs are not the same. We might distinguish two grades of this deflection from the abstract ideal: (I) where the interval of time between two revisions being very small the variations in the amounts consumed are slight, *differentials*, we might call them; and (II) *integral* or considerable changes which occur in the course of a long interval of time.

I. The method of procedure in the first case may thus be symbolised: Let α, β, γ, etc., be the quantities of commodities consumed [1] at the initial epoch, and α', β', γ', etc., at a subsequent epoch; it is assumed that $\frac{\alpha'}{\alpha} = \frac{\beta'}{\beta} = \frac{\gamma'}{\gamma} =$ etc. $= 1$ nearly. And similarly for a second subsequent epoch $\frac{\alpha''}{\alpha} = \frac{\beta''}{\beta} = 1$ nearly. Upon these assumptions several methods of determining the Unit present themselves. Let us designate the prices at the initial epoch by p_α, p_β, p_γ, etc., and at a subsequent epoch p_α', p_β', p_γ', etc. Then,

(1) We may take the type which first presents itself upon Professor Sidgwick's view of the problem, viz.—

$$\frac{\alpha p'_\alpha + \beta p'_\beta + \text{etc.}}{\alpha p_\alpha + \beta p_\beta + \text{etc.}}.$$

This method is (in effect) adopted by Mr. Sauerbeck for years earlier than 1866–77 (*Journal of the Statistical Society*, 1866, pp. 595–613).

The method is also exemplified by Mr. Giffen's retrospective estimate of the change in the value of money between 1873 (and 1883), and *earlier* years (Report on Prices of Exports and Imports, 1885, Table V.).

[1] Agreeably to this definition the prices on which the Consumption Standard is based should theoretically be the prices paid by consumers—retail prices. For *this* purpose wholesale prices are to be employed only in the absence of the proper statistics, as an index of prices paid for the finished products—a very imperfect index, as Dr. Scharling, in his excellent paper on retail prices, and other authorities have shown.

(2) The next type, also given by Professor Sidgwick,[1] is the converse of the first, viz.—

$$\frac{\alpha' p'_\alpha + \beta' p'_\beta + \text{etc.}}{\alpha' p_\alpha + \beta' p_\beta + \text{etc.}}.$$

This method is exemplified by Mr. Giffen in his Table IV. (Reports 1881, 1885), by Mr. Mulhall, and by Mr. Sauerbeck (for years after period 1867–77), (*Journal of the Statistical Society*, 1886, p. 595).

(3) The third type is a mean between the first two, viz.—

$$\tfrac{1}{2}\frac{\alpha p'_\alpha + \beta p'_\beta + \text{etc.}}{\alpha p_\alpha + \beta p_\beta + \text{etc.}} + \tfrac{1}{2}\frac{\alpha' p'_\alpha + \beta' p'_\beta}{\alpha' p_\alpha + \beta' p_\beta + \text{etc.}}.$$

Professor Sidgwick has suggested and remarked upon this procedure in a note. It has been noticed also by Drobisch.

(4) The next type is also a mean :—

$$\frac{\tfrac{1}{2}(\alpha + \alpha') \times p'_\alpha + \tfrac{1}{2}(\beta + \beta')\, p'_\beta + \text{etc.}}{\tfrac{1}{2}(\alpha + \alpha')\, p_\alpha + \tfrac{1}{2}(\beta + \beta')\, p_\beta + \text{etc.}},$$

suggested independently by Professor Marshall and the present writer.

(5) The next type is one adopted by Mr. Palgrave :—

$$\frac{\alpha' p'_\alpha \times \dfrac{p'_\alpha}{p_\alpha} + \beta' p'_\beta \times \dfrac{p'_\beta}{p_\beta} + \text{etc.}}{\alpha' p'_\alpha + \beta' p'_\alpha + \text{etc.}}.$$

(6) The sixth type is that which Mr. Giffen has employed in his Table III. Put $_{,}\alpha$ and $_{,}p_\alpha$ for the quantity and price of the first commodity in 1875 (or other year selected as representative). Then for the increase in the value of money in the year whose symbols are α', p'_α, as compared with year α, p_α, write :—

$$\frac{{}_{,}\alpha_{,}p_\alpha \times \dfrac{p'_\alpha - p_\alpha}{p_\alpha} + {}_{,}\beta_{,}p_\alpha \times \dfrac{p'_\beta - p_\beta}{p_\beta}}{{}_{,}\alpha_{,}p_\alpha + {}_{,}\beta_{,}p_\beta + \text{etc.}}.$$

The expression for what we have called the Unit is found by adding *unity* to the above $\left(\text{substituting } \dfrac{p'_\alpha}{p_\alpha} \text{ for } \dfrac{p'_\alpha - p_\alpha}{p_\alpha}\right)$.

(7) Next we may place the formula of Drobisch, of which the principle is to compare the price at different epochs of an objective unit, such as a hundredweight, supposed to be made up of all sorts of articles in the proportion in which they enter into national

[1] See the passage above referred to.

consumption. In our notation the formula (for what is here called the unit) becomes

$$\frac{\alpha' p'_\alpha + \beta' p'_\beta + \text{etc.}}{\alpha' + \beta' + \text{etc.}} \div \frac{\alpha p_\alpha + \beta p_\beta + \text{etc.}}{\alpha + \beta + \text{etc.}}.$$

(8) Last, but not least, either in respect of bulk or of theoretic weight, occurs the formula of Dr. Julius Lehr (referred to above, p. 211), of which the principle is to compare the price at different epochs of a *pleasure-unit*, or unit of final utility. The formula may be thus conveyed in our notation :—The mean " Genusseinheit," or *final utility*, of the first commodity is $\frac{\alpha + \alpha'}{\alpha p_\alpha + \alpha' p'_\alpha}$. Of such units these came into consumption, α at the first epoch, and α' at the second. Now sum up all the *Genusseinheiten* for all the commodities which came into consumption at the initial epoch, and divide the national expenditure ($\alpha p_\alpha + \beta p_\beta +$ etc.) by the sum of *Genusseinheiten*. Thus you have the average price at the initial epoch of a *Genusseinheit;* say P_1. Similarly determine P_2 for the posterior epoch. Then $P_2 \div P_1$ is the required unit.[1]

Of these methods it may be remarked that the first four seem to have an advantage over the remaining two, in that the former make no assumption as to the extent of the change of price, while the latter proceed on the supposition that those changes are small. The fifth method seems to assume that we may write for p'_α $p_\alpha (1 + \Delta'_\alpha)$, where the second powers of Δ'_α are negligible. And similarly in the sixth method we must be allowed to write for ${}_{,}\alpha_{,}p_\alpha$ ${}_{,}\alpha p_\alpha (1 + {}_{,}\Delta_\alpha)$, where ${}_{,}\Delta_\alpha \times \Delta'_\alpha$, ${}_{,}\Delta_\beta \times \Delta'_\beta$, etc., are negligible. No doubt, when we grant the steadiness of the

[1] With regard to the formula proposed by Dr. Lehr, the present writer agrees with the criticism expressed by Professor Lexis in a recent number of Conrad's *Jahrbuch*. The received formulæ and Dr. Lehr's formula are equal as touching their theoretical validity; but the former (including our A B c D) have the advantage of practical simplicity.

Dr. Lehr's treatment of the *variables* as distinguished from the formula also calls for remark. His object being to discover how far the power of money to purchase *Genusseinheiten* has varied, it is not quite clear why he should insist on including wages, the wages of ordinary industrial or productive labour as well as of stipendiary services, among the data. Do we not take sufficient account of productive labour when we take account of the finished products? *Either*, but not *both*, those items should figure in the expression of our Unit.

One more remark seems called for in justice to the reader whom our notice of this work may have attracted. He must not be discouraged by the opening paragraphs, which are both extremely obscure and not directly relevant to our present purpose. The general reader is advised to begin at p. 10 (" Der begriff Durchschnittpreis "), or even at p. 28 (" Das Verfahren zur Ermittelung des Geldpreises," etc.).

proportions $\frac{a'}{a}$, $\frac{,a}{a}$, etc., we can hardly refuse this additional postulate.

The first four methods are all equally good if our fundamental hypothesis is strictly true. Where, as in fact, the hypothesis is only hypothetically true, the third and fourth methods, being of the nature of *means*, are apt to minimise error.

On the whole, the fourth method may appear the best; abstracting the difficulty of obtaining the proper numerical data, which is beyond the scope of this paper.

The seventh method is exposed to the objection (noticed by Dr. Lehr) that services cannot be weighed by hundredweights. Dr. Lehr's own formula is objectionable chiefly on account of its bulkiness.

It might be a good plan to take the mean of the numerical results of all the methods that are equally entitled to confidence (? the third, fourth, seventh, eighth, and—in the absence of violent price-variations—the fifth and seventh). We might thus obtain not only a better result, but also the opportunity of forming an opinion upon the *error* incident to the calculation: by how much it is likely, and by how much it is unlikely, that the result should be wide of the mark.

There are some other concrete circumstances which may entail some modifications of the general rule: (1) Unless the interval between the revisions of the units be very short indeed we must suppose that the unit is employed at times when, owing to the movements of prices (since revision), it has ceased to be exact.[1] Ideally it might be best, instead of p'_α, p'_β, etc., present prices, to take for each article the mean of its present price and its prices in the proximate future for all the period that the unit has to function unrevised. But of course we cannot know the future prices, and therefore we must be content with taking present prices (or it may be means of the present and the immediate past) as the best representatives of the ideally preferable *mean*.* Now, considering the fluctuations of each price between two periods of revision, we see by the theory of errors that the price which fluctuates least is (*ceteris paribus*) the best representative of the mean price. And accordingly, in the combination of the different

[1] This obvious circumstance is explained at some length by Held in Conrad's *Jahrbuch* for 1871.

* It is interesting to observe that even in this section where no hypothesis securing sporadic dispersion of the prices (at any the same time) has been entertained, there still intrudes the principle of *sampling* (cp. *Journal of the Statistical Society*, 1923, p. 581, par. 2, *sub finem*.

indications of change in the value of money, there is a *prima facie* presumption that peculiar weight should be assigned to those indications which are peculiarly accurate.

But the validity of this principle turns upon very nice considerations. Where we have several measurements of one and the same thing it is indisputable that more weight attaches to the less fluctuating measures. This is true not only in the case of a real objective measurable, such as the distance between two points, but also where the *quaesitum* is a subjective mean, such as *l'homme moyen*. If, as in a case mentioned by Dr. Baxter,[1] we have two sets of measurements of heights of American citizens, the one executed with the utmost precision, the other rough-and-ready, then, in order to obtain the best value for the mean height of the American man, it would be best to affect those careless measurements with inferior weight.

But it may be otherwise when we are seeking not a single mean, but the sum of two or more. If we have to determine the distance from Dover to York *via* London, and we have very good measurements for the first distance, and very bad for the second, the best that we can do, though bad may be the best, is to add together without qualification the two means. So if we have to determine the income of a nation consisting, say, of two classes, upper and lower, for one of which the returns are very accurate, for the other very loose, still the best combination of data which is available is the simple addition of the two estimates.[2]

Yet again, if we have several estimates of such a compound mean as has been supposed, the principle of *weight* may again make its appearance. Suppose that, as Laplace proposes[3] (in the case of birth-rates), it were the practice to ascertain the statistics of " a great empire " by way of *sample*. Let observations be taken on several villages or districts, consisting each of an upper, middle, and lower class. In combining these observations so as to obtain the mean income for the empire, it would be proper to assign less weight to those localities where the returns were obtained in a more summary fashion, by a less accurate method. Further, although each estimate might not be based upon all the classes in each district, but only on a miscellaneous selection from them, still if we could divide such estimates into two classes, contrasted in respect of accuracy and differentiated

[1] United States Sanitary Commission.

[2] Supposing, of course, no wrong *animus mensurandi* or *constant error* in one direction, such as that of underrating income.

[3] *Théorie Analytique*, liv. II, *cap.*

by no other attribute, the best method of combination would be a weighted mean.

To apply these principles: (1) if, like Jevons, we content ourselves with taking *samples* of commodities rather than all commodities—a perfectly legitimate procedure, and justified alike by the theory of Laplace and the practice of statisticians, *e. g.* Jevons in his enumeration of sovereigns—then undoubtedly, the principles of *inverse probability* becoming applicable to this mode of measurement, greater weight should attach to the less fluctuating species of returns. It might indeed be a nice question how much the principle of *quantity* should be cut into by the consideration of fluctuation. Thus, if we took Mr. Giffen's [1] statistics of the variation in the prices of exports and imports as a sample (or part of one) of the change in the purchasing power of money, cotton perhaps, on account of its unique importance in respect of quantity, stands out by itself, and ought to receive full weight. But if we have several articles of about the same importance in respect of quantity but differing in fluctuation, a higher combination-weight should be assigned to the less fluctuating mass of value.

(2) A similar principle should govern our procedure, if we had to base our calculation upon returns relating not to the whole population, but only to specimens thereof. Suppose, for instance, it was sought to determine the change in the value of money in China, and that statistics could only be obtained for certain representative localities. If we make a complete enumeration of commodities we ought to take account of all articles, without regarding whether they are consumed in the same proportions or in different proportions by different persons. But if we proceed by way of sample, then we ought to assign special weight to those articles which, as Engel's law and the American labour statistics have established, are consumed in nearly equal proportions by each household throughout a large class of the community. Less weight should attach to those articles, the "sundries" of the statistics referred to, which appear more fitfully in the household budgets. How far in England we have to proceed by way of samples afforded by certain markets and certain commodities is a question not to be decided in this Memorandum. The difference upon which these distinctions turn is that which the writer, in treating of the theory of errors, has drawn between simple induction and inverse probability (see "Observations and Statistics," *Camb. Phil. Trans.*, 1885).

(3) A more obvious ground of selection is that some articles

[1] *Parl. Papers*, 1881–85.

(however large their money value) interest only a comparatively few (rich) persons. Accordingly, in constructing a standard adapted to the general requirements of the community, we ought upon utilitarian principles to treat the variations in the price of that class of articles as of comparatively little account.[1]

It may be doubted whether the practical worth of these subordinate modifications corresponds to their theoretic interest. For to assign less importance to some of the data on the ground of a deficiency of weight which is not susceptible of numerical evaluation is a practice which, though countenanced by the example of physicists in their reduction of observations, is apt to diminish confidence in sociological calculations. For the sake of a little additional accuracy it may not be worth while incurring the suspicion of cookery :—

Denique sit quidvis, simplex dumtaxat et unum.

II. We come now to the case where, the interval between the compared epochs being considerable, the quantities consumed at the two epochs are materially different, and the ratio of the quantity consumed at one epoch to the quantity consumed at the other is no longer even approximately the same for the different commodities. The difficulties presented by this case, which seemed to defy science, have been triumphed over by Professor Marshall.[2] The incommensurable proportions of the dissimilar expenditures he manages to compare by means of a series of the intercalated intermediate forms presented by the changing national inventory. Equating each term of this series to its

[1] Another modification which might be suggested is that less weight should be attached to those commodities of which the price-variations affect the general public and a particular class in different senses—a fall, for instance, benefiting the consumer, but ruining the producer. It will be found, however, a difficult and endless task to carry out this principle. For what commodities would be excepted from it? Imports perhaps, in so far as it is the foreign producer chiefly who is damaged by the fall and benefited by the rise of those prices. But with regard to the home industries, in order that the interest of the producer and the consumer should vary in opposite directions, we must suppose an equilibrium of profits to be transmitted from trade to trade, according to Ricardian principles, with a rapidity that is not supposable.

But not only is the working of the proposed principle difficult, but also it is incorrect; *here*, in this section, where our object is that the unit should afford a constant quantity of valuables to the average consumer, without reference to the number of units which the different classes of consumers have to spend. To tamper with certain items of expenditure, such as wages of Domestic Service, on the ground that these transactions belong to *distribution*, as distinguished from *exchange*, is virtually to introduce the principle of the *sliding scale*, to substitute the attribute c for C.

The exclusion of "unproductive" labour has been maintained on other grounds considered in note to p. 211.

[2] *Contemporary Review*, March 1887.

predecessor and its successor, he brings the first term into relation with the last term. Though the final and initial shapes of the *Unit* cannot even approximately be superposed, yet its content of utility is preserved constant. It is true that at each step of this process some deviation may occur. At each act of weighing something may fall out of the balance. But something may fall in also. And thus, in the absence of a constant bias towards error in one direction, there is reason to believe that—except for very long deferred payments—the result will be as accurate as that which is attainable under more favourable conditions.

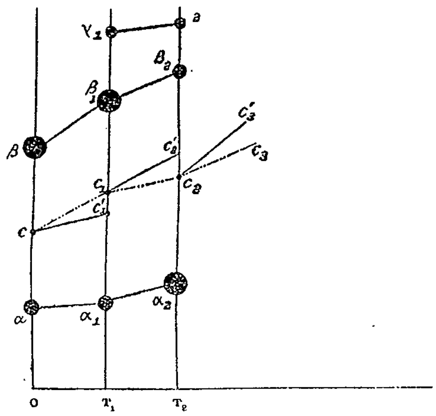


FIG. 2.

Professor Marshall's method may thus be illustrated Let us with Cournot represent ratios by logarithms, and logarithms by linear distance. But, unlike Cournot, let us take account, not only of the price, but also of the quantity of each article. Let the distance of the dot a from the abscissa represent the price of the first commodity, and the size of the dot the quantity consumed (per unit of time). Let the abscissa represent time. At the initial epoch, corresponding to the origin, the purchasing power of money, the denominator of the sought unit, is represented by Oc, where c is the centre of gravity of the system initially. Now, if, during the interval O T_1, only money and prices were affected, other things being constant, the required (numerator of the) *Unit* would be $T_1c'_1$, where c'_1 is the centre of gravity of the system in its new position. But other things are not constant. There occur variations, not only in the relative positions of the particles, but also in their masses (as shown by the varying size of the dots). Also new particles enter the system (*e. g.* γ_1 at the time T_1), and old ones drop out. Thus the true centre of gravity at the time, T_1 is not c'_1 but c_1. This point can be found at that time; but it is not available for our first edition of a tabular standard. The second edition at the time T_2 is similarly obtained by comparing $T_2c'_2$, the height of the apparent centre of gravity at the later epoch, with T_1c_1, the height of the real centre at the earlier epoch. If we join the points $cc'_1c'_2$, etc., we have the locus of apparent unit hugging the corrected curve cc_1c_2.

At every step there is incurred an error, say a "probable error," Δu, and accordingly what may be called an improbable error (about $4\Delta u$).[1] These errors being presumably independent, without bias in excess or defect, it follows, from the theory of errors, that the total error incurred in the course of n steps is $\sqrt{n}\Delta u$. It is a nice question how frequent the revisions of the standard should be, in order that this error may be minimised. Let Δt be that interval of time within which there cannot possibly or probably occur a change of sign in Δu, owing to a variation in those disturbances of the economic fabric which cause our standard to be inaccurate. Then it is expedient that the revisions shall take place as often as, but not oftener than, once in every such short interval. This condition points to the frequent revisals[2] contemplated by Professor Marshall.

It may be observed that Professor Marshall's solution is largely applicable to a problem kindred to ours, but which we have not supposed to be comprehended in the question set to us; namely, to measure differences in the value of money between different *places*. For instance, if the economic habits of the peoples of the Austrian empire varied by gentle gradations along a line trending from north-west to south, very much as the vital statistics of the empire are shown by Hain (in his important work on "Das Oesterreichische Reich") to vary gradually, then it might be possible, so to speak, to carry the equation of utility from Bohemia along to the Military Frontier. It is otherwise where natural and political barriers produce discontinuity; for instance, in the case of the United Kingdom compared with the United States.[3]

[1] The reader, according to his habits of thought, may regard u as standing either for the sought unit or the utility which it is required to keep constant.

[2] *Contemporary Review*, March 1887.

[3] It is difficult to understand the rationale of the method by which it is proposed in the Massachusetts Labour Report for 1884 to bring together for comparison the purchasing power of wages in England and the United States.

Section III.

Determination of a Standard for Deferred Payments; not based upon the items of national consumption; calculated to afford to the consumer a constant value-in-use; no hypothesis being made as to the causes of the change in prices. (A B C d.)

We come next to the case where the items which enter into our Unit are not copied from the statistics of national expenditure, but are selected on some other principle. Although the rule in this case is different, the ground of the rule will be found to be much the same, namely, the desirability that the advantage derived from the expenditure of a unit should be as far as possible constant. To those who admit the utilitarian character of the problem (as defined by the attributes A B C) it will appear evident that a formula other than the direct solution can only recommend itself as being a workable approximation thereto.

Among methods which may seem to have a claim to that character we may distinguish the three following :—

(1) There is first what may be called *polymetallism*, the Unit based upon the price of an aggregate of specified quantities of specified metals; and not only metals but other substances which possess an attribute ascribed to the precious metals, peculiar fixity of value.

(2) Next we place the index numbers of the *Economist*, the simple average of a number of prices, especially if, as Mr. Bourne has pointed out, care be taken to exclude the repetition of the same article in different forms.

(3) Another foundation may be afforded by a basis which Professor Nicholson (*aliud agens*, or at least not confining himself to the purpose specified in the present section) has lately laid down in the able and highly original paper which he has contributed to the March number of the *Journal of the Statistical Society*. The new basis may be described as (the value of) the " total mass of purchasable 'things,' " (" the aggregate of purchasable commodities in the widest sense " of the term). We shall sometimes, for the sake of brevity, describe Professor Nicholson's invention as the *capital* standard.

Of these secondary methods the first and second at least have some advantage in respect of convenience over the direct solution. It is quite possible that their disadvantage in respect of inaccuracy should not be very great. The error which we incur by taking some sample commodities instead of all the items of national expenditure might be not worth correcting in view of another error with which our calculation is unavoidably affected. This is the error incident to the misfit between the consumption of the individual and that of the community. As, however, individuals resemble each other considerably in respect of consumption, there is reason to believe that this species of defect is not so important here as in the following section, where we are concerned with income derived from production.

Section IV.

Determination of a Standard for Deferred Payments; based upon the items of national consumption; calculated to afford to the consumer a value-in-use, varying with the national affluence, after the manner of a sliding scale; no hypothesis being made as to the causes of the change in prices. (A B c D.)

We now abandon the idea of a fixed standard, and attempt to construct a *sliding scale.*[1] We have hitherto supposed that the average man in paying or receiving a Unit should give or take the same quantity of wealth. But is it just, is it expedient, that, when the national wealth is increasing, the creditor should demand, the debtor pay, a *constant* quantity, or quantity proportioned to the increase of general prosperity? Probably most persons would answer in favour of the former alternative.[2] But they might be embarrassed if the principle were extended to the case of declining prosperity. Would it seriously be proposed that, if money were depreciated by the decrease of goods other than money, the debtor should pay an ever-increasing amount of currency? This seems to be one of those questions of *la haute politique* which it is not our business to decide.

If it is judged desirable that the Unit should represent a quantity of wealth varying with the national affluence, a simple

[1] The idea of a *sliding scale* may not seem at first sight to be suggested by the question set to us. It will be found, however, to be implicit in much that is written on our subject by the ablest writers—those, for instance, who, in estimating the depreciation of money, dwell upon the fact that the style of living expected in each class of life, the *Lebensansprüche*, have become heightened; those, again, who, *without entertaining an hypothesis such as that which forms the definition of our section* a, still insist on including among the constituents of the Unit industrial, as distinguished from stipendiary wages, and material in addition to finished products, and exports and imports, without reference to the amount of home consumption; in fine, those who would exclude wages of domestic servants, rents, and generally *distribution* as distinguished from *exchange* on the grounds specified in note to p. 211.

[2] Cf. Poulett Scrope, *Political Economy*, (ed. 1833), p. 410.

method of effecting that condition is to put for the *Unit* the ratio of the national expenditure on articles of consumption at the later epoch to the corresponding expenditure at the earlier epoch. Employing the same notation as before, we have now the formula

$$\frac{\alpha' p'_\alpha + \beta' p'_\beta + \text{etc.}}{\alpha p_\alpha + \beta p_\beta + \text{etc.}}.$$

If it is judged desirable to compare not the absolute expenditure, but the amount relative to the number of the population, we ought to multiply the above written expression by the factor $\frac{N}{N'}$, N and N' representing the number of the population and the earlier and later epochs respectively.

This method appears to the writer to deserve more attention than it has received. The result would probably be much the same (in the case of short intervals at least) as for the more familiar formula. But the construction would be simpler as not requiring a mean to be taken [1] between the quantities consumed at different epochs, and the philosophic basis would be free from the difficulty which besets the equation of utility.

SECTION V.

Determination of a Standard for Deferred Payments; based upon the amount of national income or upon prices which affect the income of any class; varying with such income or prices, after the manner of a sliding scale; no hypothesis being made as to the causes of the change in prices. (A B c d E.)

Another method of accommodating debt to the resources of the debtor is to take income as our sliding scale.[2] The received estimates of national income may be employed for this purpose. In this case the Unit might be in effect an assigned proportion of the national income per head of the population.

It should be observed that this standard, revised at most once a year, would not be adapted to the more transient fluctuations

[1] See above, pp. 212, 213.

[2] The principle of the sliding scale may be contrasted with the "Consumption standard" in two distinguishable cases—(1) First, we may suppose national wealth, the average income, to increase (or decrease) *ceteris paribus*. In this case the proper items on which the sliding scale Unit should be based appear to consist of the expenditure on finished products (our A B c D). (2) Secondly, distribution may be supposed to vary. To adjust the Unit to this variation we have to take account of wages and other distributional transactions; also of materials as affecting the incomes of certain classes

of industry. Accordingly it might be worth while to consider whether we could derive a more flexible measure of income from the prices of certain articles.* Let us begin with a simple case—an importer of articles of consumption, say of the species α, who might be considered as paid by commission on the amount of his dealing. His income then varies with the price of α in the ratio $\frac{p'_\alpha}{p_\alpha}$. In the interest of this class exclusively the unit ought to be $\frac{p'}{p_\alpha}$. Or, if we suppose several such dealers, we have the weighted mean $\frac{\alpha p'_\alpha + \beta p'_\beta + \text{etc.}}{\alpha p_\alpha + \beta p_\beta + \text{etc.}}$ (assuming that the *quantities* have not materially varied between two revisions, and that the " commission " of all the dealers may be regarded as the same).

Consider next residential rent and stipendiary wages. The incomes of certain classes vary directly with these payments; yet, as these incomes are not, like the preceding, equal to a small fraction, but to the entire volume, of the transactions in question, it will not be easy to combine these data with the preceding into a properly weighted mean.

Again, when we take in ordinary wages and industrial rent, we are met by the fact that, while the income of some classes varies directly with these amounts, the interest of another class, entrepreneurs, varies *inversely*—not indeed in exact inverse ratio, but in an opposite direction to the same quantities. Again, the materials of one manufacturer are frequently the finished products of another. Accordingly the price of such articles constitutes a very bad measure of the income of all the parties concerned.

It follows from these considerations that from an examination of prices we can obtain at most a very rough and precarious indication of the variation of resources. Such a method would be related to the more exact calculation of income very much as our method A B C d was related to A B C D.

At the same time, when we consider the purpose of our sliding scale—to mitigate the evil of industrial fluctuations—it may be doubted whether this end is not realised nearly as well by a rough-and-ready method as by the most exact calculation. For a standard based upon the vicissitudes of all cannot well be adapted to the vicissitudes of each. The fit is at best so bad that it is

* It may well be doubted whether the proposed method of measuring income —which has some affinity to the method pursued in the Census of Production (190)—is better adapted than other methods to measure fluctuations of income.

not made much worse by some additional imperfections of measurement.

The character and worth of such a mean variation of price as we here desiderate might be illustrated by an imaginary example of another sort of mean, one obtained by taking the average temperature for the same day over a period of years. We have known old ladies who each year discontinued and resumed fires on the same days of the year. Suppose that they had affected even greater precision, and had burned each day a quantity of fuel based upon the mean temperature for that day averaged over a period of years. It is clear that in a climate like ours those who adopted this arrangement would some days suffer from too great heat and other days from too great cold. The arrangement would be so very defective that it would not be sensibly deteriorated by some imperfections in the method of averaging the temperatures. Suppose, for instance, that in the different years the thermometrical measurements had been effected with different degrees of completeness. For the earlier years there might be (for a given day) only sample readings of the thermometer, made two or three times a day. For the later period there might be a more continuous record of temperature. Theoretically, in combining such data more weight should be given to the more complete measurements. But practically for the purpose in view such elaboration would be nugatory.

To look at the matter more closely, let us suppose with sufficient accuracy that the income of a particular class of producers depends mainly on the prices of a certain group of articles, so that it would be convenient for that particular class that the standard for deferred payment should be regulated by the movement of those particular prices. Roughly speaking, the desideratum for that class is that the unit should be proportioned to some mean of those prices; say $\frac{mp' - \mu\pi'}{mp - \pi}$, where p and π are prices of products and agents of production respectively. But in fact the unit must be based on the prices (and quantities) of all kinds of articles. In view of the considerations touched in the text the ideally best combination of prices must be a complicated function, say of the form $\frac{F(p'_a, p'_b, \text{etc.})}{F(p_a, p_b, \text{etc.})}$. By an approximation admitted in mathematics, this expression may be written $\frac{ap'_a + bp'_b + \text{etc.}}{ap_a + bp_b + \text{etc.}}$, where the weights a, b, etc., are not (like our old friends α, β) quantities, but coefficients deduced from the quantities by the solution of a stupendous utilitarian problem. The varying relations between the quantities of things consumed or " used up " in manufacture, and the income of different classes—such as the importers and manufacturers of an article—all these complex correlations must be supposed duly expressed by the function F and the derived simpler form. By an allowable abstraction we may suppose the course of industry so uniform that the coefficients a, b, etc., remain constant during the interval under consideration. We shall now show that for the purpose in hand—to mitigate the vicissitudes in each industry—it does not much matter what values (within wide limits) we assign to the weights a, b, etc. As announced in the Synopsis, almost any combination of the more important articles of trade is likely to be equally imperfect and equally serviceable.

Put for p'_α, p'_β, etc., the following: $p_\alpha(1 + E_\alpha)$, $p_\beta(1 + E_\beta)$, etc. And let the displacements E_α, E_β, etc., be made up of two portions, one affecting all articles equally, the other proper to each. Call the former ε, and let $E_\alpha = \varepsilon + e_\alpha$, $E_\beta = \varepsilon + e_\beta$, and so on. The unit which would be most desirable in the interest of a single class becomes of the form $1 + \varepsilon + e_\alpha$ (putting a single article as the representative of a small group). Meanwhile the general standard is of the form $1 + \varepsilon + \dfrac{ap_\alpha e_\alpha + bp_\beta e_\beta}{ap_\alpha + bp_\beta} + \text{etc.}$ The first part of both expressions coincides. But it is only by accident that the remainders can be of a piece. For *by the theory of errors* the displacement (E_α) incident to a single article is likely to be of an order much greater than almost any mean of the proper displacements independently incident to n articles. As this proposition turns upon a matter of fact, the *independence* of the proper displacements of several articles, it may be well to illustrate it by some actual statistics. In the following example (p. 227) afforded by the immense drop of prices during the crisis of 1857, ε, the common displacement, is considerable.

In this table the first column contains the percentage decrease for each article. The next two columns contain the differences between the average decrease (27) and the individual decreases. The modulus, or measure of fluctuation, is found to be about 16. Hence, by a well-known theorem, the probable error of the sum of n differences, n being large, tends to be $\sqrt{n} \times 16 \times \cdot 477$ (a theorem which does not assume that the differences are grouped according to a known curve). Suppose, for instance, $n = 9$. The probable error of the sum of n differences taken at random

PERCENTAGE DECREASE OF PRICES OF SEVERAL ARTICLES WITHIN A FORTNIGHT, NOVEMBER 1857.

(Based upon "Commercial Daily List," cited by Patterson, *Economy of Capital*, p. 191).

		Differences —	Differences +	Squares	
Tallow	17	10	—	100	
Sugar	36	—	9	81	
Cotton	14	13	—	169	
Scotch pig	16	11	—	121	
Saltpetre	31	—	4	16	
Rice	33	—	6	36	
Silk	33	—	6	36	
Linseed	17	10	—	100	
Linseed oil	20	7	—	49	
Tin	10	17	—	289	
Tea	25	2	—	4	
Pimento	40	—	13	169	
Turmeric	50	—	23	529	
Shellac	33	—	6	36	
Jute	40	—	13	169	
Hemp	16	11	—	121	
Sums	431	79	80	2,025	
Means	27	10	10	127	= Mean square of error
				254	= Modulus squared

should be about 23. This may be illustrated by actually taking some batches of nine, say the first nine, tallow to linseed oil, the last nine, linseed to hemp, and a central nine. The sum of the first set of differences is $- 51 + 25 = - 26$. The sum of the second set of differences is $- 47 + 55 = 8$. The sum of a third set, from Scotch pig to pimento, is $- 58 + 29 = - 29$; while if we put out the Scotch pig and take in turmeric we obtain $+ 5$. These observed results are very consonant with the theory that the probable error is 23. Hence the probable error of the *mean* of nine differences is $2\frac{5}{9}$. Meanwhile the probable error of *any single* difference may be found by observing that the "quartiles," in Mr. Galton's phrase, occur on the one side between $- 10$ and $- 11$, and on the other side between $+ 6$ and $+ 9$, giving a probable error of, say, 9. Or we may proceed more hypothetically, and, assuming that the grouping (of the differences) is conformable to the [1] normal type, find the probable error ($\cdot 477 \times$ modulus) about 8. Thus the displacement of the single article is seen to exceed the mean displacement of several articles in about the degree required by theory.

We have taken the simple (arithmetical) mean. But much the same would be true if we had taken *any* weighted mean * of all prices, in particular the ideally best, whose weights are ap_a,

[1] The probability-curve [normal law of error].

* Within limits; as shown in the Second Memorandum.

bp_β, etc. (provided at least those coefficients are not extremely unequal). The deviation of the particular standard from the general standard is apt to be so considerable that it does not much matter by what system of weights we determine the general standard. The unit best in the indivdual interest is as we have seen above (p. 226), $1 + \epsilon + \epsilon_\alpha$. The unit in the general interest is of the form $1 + \epsilon + \frac{A\epsilon_\alpha + B\epsilon_\beta + \text{etc.}}{A + B + \text{etc.}}$ (putting $A = ap_\alpha$, and similarly B). The deviation of the former from the latter is of the form $\epsilon_\alpha - \frac{Ae_\alpha + Be_\beta + \text{etc.}}{A + B + \text{etc.}}$. Now, if e_α, e_β, etc., be on an average of the order e, then by the theory of errors their weighted mean, the latter part of the expression just written, will be of the order $e \frac{\sqrt{A^2 + B^2 + \text{etc.}}}{(A + B + C + \text{etc.})}$, an expression which tends to zero as the number of the coefficients is increased. The unavoidable discrepancy between the particular and general interest is therefore not likely to be much diminished by a more exact calculation of weights when those weights are numerous.—Q.E.D.

Take, for example, the statistics above cited, where there are only sixteen items, and let us suppose the weights so disparate as the cardinal numbers 1, 2, . . . 16. If we based our unit on the simple arithmetic mean, we have $\epsilon = \cdot 27$, and for the *Unit* 1·27. Now this *Unit*, as applied to each particular interest, is apt to be out by about ·1, or 10 per cent. In the tallow interest, for instance, 1·17 would have been the best unit; if we legislated exclusively in the sugar interest the unit would be 1·36. Let us see now how these misfits would have been mended by a more elaborate adjustment of the standard. The expression $\frac{\sqrt{A^2 + B^2 + \text{etc.}}}{A + B + \text{etc.}}$ becomes when $A = 1$, $B = 2$, etc., about ·3. The correction then upon the arithmetic mean ·27 would be of the order $\cdot 3 \times \cdot 1$ (e being of the order ·1),[1] that is, ·03, or 3 per cent. This theorem may be verified by actually assigning the weights 1, 2, 3, etc., to the percentages above cited. The weighted mean $\frac{1 \times 17 + 2 \times 36 + 3 \times 14 + \text{etc.} + 16 \times 16}{1 + 2 + 3 + \text{etc.} + 16} = 28{\cdot}8$. If we reverse the order of importance, and, beginning at the bottom of the list, assign a weight 1 to hemp, 2 to jute, 3 to shellac, etc., we obtain for the weighted mean 25·6. The difference in each

[1] Assuming that each of the articles (tallow, sugar, etc.) is subject to the same law of fluctuation, we may conclude (from an examination of the table) that the average error for any article is 10 per cent.

case between the simple and weighted mean is even less than theory predicts. Suppose the corrected unit becomes 1·25, the tallow interest will now be out by *eight* per cent. instead of *ten* per cent. from the standard best for them exclusively—no very great gain, and partly (by hypothesis of course, not wholly) [1] balanced by the loss of the sugar interest, who are now more out than before. *A fortiori* when the number of articles is greater than *sixteen*.

The general conclusion is that for the purpose in hand it is not much matter what sort of mean we take; provided that the weights assigned to the different articles are not very unequal, and provided that there is no reason to think that the ideally best system of weights would be very unequal. The test that factors A, B, etc., are not sensibly unequal is the condition that $\sqrt{A^2 + B^2 + \text{etc.}} \div (A + B + \text{etc.})$ should be small; which is true enough within very wide limits (*e. g.* in the case of sixteen weights being respectively 1, 2, 3, etc., 16). When there are a few relatively very large interests, such as possibly in England cotton, iron, and ordinary wages, then in constructing our general sliding-scale we should pay special attention to those interests; though from the considerations mentioned above (p. 204) we are not entitled to assume that the weight to be attached to (the price-variation for) each interest is *directly proportioned* to the magnitude of the transactions.

It will be observed that this reasoning turns upon the unique interest of particular groups of persons in the prices of particular articles, on the circumstance of *division of labour*.[2] The conclusion as to the worth of our result is therefore not equally applicable to what may be called the *consumption* (A B C) as distinguished from the *production* (A B c) standard. For the rest the latter calculation resembles the former in being amenable to similar secondary modifications (see above, p. 217). For instance, upon the third of the principles referred to a variation of wages ought to affect the *Unit* more than an equal variation of profits as concerning a greater number of persons.

[1] If we suppose the *weights* 1, 2, . . . 16 to constitute the ideally best system, that which affords the maximum sum total of advantage to all.

[2] Compare the remarks of Von Jacob cited by Mr. Horton in his admirable chapter on the *Standard of Desiderata ; Silver and Gold*, p. 39

SECTION VI.

Determination of a Standard for Deferred Payments; based upon the amount of national capital; varying with such amount, after the manner of a sliding scale; no hypothesis being made as to the causes of the change in prices. (A B c d e.)

The next category is distinguished by the condition that the basis of the required sliding scale is capital rather than income. This Unit might be specially adapted to certain debts; for instance, in estimating the capital (but not the interest) of sums raised upon mortgage of fixed capital. It is interesting to inquire what sort of weight should be assigned to wages for the purpose here defined. May we measure the importance of wages as a means for paying off capital by the lump sum which the wage-earner is able to raise upon the prospect of his earnings by way of insurance?

With reference to this most important application of Professor Nicholson's method, it may be proper here to introduce a remark which is applicable also to other uses of that method. When its originator is met with the difficulty that articles do not increase uniformly, he argues that "the change in the purchasing power of the standard is found by dividing the value of the new inventory at the old prices by its value at the new." And he is understood to regard this method as preferable to the converse method, dividing the value of the *old* inventory at the old prices by its value at the new. His reasoning turns upon the postulate, "Let the total value of the new inventory (consisting of different quantities of the old items) reckoned at the old prices be v_1 and the total value of the old inventory, also at old prices, be w_1; then $\frac{v_1}{w_1}$ is the measure of the increase in the quantity of wealth." In this passage read for "old prices" *new prices*, for v_1 read w_2, and for w_1 a new symbol v_2, and you will have a postulate no less true, or no more arbitrary. According to the substituted principle, "the measure of the increase in the quantity of wealth" is $\frac{w_2}{v_2}$; which being multiplied by $\frac{w_1}{w_2}$, by parity of reasoning with that employed by the author on the page referred to, gives for the "measure of the new purchasing power compared with the old" $\frac{w_1}{w_2} \times \frac{w_2}{v_2} = \frac{w_1}{v_2}$; which being interpreted means dividing the *old* inventory at the old prices by the value of the same inventory at the new prices.

Observing that the " change in the purchasing power of the standard " is the reciprocal of what we have elsewhere called the Unit, we see that the two methods just reached correspond to the formulæ (2) and (1) of our section A B C D (above, p. 212). It is important to point out that neither of these solutions is afore nor after the other.[1] Otherwise there might be an objection to the use of a symmetrical mean between the two, such as has been recommended.

SECTION VII

Definition of the Appreciation [or Depreciation] which it is the object of Bimetallism and similar projects to correct; no hypothesis being made as to the causes of the change in prices.

The variation in the value of money which we have been hitherto considering is that which is corrigible by the adoption of a " Unit " for deferred payments. For different purposes different formulæ are appropriate. The purpose next in importance to the construction of a Unit (if not indeed, as some think, prior in importance and the main scope of the task set to us) is to correct the instability of trade, to restore the level of prices by augmenting the quantity of legal-tender currency, whether by Bimetallism or the increase[2] of paper-money.

Now, if we might assume all prices diminished uniformly, like the shadows of objects as the sun advances from the east, the problem would be very simple. It is an intelligible proposition that the *status quo* might be restored by an elevation of the objects all round. And the significance of the proposition need not be impaired if we suppose the objects waving and oscillating, and some of them depressed, others elevated in random fashion between the two epochs at which the shadow-lengths are observed. But we are not entitled *here* to make an assumption, which is the characteristic of the following section. We must rather seek a rule available even in the case in which one large category of objects may be considerably and uniformly elevated, another depressed;

[1] The question whether it is easier to get present quantities at old prices than old quantities at new prices does not come within the scope of this memorandum.

[2] *E. g.* By introducing £1 notes in England, or according to some more daring plan, such as those proposed by Professor Marshall (*Contemporary Review* March 1887, note near end), Faucher (*Jahrbuch für Gesetzgebung*, 1868), and others.

where the variations do not present any true mean or normal type. Our formula should be irrespective of such an hypothesis here equally as in the previous sections.*

The operation of augmenting the currency proper to the present section, as contrasted with the method of making contracts in Units, presents the following four distinguishing characteristics :—

(1) The infusion of money is not adapted to correct the more transient fluctuations of prices due to the oscillations of credit; whereas our Producers' Unit—including commodities other than finished products—is specially adapted to the correction of transient fluctuations.**

(2) The operation of the proposed remedy requires time. The detection of the evil—the secular as distinguished from the tidal variation of price-level—also requires time. It follows that the epochs which are to be compared in respect of purchasing power are separated by a considerable interval. Hence the calculation of a Unit to express change in the purchasing power of money must be of the less exact sort, which might be distinguished as *integral*.

(3) Again, the area which is affected by the augmentation of currency is very extensive, at least when (as in the case of Bimetallism) the added circulation consists of precious metal. Accordingly, the appreciation which is to be corrected by that remedy must relate to a very wide area, the whole system of states in monetary communication; that is, the greater part of the civilised and uncivilised world. Now, the larger and more diversified the public to which there is applied any regulation based upon the mean requirements of the average man, the less perfectly is that type or norm likely to be adapted to the requirements of the individual. The correction of appreciation, which may be effected by the infusion of metallic money, is therefore likely to be of less benefit than that which attends the method of contracting in Units.

(4) Moreover, in the latter case the measure of the evil and of the remedy is the same. The same calculation which gives the appreciation assigns the Unit in terms of which debts are to be paid. But it is not so where the remedy is the augmentation of legal-tender money. The extent of the evil (the appreciation) having been found, the extent of the remedy is still to seek.

* Here, and in the context, there are omitted sentences identifying the desired ormula with conceptions already defined.

** But see note added to Section V above, p. 224.

For it is a very naïve [1] conception that, in order to increase prices all round in a certain ratio, it is necessary and sufficient to increase the quantity of legal-tender money in that ratio.

These imperfections of the method under consideration may be thus summed up: (1) It cannot even aim at certain objects which are within the range of the alternative method. (2) The objects which it does aim at are not sighted so clearly; its shots are apt to be very wide of the mark. (3) The advantage of hitting the mark, the prize to be won, the quarry to be brought down, is not so considerable as in the case of the alternative method. (4) Lastly, in the one case we shoot point-blank; having discovered the position of the object, we have the direction in which we ought to aim. But in the other case the trajectory has yet to be calculated, in virtue of which, being given the position of the object, we can deduce the direction of our aim.*

Section VIII.

Determination of an Index irrespective of the quantities of commodities; upon the hypothesis that there is a numerous group of articles whose prices vary after the manner of a perfect market, with changes affecting the supply of money. (a F.)

So far we have made no supposition as to the cause of the phenomenon which is under measurement. As far as we have been concerned there might have been a number of heterogeneous causes, or, what is even more unfavourable to calculation, a few great causes; as if one large class of prices were heightened according to the law of diminishing returns, while other prices, also forming a large class, were lowered by increased division of labour, and others by improved means of transport. We are now to entertain an hypothesis, namely, that there is an effect capable of being discovered and worth discovering, due to [2] "causes which operate upon all goods whatever," or at least upon a considerable group of goods; for instance, the increased

[1] See below, Section IX.

* There is here omitted as not particularly appropriate here an elaborate metaphor designed to illustrate the significance of an hypothesis implying the presence of a true mean or good average. Suffice it to repeat "if one great group of commodities varies pretty uniformly in one direction, and another in a different direction (or even in the same direction, but in a markedly different degree), then the task of restoring the level of prices can no longer be regarded as a purely objective *quæsitum*, a currency problem.

[2] Mill, *Political Economy*, Book. III. chap. viii. § 2.

quantity, or efficiency, of legal-tender money, or the improvement of money-saving expedients.[1]

The simplest hypothesis of this sort is the proposition in the text-books that prices vary inversely with the quantity of money, other things being equal. But we are not restricted to the "Quantity Theory." [2] It is sufficient for our purpose that there should be a circle of commodities, including money, such that the equilibrium of exchange between them should continually be readjusted by a comparatively frictionless play of market-forces. That this condition does hold approximately with respect to a large group of articles is shown in the case of Austria by Dr. Kraemer in his important work on Austrian Paper-money. From the statistics given in his Chapter III. there can be no doubt that a change in the "valuta" of currency does enter into, and might be extricated from, the prices of a certain set of commodities. The following articles may be instanced as particularly sensitive :—*Wool, spirits, rape-seed, undressed leather*, and, in general, articles of foreign trade. These observations are supported by the copious statistics adduced by Herska, Bela Földes, and others. The only question is whether we ought not to regard all commodities, rather than only some commodities, as varying with the *agio*. No doubt it is a delicate question, and only to be decided by the proper mathematical methods of statistics, whether it is possible to extricate a mean variation in the value of money from the changes of particular prices. It seems to be so in the case of Austria. In the case of the United States, if we could accept the law laid down by Mr. Delmar as to the propagation of a change in price, we could not hope for a sufficiently large group to afford a real average. But the statistics adduced by Hock, in his history of the finance of the United States, show conclusively that in correspondence with the condition of the inconvertible currency and the state of credit there did extend pretty uniform waves of disturbance over a part, if not indeed the whole, of American industry.

The proposition which has been proved for inconvertible currency is shown to be true for metallic money—as regards, at least, a certain zone of industry—by the index numbers of the *Economist*, the statistics adduced by Soetbeer, Laspeyres, and others.

[1] A good enumeration of causes that are apt to cause a general variation of prices in the case of Inconvertible currency is made by Bela Földes in the *Jahrbuecher für Natl. Oekonomie*, 1882.

[2] The discussions in Section IX will show how far the writer is from regarding this theory as generally applicable.

Assuming, then, that there is, or may be, over a certain region of the industrial world a mean disturbance of the sort described, it would be a significant operation to take the average of all the relative prices, *irrespective of the quantities of the corresponding commodities*. We should thus obtain a mean elevation or depression which may be described as a figure such that, if we took any ware at random, that figure [1] would be more likely than any other to be equal to the relative price of the selected ware. A similar typical mean of human heights (irrespective of other attributes) has proved a useful implement of statistical induction in the hands of Mr. Galton, Dr. Charles Roberts, and others.

A more exact illustration is afforded by the following physical analogies. Suppose it were required to measure the force of gravitation in the neighbourhood of a mountain. Our data might consist of a set of pendulums, all disturbed from the vertical by the attraction of the mountain, and each further subject to proper disturbances. The displacement from the vertical constituting the required measurement might be found by taking a mean of the displacements suffered by all the pendulums. Now, *from what we know of the action of gravity*, there is no reason to think that the displacement of a larger mass gives in general a better measure of the common disturbing agency, the gravitation force, than a smaller mass does. Hence, in taking the mean of the displacements, there is no propriety in assigning more importance to the displacement of the more massive pendulum. If we do assign preferential importance, it should be on other grounds, namely, that the proper disturbances of some pendulums are apt to be less serious than those of others. The *combination weights* (or "multiplier weights," in Sir G. Airy's phrase) determined by such considerations must be carefully distinguished from the "weight" in the ordinary sense. The pendulum weightiest in the former sense might be lightest in the latter sense. Another caution is to distinguish the present investigation from that whose object is the displacement of the *centre of gravity* of the system,[2] a *quæsitum* which does not presuppose any common disturbing agency.

Again the problem special to this section has been likened to the problem of discovering the proper motion of the solar system by means of the apparent movements of the stars. Let us

[1] In short, the greatest ordinate of the curve of price-variations.

[2] Analogous to the calculation of Units in our earlier unhypothetical sections.

suppose, for the sake of illustration, that the *line* in which the solar system moves has been ascertained. The only questions are in which direction of that line, positive or negative, say towards or from a certain star in Hercules, and at what rate, we are moving; how far we have moved between two given epochs. Now, if we take several groups of stars at random, say (as in fact is done) some groups in the northern hemisphere, and others in the southern, and for each of these groups we take the mean of the apparent motion of the stars along the given line; then, if the mean resultant is much the same [1] for every group, we may be reasonably certain that the phenomenon is due to a common cause, which is doubtless no other than the proper motion of the solar system. Suppose, however, that the motions of the stars did not conform to what may be called a true mean. Suppose that what Mr. Proctor calls "star-drift" was prevalent on a much greater scale than he has found to be the case; that the Milky Way, together with other zones, moved off *en bloc* in one direction, while the Great Bear carried off another half of the heavenly host in the opposite direction. In this case we should no longer be able to detect the motion proper to the solar system. The peculiar grip which a plurality of independent events affords to the calculus of probabilities now becomes wanting.

It is to be observed that, in assigning importance to the different indications given by the apparent motions, the criterion is not the *mass* of the star, but its "weight" [2] in the sense of affording a better measure of the *quæsitum*, the motion of the solar system.

Similarly, in the problem before us it must be either given by previous experience (as in the case of our first illustration), or discoverable from the data themselves (as in our second illustration), that there is a *true mean;* that one set of commodities, such as the products of extractive labour, has not risen *en bloc*, while another set, as manufactures, has fallen. Without that condition we cannot follow Jevons in reasoning by the principles of probabilities that gold has been depreciated (or appreciated) to a certain extent. With that condition we may follow Jevons in taking a mean of price-variations, *irrespective of the quantities of the commodities.*

The problem before us may be thus defined. Given a number

[1] If it be asked what extent of difference between the means of different groups is to be expected and may be regarded as insignificant, the answer is supplied by the mathematical Theory of Errors. See the writer's paper on *Methods of Statistics.*

[2] Depending on considerations not here relevant.

of observations consisting each of the ratio between the new price and the old price of an article, to find the mean of these observations—the objective or quasi-objective mean—as distinguished from those combinations in the preceding sections which were prescribed by considerations of utility. The problem as thus conceived belongs to that higher branch of the calculus of probabilities which may be called the doctrine of errors. Upon the theory of errors are based two kinds of problem; of which the first is exemplified by the method of determining the true position of a star from a number of separately erroneous observations, the second, by the method of constructing the typical stature of a people, *l'homme moyen*, from the measurement of a great number of individuals. To which of these analogies—the more, or the less, "objective" species of mean—our case most corresponds is a nice inquiry, varying with the shades of hypothesis.[1] Upon either view the practical rules for extricating the mean are much the same. They may be arranged under two headings, relating (1) to the form in which the given observations are to be combined; and (2) the relative importance to be assigned to the different observations.

(1) As to the first point the general rule is that in the absence of special presumptions to the contrary an arithmetical mean (or linear function) of the given measurements is the proper combination.[2] That is to say, if the different measurements are r_1, r_2, etc., each purporting to represent one and the same object, in our case the appreciation or depreciation of money, the proper combination of these data is—

$$\frac{w_1 r_1 + w_2 r_2 + \text{etc.}}{w_1 + w_2 + \text{etc.}};$$

where the factors w_1, w_2, etc., are *weights*, such that if w_1 is greater than w_2 then r_1 contributes more to the result than r_2.

This general presumption in favour of the arithmetic mean may, however, be rebutted by specific evidence in favour of some other mean, and it is here submitted that in the case of prices there does exist such specific evidence in favour of the *geometric mean*.

It appears that prices group themselves about a mean, not

[1] Consider the illustrations given below at p. 247.

[2] The ground of this presumption is partly that the arithmetic mean is one of the *simplest* methods of combination; partly that it is specially adapted to a species of observation which is very extensive *in rerum naturâ*, which may be said to be always tending to be realised, the exponential law of error, or probability-curve.

according to a symmetrical curve like that which corresponds to [1] the arithmetic mean, but according to an unsymmetrical curve like [2] that which corresponds to the geometric mean. Before adducing the empirical proof of this proposition it may be well to consider what *a priori* grounds we might have for preferring the geometric mean. There are [3] those who consider that the mere accumulation of agreeing experiences can seldom suffice, without some antecedent probability, to establish an inductive conclusion.

It has been shown by Mr. Galton and others that the geometric mean is adapted to a particular species [4] of observations, which may be described as *estimates*. For instance, the estimates which different persons (or the same person at different times) might make of a certain weight would be likely to err more in excess than in defect of the true objective weight, and in such wise as to render the geometric mean of such a series of estimates the proper method of reduction. This law of prizing may well extend to prices. The fluctuating estimates which from time to time a person might make of the [5] utility of an object, as measured by the quantity of some other object, *e. g.* money, might well fluctuate according to the law which has affinities to the geometric mean. So far then as changes in price might depend upon fluctuations in demand,[6] there is something to be said in favour of our proposition.

Again, there exists a simple reason why prices are apt to deviate much more in excess than in defect: [7] namely, that a price may rise to any amount, but cannot sink below zero.[8]

Lastly, the supposed tacit combination which everywhere

[1] The probability-curve [normal law of error].

[2] The curve described by Dr. Macalister in his paper on *The Law of the Geometric Mean* in the *Philosophical Transactions*, 1879.

[3] G. C. Lewis as quoted by Dr. Bain in his *Logic*.

[4] Wherever the law of Fechner applies. See papers by Mr. Galton and Dr. Macalister, *Proc. Royal Soc.* 1879.

[5] *I. e.* the "final utility."

[6] Variations in what is technically called the demand-curve.

[7] As in the annexed diagram.

[8] That price should be, in Dr. Venn's phrase, a "one-ended phenomenon" may raise a presumption in favour of an asymmetrical grouping, but by no means dispenses with empirical verification. For the same presumption exists not only in the case of many anthropometrical and other statistics which prove to be symmetrical, but also in cases where there is an asymmetry in the sense contrary to the theory, an extension of the *lower* limits of the representative curve. Such are the statistics of barometrical height arranged by Dr. Venn in *Nature*, Sept. 1, 1887; the statistics of eyesight given by Dr. Charles Roberts in the *Medical Times*, Feb. 1885; the grouping of Italian recruits by Signor Perozzo in *Annali di Statistica*, 1878.

exists between dealers may prevent prices falling as low as from time to time they otherwise would according to the play of supply and demand.

There is therefore at any rate no *a priori* presumption against the proposition that price-returns are apt to group themselves in an unsymmetrical curve of which the range in excess is greater than in defect. In favour of this proposition the following empirical evidence is adduced :—

In the first table are examined the prices of twelve commodities during the two periods 1782–1820, 1820–1865. The maximum and minimum entry for each series having been noted, it is found that the number of entries above the "middle point," half-way between the maximum and minimum, is in every instance less, and in some instances very much less, than half the total number of entries in the series. In the twenty-four trials there is only one exception to the rule, and in very few

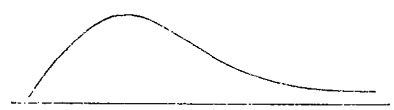


Fig. 3.

cases even an approach to an exception. We may presume then that the curves are of the lopsided character indicated by the accompanying diagram. For the "median" [or point having as many entries above as below it], which upon the supposition of symmetry ought to be about coincident with the "middle point" as above defined, or at any rate as often above as below it—this median is in every instance but one (fodder, 1798–1820) below the middle point.

Fig. 3 very well represents the prices of corn during the periods 1261–1400, and 1401–1540 given in Professor Rogers' *History of Agriculture*. The abscissa in the figure represents prices, and the ordinate the number of years in which the corresponding price was enjoyed. It will be found that in both cases the maximum elevation, the greatest ordinate of the curve, occurs between five and six (shillings). Below that maximum-point, in both cases the curve does not sink more than two or three shillings (2*s.* 10¾*d.* is the lowest entry), while above that point one curve stretches out to 14, the other to 16. There can be no doubt about the fulfilment of an unsymmetrical law. Further verification of the law may thus be obtained from the earlier series of statistics. Compare the decennial averages (of corn prices) given by Professor Rogers with the annual returns on which they are based. The "middle point," half-way between the maximum and minimum of each decade, is in almost every case above the average. There are only three exceptions out of the fourteen decades, viz., 1271–1281, 1281–1291, 1371–1381; and one of these exceptions is not an instance to the contrary, the middle point exactly coinciding with the average.

If the prices are similarly examined by decades for linen (Vol. I. p. 593), clouts, and other commodities, it will be found that the rule holds, with no exceptions, or trifling ones. Thus for clouts there is not a single exception during twelve decades, 1271–1390. The only exception which Professor Rogers' statistics show is the decade 1391–1400.

Similar results are presented by the table of price fluctuations in the *Massachusetts Labour Report*, 1885, p. 459. Out of seventy-eight commodities nine only have the minimum further below the average than the maximum is above it. And those exceptions are slight in respect of extent, while the exemplifications are often marked.

Examination of Variation of Prices, 1782–1865.

(See Jevons, *Currency and Finance*, Table VIII. p. 144.)

		Minimum.	Maximum.	Middle point between Max. and Min.	No. of returns above middle.	Median.
Oriental products	1782–1820	65	107	86	15	84
	1821–1865	30	80	55	11	45
Tropical food	1782–1820	60	102	81	8	65
	1821–1865	34	65	49½	14	48
Metals	1782–1820	89	169	129	12	113
	1821–1865	71	123	97	13	87
Iron	1782–1820	67	139	103	16	99
	1821–1865	35	114	74½	10	55
Timber	1782–1820	54	533	293½	4	116
	1821–1865	62	137	99½	9	90
Oils	1782–1820	81	166	123½	14	105
	1821–1865	75	121	98	10	90
Dye materials	1782–1820	64	157	110½	10	98
	1821–1865	30	98	63	7	36
Fibres, cotton, wool, etc.	1782–1820	88	214	151	4	130
	1821–1860	61	121	86½	17	78
	1821–1865	61	129	95	12	88
Cotton	1782–1820	57	204	130½	2	87
	1821–1860	21	63	42	9	33
	1821–1865	21	128	74½	4	36
Corn	1782–1820	99	252	175½	9	134
	1821–1865	92	176	134	18	128
Wheat	1782–1820	81	231	156	12	131
	1821–1865	78	151	114½	18	113
Fodder	1798*–1820	118	308	213	12 (out of 23)	214
	1821–1865	156	250	203	20	199

* Return previous to 1798 wanting.

The next statistics present not time fluctuations, but place fluctuations. In the *Illinois Statistics of Labour Report*, vol. iii. p. 340, are given the prices of thirty-eight articles in 34 different [1] towns. Examining the series of prices for each article, we find that there is fulfilled in almost every case the law that the maximum is further from the average than the minimum is. Most of the exceptions are very slight, and disappear if we take in the penultimate the observations *penemaximum* and *peneminimum*. The only real exceptions are mackerel, fresh fish, cheese, butter, and crackers, five articles out of thirty-eight. The odds against such a phenomenon occurring by accident are hundreds of thousands to one.

Lastly, let us take price returns for the same time and locality, but for different articles.

This table is extracted from Jevons' table of Proportional Variation of Prices, *Currency and Finance*, p. 144. The "median" is the point which has as many observations above as below it. Where, as in the majority of the rows above, the number of entries is even twelve, namely 12, the point half-way between the sixth and seventh has been taken as the median. The sixth and seventh being in almost every case close together, there is very little of arbitrariness in this procedure. The fact that the maximum is in every case further from the median than the minimum shows the lopsided character of the price-curves. The median has been used instead of the arithmetic mean only for convenience of calculation. Much the same conclusions would evidently have followed from the use of the arithmetic mean,

	Oriental Products.	Tropical Food.	Metals.	Iron.	Timber.	Oils.	Dyes.	Fibres.	Cotton.	Corn.	Wheat.	Fodder.	Max.	Min.	Med.	Distance from Med. to Max. compared with Distance from Min.
1783	101	87	100	97	108	94	92	112	102	127	110	—	127	87	102	+
1791	89	72	100	92	85	82	77	96	64	112	99	—	112	64	85	+
1801	80	73	139	139	167	134	108	142	114	232	222	244	244	73	139	+
1811	74	60	148	106	381	136	107	149	66	167	178	308	381	60	148·5	+
1821	68	63	101	82	116	89	74	112	53	116	114	182	182	53	113	+
1831	49	48	80	63	104	90	49	87	33	150	135	199	199	33	88·5	+
1841	51	63	90	61	113	97	40	88	37	140	131	227	227	40	89	+
1851	36	41	73	36	68	90	31	77	27	98	78	163	163	27	77·5	+
1861	36	38	88	37	69	111	32	96	39	136	113	213	213	32	78·5	+

[1] For some few of the towns more than one price is quoted.

as the writer has verified for the years 1801, 1821, 1831, 1851. The figures in each row overlined and underlined respectively are the penemaximum and peneminimum. If we compare the distances between each of these and the median the series of signs is found to become $0 + + + - + + + +$. The exceptional year is 1821. If we examine the arithmetic mean for that year the exception still exists, but in a less marked degree.

Such a curve is well represented by the equation

$$y = \frac{h}{x\sqrt{\pi}} e^{-h^2(\log x)^2},$$

where h is a constant corresponding to the dispersion, or *écart*, of the curve (see Dr. Macalister's paper "On the Law of the Geometric Mean" *Proc. Roy. Soc.*, 1879), and compare the present writer's "Observations and Statistics" (*Cam. Phil. Trans.*, p. 149). Hence, given a number of observations deviating from the mean about which they are grouped, each according to a law of the general form above stated, the most probable value of the mean deduced from these observations will be the *weighted geometric mean* give by the equation

$$\log x = \frac{h_1 \log x_1 + h_2 \log x_2 + \text{etc. } h_n \log x_n}{h_1 + h_2 + \text{etc.} + h},$$

where x is the sought mean, x_1, x_2, etc., are the given observations, and h_1, h_2, etc., are the weights, of which more hereafter.

It must be remembered, however, that there may be other means adapted to represent the bias which has been observed, in particular what may be called the unsymmetrical probability-curve, elsewhere described by the present writer (*Lond. Phil. Mag.*, April 1886). Nor, again, is it to be supposed that *all* statistics of prices are grouped unsymmetrically. Where the entries are average prices based on a great number of items it is agreeable both to [1] theory and the writer's observations that the normal symmetrical "probability"-curve will set in. It will be found difficult, for instance, to trace evidence of lopsidedness in the five-year averages given by Soetbeer.[2]

The evidence adduced appears to afford a reasonable presumption that the required method of combination is some form other than the arithmetic mean, of the general character of the geometric mean. Those who have followed Jevons' investigations will be

[1] See in *Methods of Statistics* the statement of the proposition that the average of a *large* number of returns obeying individually *any* law of grouping tends to conform to the probability-curve.

[2] *Materialen*, pp. 99-114.

familiar with the proposal that the logarithm of the required mean or general percentage should be equated to the arithmetic mean of the logarithms of the percentages special to each article. To which it is now to be added that this arithmetic mean need not be *simple*, but may be *weighted* in the sense above indicated (p. 242); *e. g.*—

$$\log x = \frac{w_1 \log x_1 + w_2 \log x_2 + \text{etc.}}{w_1 + w_2 + \text{etc.}}$$

What then are these weights to be? is our *second* inquiry.

(2) The theory of errors supplies the following rules—of which the first two have been already implied in our statement of the problem—(a) In the first place no weight should be attached to a class of observations known to be affected with what is called a *constant error*, or uniform bias in one direction. It is supposed of course that only the fact, but not the amount, of the error is known; otherwise it would be possible to get rid of it. In our case this rule dictates to reject all prices which are not amenable to that play of a perfect market whose change of level we have to investigate. The writer is far from pretending that this region of permeability can at present be marked off with precision. However, a rough delimitation may be effected by researches like Dr. Kraemer's.

Assuming then that we have selected a set of percentages which may be regarded as accidental deviations from a common mean, on what principle should more importance be attached to one indication of change rather than another? The second (β) maxim which we have to apply is that the observations should be independent. This condition excludes the prices of the same commodity at different stages of production, since these prices are closely interdependent. Or, if we must take account that at each stage some fresh cause of fluctuation—source of " error " —is introduced, at any rate each price-return is not to count for one, but only for a fraction.

Here arises the question whether a commodity extensively consumed like meat or cotton ought not to count for more, in so far as its price is a mean of a greater number of transactions, than Cloves and Peppper. The answer is that those transactions are not *independent*. The law that there can be only one price in a market *prima facie* removes the presumption in favour of the more largely consumed commodity. There is no analogy between the average price of such a commodity and a mean founded upon a specially large number of independent observations in theory at least, and for the purpose of a first approximation; for it will appear in the next section that this abstract proposition is qualified by the inevitable imperfections of our statistical data.

(γ) A third principle is that less weight should be attached to observations belonging to a class which are subject to a wider deviation from the mean. Such, in our case, would be the prices of articles which, exclusive of the common price-movement of all the selected articles, are liable to peculiarly large *proper* fluctuations. Cotton and Iron, for example, fluctuate in this sense much more than Pepper and Cloves.

The weighting of a geometric mean is a delicate matter, but not beyond the resources of science. A general rule is given by Dr. Macalister in the important paper already frequently referred to. Suppose we have a considerable series of observations belonging to a certain class, we can extract a constant which may be described as the measure of fluctuation for that series or class of observations. The constant thus given constitutes the *weight* with which we ought to affect the logarithm of an observation when we combine it, according to the arithmetic mean, with others (of a different degree of precision) in order to obtain the best possible measure. The data for determining this constant are afforded by series of prices for successive years, such as those in Mr. Giffen's *Report to the Board of Trade on Prices of Exports and Imports*, 1881–85.

If in the present state of statistics and public opinion it appears too difficult and delicate a matter to weight the data on the principle of fluctuation, the practical result of this section may be thus summed up. After the manner of Dr. Kraemer, select a number of (independently fluctuating) articles which are found to be particularly sensitive to changes in the value of money. After the manner of Jevons, find the percentage indicating the price-variation in each article, and put the geometric mean of those percentages as the required unit, or standard, or measure of depreciation. Or rather, if we must treat as equal weights certain to be unequal, it is better (for reasons which will be more fully stated in the next section) to employ a formula which is specially adapted to such jumbling of different weights: to wit, *the Median*. Examples of this species of Mean have been given above.

So far on the hypothesis that the widening circle of price-disturbance has not yet spread beyond a limited area; a case which is almost too restricted and particular to be the subject of our consideration.[1] If we suppose that the circle has completely

[1] Compare the last paragraph of the *Introductory Synopsis*.

spread, that all the compartments of the economic fabric are equally penetrated by the influence of some change in the supply of money, we have then a limiting case of the problem just discussed.

The objection to this supposition is that, for an all-pervading percolation, considerable time must, in general, be required. And then it happens—what is not necessarily true of more transient oscillations, such as those of an inconvertible currency —that the changes in prices are apt to be referable to one or two leading categories : *e. g.* of articles which follow the law of decreasing or increasing returns, after the manner exhibited by Laspeyres in his classical paper [1] on the prices of Hamburg wares.

If we examine some of the statistics adduced by Laspeyres, according to the appropriate mathematical methods, we shall not discover a very serious hiatus between the different categories of wares. The *modulus* for the fluctuation of the price-variations about their average may be (roughly) estimated to be about 40 for any of the eleven categories discussed by Laspeyres in the masterly paper entitled " Welche Waaren." . . . Hence we can calculate the probability that the differences between the various categories are really significant, and not merely accidental. It will be found, if, with Laspeyres, we dispose the data in three main divisions—*Urproductionen*, *Colonialwaaren*, *Manufacte*, etc. —that the cleavages *within* those divisions are not important. The separation between the divisions is marked, yet not very serious, not more serious than is found to exist within the most perfect groups which are known to exist; for instance, the proportion of male to female births. The mean (percentage) for the first division (*Urproductionen*), containing 129 wares, is 128; for the second division, containing 85 wares, 118; for the third, containing 98 wares, 108. The modulus of comparison between the first and second mean is (see the writer's " Methods of Statistics ") about $40\sqrt{\frac{1}{129}+\frac{1}{85}}=$ about 5·5; while the observed difference is 10, nearly twice the corresponding modulus. Which constitutes a real, yet not enormous, difference; not greater than the differences in stature which exist between the sub-classes of a nation constituting a perfect type. Similar statements are true of the comparison between the second and third means.

If in the light of these conceptions we actually plot the 312

[1] *Jahrb. f. Nat. Oekon.* vol. iii. See also *Zeitschrift f. Staatswissenschaft*, 1872.

price-variations, it will be difficult to resist the impression that we have here a *typical mean* as perfect as any presented in concrete statistics, with the exception of the circumstance not relevant to the point now examined, that the curve representing the 312 wares, however continuous, and far from being saddle-backed, is not symmetrical about its greatest ordinate; the law of price statistics above announced making itself markedly felt.

The evidence that the general average rise for the whole group of 312 articles, namely, from 100 to 118, is no mere accidental appearance, but indicative of a real agency, is mathematically estimated by odds of trillions to one.

So nearly complete a fulfilment of our hypothesis is doubtless not presented by certain other statistics, *e.g.* some of those adduced by Dr. Forsell in his interesting brochure. But it may be safely said that no statistical argument would stand tests so severe as he applies. Consider the evidence in favour of the motion of the solar system, as marshalled in the masterly papers of Sir J. Airy and Messrs. Dunkin and Plummer in the *Memoirs of the Astronomical Society*. It will be found that, if you omit here, and stick in there, some star of peculiarly large apparent motion, the general conclusion as to the sun's movement will be most materially altered. *E pur si muove.*

We see in the case of one example presented by one country that the hypothesis is fairly well realised by the price-variations of the majority of wholesale commodities. But it is a long step from one set of statistics to others, from wholesale commodities to the whole field of industry, and from a single country to the entire system of countries in monetary communication. Over a large area (as Leslie, Knies, and others have pointed out) there is apt to arise a marked diversity between the price-variations of different localities; a diversity which may well be inconsistent with the hypothesis of a unique and general mean type. There is no doubt that these considerations materially restrict the fulfilment of the conditions which are prefixed to this and the following section. It is possible, however, that an hypothesis, though known to be inexact, may correspond with the facts sufficiently well for the purpose in hand.

SECTION IX.

Determination of an index utilizing quantities of commodities: upon the hypothesis that a common cause has produced a general variation of prices. (a f.) [1]

We have seen that, upon the supposition of a change in the supply of money, Jevons' method of combining the variations of prices without regard to the corresponding volumes of transactions is by no means so absurd as has been thought by some. The case is, as if we wanted to discover the change in the length of shadows, due to the advance of day. If the objects casting shadows were unsteady—waving trees, for instance—a single measurement might be insufficient. We might have to take the mean of several shadows. Now for our purpose the *breadth* of the upright object casting the shadow would be unimportant. The "wide-spreading beech" and the mast-like pine would serve equally well as a rude chronometer.

Suppose, however, that the top of the broader tree was not level but serrated, each apex oscillating more or less independently. If by the shadow of a tree was understood the mean length of the shadows cast by all its apices, in that case the broad tree should count for more than a bare pole. How much more would depend upon the connection between the projecting branches. The more independent the oscillations of each apex, the better the measure afforded by their mean shadow.

This image seems appropriate to our problem. Each price which enters into our formula is to be regarded as the mean of several prices, which vary with the differences of time, of place, and of quality; by the mere friction of the market, and, in the case of "declared values," through errors of estimation, it is reasonable to suppose that this heterogeneity is greater the larger the volume of transactions. On this account, therefore, and irrespective of those considerations of utility which were proper to our earlier sections, greater weight should attach to the prices of those commodities whose quantities are larger. It does not follow that the weights should be proportionate to the masses. The proper coefficients could be ascertained by scientifically examining the detailed statistics of each market. But it is agreeable to the Theory of Errors [2] and to the successful practice of

[1] In the preparation of this section the writer has derived much assistance from repeated conversations with Professor Foxwell.

[2] An improvement in weighting can only diminish, very often only slightly diminish, the error inevitably incident to the result of any measurement.

physicists to employ a discretionary good sense in assigning "weights" when a precise determination is difficult or impossible. In our case a good system of weights appears to be afforded by the quantities of commodities sold (once, and exclusive of resales) per unit of time. The weight so assigned would doubtless often be too large. It might sometimes be too small in the case of commodities much resold. On the whole it would be a good and safe system. This principle of ponderation is to be combined with those which have been given in the last section.[1] If we suppose the variation of prices not confined to a particular zone, but propagated over the whole sphere of industry, then we shall obtain a set of weights almost coincident with those prescribed (upon a different ground) by the standard based on National Consumption (Section III.). For the condition that the observations should be independent[2] leads us to exclude, or at least take little account of, the same commodity at different stages of production.[3]

But though in the present operation the weights would be much the same as before, the balance, the method of combination, is different. In view of the evidence adduced in the last section that price-variations are apt to be grouped asymmetrically, the "arithmetic" species of mean becomes precarious when our *quæsitum* is a quasi-objective type. The additional complexities which have been introduced in this section make against the geometric mean which was above recommended a certain hypothesis. There exists another species of mean more adapted to the rough character of our calculation, the Median; that is, in the simpler cases, that quantity which has as many of the given observations above it as below it, but a certain analogue of this operation, when the observations have different weights. *The*

[1] See the headings, α, β, γ, pp. 243–4.

[2] See β, *loc. cit.*

[3] It would be a question whether industrial wages and industrial rent should be included, in addition to, and otherwise than as representative of, the corresponding products. At any rate their weights ought not to be proportionate to their volumes; partly on account of their close connection with commodities, partly on account of the magnitude of these volumes. In the case of transactions so extensive, and perhaps we may add some other large interests such as cotton and iron, it would be best to determine the proper coefficients by specially examining the detailed statistics of each market in the light of the Theory of Errors. A summary method would be to assign to these enormous masses an averagely large weight about as large as any other weight employed in our operation. The ideally best weight is not likely to be very different from the arbitrarily assigned one, and slight differences of weight do not appreciably affect the result; as may be seen by comparing the results corresponding to two different systems of weights.

required formula is the Weighted Median, the operation designated by Laplace [1] as the "Method of Situation."

The reasons in favour of the Median may thus be summed up. If, in spite of the evidence above adduced, the normal probability-curve should after all turn out to be the most appropriate representative of the group under treatment, the Median is a reduction well adapted to this case, affected as it is with a probable error only slightly larger than the arithmetic mean (Laplace, *loc. cit.* See "Problems in Probabilities," *Phil. Mag.*, Oct. 1886). But if the grouping is of the geometrical (Galton-Macalister) species, the Median is still a very good reduction, coinciding as it does with the greatest ordinate of the curve denoted. Moreover, it has been shown by the writer ("On the Choice of Means," *Phil. Mag.*, Sept. 1887) that there is a peculiar propriety in the use of the Median when the observations are "discordant," when their facility-curve may be regarded as a compound made up of different families, or different members of the same family, of symmetrical curves. It is now to be added that this prerogative of the Median is retained when some or all the discordant elements are of the geometrical species. Now the phenomenon of "discordance" is remarkably evidenced by the different degrees of dispersion which series of (*e. g.* yearly) price-returns present in the case of different commodities. Cotton, for instance, appears to have a much larger modulus of fluctuation than Pepper. Add that this method of reducing observations is the least laborious of all, and there will remain no doubt that in the present state of our knowledge, and for the purpose in hand, the Median is the proper formula.

The method of the Weighted or Corrected Median may best be described by an example. The first column of figures given below are price-variations, expressed as percentages, for nineteen commodities, obtained by comparison of the year 1870 with the period 1865–9. The figures are taken from table 26 of the Appendix to the Memorandum contributed by Mr. Palgrave to the Third Report on the *Depression of Trade*. The percentages given by him are here rearranged in the order of magnitude. Opposite each percentage in the third column is given the proportional quantity of commodity, or "relative importance," taken from Mr. Palgrave's table 27 (year 1870). The fourth column contains the (approximate) square roots of these quanti-

[1] *Théorie Analytique*, Supplement 2. See the present writer's paper on "Observations relating to Several Quantities," *Phil. Mag.*, 1887.

ties.[1] Now for the *simple* Median the rule is to find that one of the entries in column 2 which has as many observations above as below it: that is the *ninth* in the order of magnitude; which proves to be 94. For the *weighted* or corrected Median we still seek the entry in column 2, which has as many observations above it as below it; but we proceed as if the observation 71 had been made, not once, but 19·5 times; the observation 72 made 12·8 times, and so on. There being in all nearly 177 such constructive observations, the Median is the 89th, that is 94. Or in other

Commodities.	Price-variations.	Quantities.	Square roots of quantities.
Cotton	71	381	19·5
Wool	72	164	12·8
Tobacco	75	17	4·1
Wheat	80	418	20·5
Copper	82	30	5·5
Coffee	89	8	2·8
Tea	90	66	8·1
Flax	91	82	9
Oils	94	38	6·2
Lead	95	21	4·6
Leather	97	55	7·4
Iron	97	128	11·7
Silk	98	49	7
Tallow	101	44	6·7
Meat	102	382	19·5
Timber	104	150	12·2
Indigo	107	9	3
Sugar	120	143	12
Tin	120	15	3·9
		2,200	176·5

words we have to find in the fourth column that figure which is such that the sum of all above [or below] it *with* the figure itself should be greater than half the sum of the entire column, but *without* that figure should be less than half the entire sum. The figure thus defined proves to be 6·2. For the sum of the entries above that figure is 82·3, and the half sum of the column is 88·25.

[1] The quantities of commodities taken as weights correspond to the *squares* of Laplace, p_1, p_2, p_3, etc. (*loc. cit.*). If we determine the Median by way of the third, instead of the fourth, column, we in effect assign for our system of weights the squares of the masses.* This operation, indicated by the bars in the third column, gives 91 as the Median. It is interesting to observe how small is the difference produced by the change of system—small in relation to the error incident to any Mean; which, as rudely estimated from the dispersion of the entries in the first column, is as likely as not to be as much as 2 or 3, and may not improbably be 4 or even 6. The difference between the systems is apt to be less, when the number of independent entries is greater. In the example cited from Mr. Giffen's statistics (where the number of entries is 58) the two systems of weights give *identical* results.

* In deference to physical analogies the term "mass" has sometimes been used in this connection where "volume of value" would have been more exact.

Now 82·3 is less than 88·25, while 82·3 + 6·2 is greater than 88·25. The entry in the second column which corresponds to the figure thus determined, viz. 94 (corresponding to 6·2), is the required *Weighted Median.*[1] The weighted Arithmetic Mean as calculated by Mr. Palgrave is 90.[1] By a similar operation performed on the export statistics for the year 1880, given by Mr. Giffen in his report of the year 1881, it is found that the Weighted Median (for the decline of price compared with 1861) is —7·8. Mr. Giffen's result, the corresponding Weighted Arithmetic Mean, is —5·83.[1]

The operation is much simplified by noticing that it is sufficient to arrange the percentages in the order of magnitude *in the neighbourhood of the Median.* For instance, if we are certain beforehand that the mean is below 100, we may dispose the entries above that figure in any order, just as they occur in the table from which they are taken.

We have shown how to construct a type of price-variations analogous to the *typical mean* of statures or other attributes defined as that height, or it may be weight, which appertains to a greater number of a certain population than any other height or weight does.[2] But here it may be asked, Why rest satisfied with a type if there exists a more substantial *quæsitum?* Why seek the mean variation of shadows instead of the objective movement of the bodies, that declination of the sun or revolution of the earth of which the varying shadows are the expression? * Why not penetrate beneath the superficies of shifting prices to the real relations between the quantity of money and commodities?[3]

The matter is simple as long as we keep to the abstract theory of the text-books. Imagine a purely metallic currency, the amount of which is, say, Q, and let the rapidity of circulation or duty of money be called C; then we may simply express the quantity of metallic money in terms of prices and volumes of transaction in our notation

$$Q = \frac{1}{C}[\alpha p_\alpha + \beta p_\beta + \text{etc.}].^4$$

[1] As to the import of these discrepancies see the preceding note.

[2] The Mean as defined in Dr. Charles Roberts' writings, not quite identical with Quetelet's *homme moyen* in case of asymmetrical curves like that on p. 239.

* This topic comes under the heading of the section inasmuch as the proposed calculation involves the quantities of commodities.

[3] What we have so far found is a mere ratio, comparable in point of objectivity to the ratio between male and female births (about 1,040 : 1,000 in England). But might the analogue be the proportion of black and white balls in large groups of balls which have been drawn at random from a huge urn? Beneath the tyical mean presented by those groups there is a more objective fact; the relative numbers of black and white balls, the masses of ebon and ivory.

[4] By α, α', etc., for the purpose in hand we should understand not so much the amount of things sold as the amount of sales (per unit of time).

Now let prices vary with the quantity of money, other things being constant, and we have for the variation in the quantity of money the simple expression

$$\frac{a p_\alpha + \beta p_\beta + \text{etc.}}{a' p'_\alpha + \beta' p'_\beta + \text{etc.}} = \frac{Q}{Q'},$$

where $\frac{a}{a'} = \frac{\beta}{\beta'} = 1$, etc., nearly, or upon an average.

Let us now introduce the several concrete circumstances, *first* that a proportion, say the ratio K, of transactions is effected by credit; *secondly*, that the volume of transactions varies between the epochs under comparison, say is multiplied upon an average by the factor P; *thirdly*, that the proportion of credit transactions, and *fourthly*, the duty of money, the coefficients C and K, do not remain constant.

When we introduce the first attribute alone, no difficulty is felt. The factor K disappears and leaves our formula in its initial simplicity. Again, when we introduce by itself the attribute of increased volumes, no great complication arises. We have only to multiply the simple formula by P in order to obtain the diminution of metallic money relative to the volume of transactions, *per unit volume*, as one may say.

This proposition may appear at first sight still to hold good when we combine the two attributes hitherto considered separately. But this presumption is negatived by the fact that legal-tender money is largely used in modern industry, by way of *reserve*, to meet the residues of claims not mutually compensated. It is shown by the present writer in his paper on *The Mathematical Theory of Banking* [1] that, theoretically and abstractedly, reserves tend to vary as the *square root* of the volume of transactions which they support. The reserve of material money and the mass of credit transactions are to each other, as Mr. Giffen says, as the little weight and the big weight at the ends of the unequal arms of a lever. But it is a lever of a very peculiar mechanism, such that, when you increase the big weight, you lengthen the long arm. It will be understood, of course, that this doctrine is quite abstract and ideal; related to banking business very much as the "quantity theory" to hard-cash transactions—"the most elementary proposition," as Mill says of the latter theory, and without which "we should have no key to any of the others."

The proper factor, therefore, is no longer P. The mildest expression for the correction now required is of the form $(1 - K)P + KJ\sqrt{P}$, where J is a new and probably unascertain-

[1] *Report of the British Association*, 1886.

able constant. That is theoretically [1] the sort of ratio by which, when the volume of trade increases, the mass of metallic money should be increased, in order to drive the trade at an unaltered level of price.

Now introduce the attribute that the ratio of credit to hard cash varies with time, and the varying ratio of the mass of metal to the volume of transactions, as we have good reason to believe.[2] Superadd the circumstance, which we have no reason to deny, that the rapidity of circulation also varies, and it is evident that the investigation which we have attempted is blocked by insurmountable statistical difficulties.

We might get a little further no doubt if we assume an additional datum, R, the ratio of gold in reserve to gold in actual circulation; then, with the help of P and K and R, as it were rail off from the industrial world a zone of hard-cash transactions to which the abstract formula of the text-books is applicable. This method has been pursued by Professor Neumann Spallart and Dr. F. Kral in the elaborate monograph *Geldwert und Preisbewegung*.[3] It certainly seems possible by this method [4] to explain the fact, if not to measure the magnitude, of a rise or fall of general prices; to predict the direction of the change, whether positive or negative, required in the amount of currency, in order that the level of prices may be restored.

It would be foreign to the spirit of this Memorandum to dwell upon ordinary statistical difficulties. But there is one scruple inherent in the nature of the metretic art which even with the progress of statistical technique does not seem likely to be removed.

[1] K and C being still supposed constant.

[2] *Cf.* Giffen, *Stock Exchange Securities*, "To give it [the abstract theory] validity, it must be assumed that a scarcity of money produces no expedients for economising money, and that an abundance of money does not lead to want of economy, which can hardly ever be the actual condtion of life."

[3] *Staatswissenschaftl. Studien*, n. v. Dr. Ludwig, Elster, Jena, 1887.

[4] The modest hope of explaining accomplished facts is not encouraged by Dr. Kral's success. For *a priori* he finds that the store of gold in Germany during the last few years has been fully adequate to the work which it has had to do—account being taken of rapidity of circulation and the amount of credit transactions. There have been no symptoms of a "Geldmangel," that is to say, no reason to expect a rise of the purchasing power of money, a general fall in prices. Yet *a posteriori* it seems to be admitted that there has been such a fall of prices. That this fall has originated "auf Seiten der Waren," that it is due to the development of industry rather than the introduction of the "Goldwährung" into Germany, may be a fact. But that fact does not seem to annul the right we have to expect a correspondence between the two lines of investigation; namely (1) the comparison between the supply of money and the amount required in order that the level of prices may be steady; and (2) the observed level of prices.

The method under consideration requires the determination of a certain residue, viz. total volume of transactions *minus* the portion effected by credit, $V - C$ in Dr. Kral's notation. Now each of these quantities, and *a fortiori* their difference, is subject to an error of measurement. And statistics must be much more perfect than there is any prospect of their being in the immediate future in order that the error incident to each of these measurables should not exceed a hundredth part of the same. But the hundredth part of the total transactions is a quantity of about the same order as that which it is sought to determine, namely, the amount of transactions in hard cash. The latter quantity, therefore, will be apt to be lost in a fringe of error. And, though the methods of determining V and C are likely to improve, yet the ratio of $V - C$ to V or C is certain to diminish, so that the precariousness of the calculation may well remain constant.

Upon the whole it seems that in the present state of science we must abandon the sort of realism which seeks an additional entity behind the phenomena of varying prices.* We must resign the fond idea of finding in the mean variation of price any quantity more objective than itself, any measure of its cause verifiable by an independent statistical investigation. We must be content with measuring the shadows; the objects behind them are beyond our reach. The cause of the observed phenomenon may be vaguely indicated as the changed relation between shining orb and opaque bodies; but there is wanting the mathematical science which should express the varying length of shadow as a definite function of the position of the sun.

The only question is whether we should not adopt a less, not a more, objective *quæsitum* than the type above described; whether, even where we can use the semi-objective type peculiar to this and the preceding section, it would not be better to use the more subjective formulæ investigated in the earlier sections. The present writer, following Laplace, has mainatined [1] that, even in the case of physical observations relating to a real thing, the proper method of combination is not so much that which is "most probably" correct, most frequently in the long run the true measure, but that which may "most advantageously" be employed.** *A fortiori*, when our *quæsitum* is at best a type, the

* The stone thus rejected has been made the corner-stone of a splendid edifice to Irving Fisher (see his *Purchasing Power of Money*, chaps. xi., xii.).

[1] *Metretike*, part ii.

** Cp. *Journal of the Royal Statistical Society*, 1908, Section I. Laplace applied the conception of "advantage" only to the determination of the best weights and species of average. It is here proposed to employ the principle of utility in proper mean may well be not the ratio which is presented by the greatest number of (independently oscillating) prices,[1] but that ratio which in reference to human uses it is best to adopt in any general regulation.[2] However a peculiar importance may be attached to the character of objectivity, when the result of the investigation is to form the basis of action for Governments or International Conventions. It is fortunate that the difference between the two species of Means is likely to be inconsiderable numerically.

SECTION X.

*Mixed Modes; compounding the ends or means of several distinct methods.**

We have now examined all the branches represented on our tree. But we have by no means exhausted all the possible ramifications; for, according to the logic of compartments or combinations, six bifurcations—the number of our principles of division—lead to sixty-four distinct branches. It is further to be observed that two or more branches may unite to form a compound arm. Two or more separate objects may be simultaneously pursued. For instance, a Unit might be required which could combine the attributes *C* and *c*, which should be adapted as far as possible to the convenience of the economic individual, both in his capacity of spender and earner. There might be sought the best possible compromise between the conditions that the creditor should receive a constant quantity of

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order to determine that point in the—supposed stable but not in general symmetrical—frequency curve pertaining to the observations which should constitute the *quæsitum*, that point to which the process of averaging indefinitely prolonged would converge (*loc. cit.*, Section V).

[1] In the case of our metaphorical shadows suppose that the scope and end of the measurement was to ascertain whether and by how much shade for the use of man and his cattle was increasing or decreasing with the change of hour. The determination of a mean variation in the length of shadows would be useful only as a step towards that end. It would be better to aim directly at the end, and combine arithmetically the length of the shadows multiplied by the corresponding breadth; this system of weights being now determined, not on the principle proper to this section, but on the ground that the broader trees are the more umbrageous.

[2] Read Professor Foxwell's very able lecture on *Irregularity of Employment and Fluctuations of Prices*, and consider *what* it is, what sort of mean or function of prices, which he requires to be kept constant: whether it is what we have called the *Producers' Unit* (A B c), or some more objective mean of all price-variations weighted by the corresponding volumes of transactions.

* The logical symbols prefixed to this section in the original have been omitted as suggesting a composition of formulæ where there is only intended a composition of purposes.

value-in-use and that the debtor should pay an amount of money varying with his resources. This middle course might be designated by the symbol $A\ B\ (C + c)$. Or, if we start with the conception of a sliding scale, and base it partly on finished products, partly on other items (as materials or wages), we have the Mixed Mode $A\ B\ c\ (D + d)$.

Again, there seem to be combined in popular thought two elements which we have sought to distinguish in analysis, namely, the conception of an objective mean variation of general prices, and the change in the power of money to purchase advantages. It is as if having to measure the intensity of a drought we were to observe the decline of rainfall in every district over the whole country, and to take the mean of those observations; while at the same time keeping an eye to the fact that peculiar interest and importance attach to the decline of rainfall in certain regions, namely, those which constitute the catchment basins of the rivers which supply the population with water. The most comprehensive combination is that represented by our last symbol, purporting to be a compromise between all the modes and purposes [1] *—*the* method, if practical exigencies impose the condition that we must employ one method, not many methods.

Doubtless, practical wisdom lies in a mean, and compromise is of the essence of common sense.' Some of the most useful plans and institutions are those recommended by a jumble of heterogeneous and incommensurable considerations, like the celebrated resolution [2] declaring the throne vacant after the flight of James II., of which Macaulay says that " its object was attained by the use of language which in a philosophical treatise would justly be regarded as inexact and confused. . . . The one beauty of the resolution is its inconsistency. There was a phrase for every subdivision of the majority."

There seems no more to be said, if what is required of us is a political measure rather than a scientific measurement. But, if otherwise, there is desiderated a *principle* by which to effect a synthesis between the purposes separated by our analysis. Perhaps

[1] Including many purposes which have not been thought worthy of a separate place here, for instance, to find the increase of National Wealth, given the total value at two epochs.

* *Cp.* Second Memorandum, Section III.

[2] " It was moved that King James the Second, having endeavoured to subvert the constitution of the kingdom by breaking the original contract between king and people, and, by the advice of Jesuits and other wicked persons, having violated the fundamental laws, and having withdrawn himself out of the kingdom, had abdicated the government, and that the throne had thereby become vacant." —*Macaulay*, chap. x.

it would be wisest frankly to acknowledge the arbitrary character of the proposed operation--

> "quæ res
> Nec modum habet neque consilium, ratione modoque
> Tractari non vult."

If a more definite answer is insisted upon, one might propose for imitation the Scotch practice of "striking the Fiars" [1] by means of a jury. A committee of experts agreed as to the general scope of the inquiry might be brought together, or put in communication.[2] Each member should independently form a numerical estimate based upon the data submitted to all. The *mean* of all these estimates constitutes the best possible value. It is thus that juries having to assess damages frequently proceed. The principle is illustrated by the following experiment. Ten gentlemen agreed each to guess the age of all the others and to state his own. The statistics so obtained evidence that a better estimate is afforded by the mean of several judgments than by the individual opinion. (For details see *Mind*, Jan. 1888).

No doubt it is a delicate problem in the higher *Metretics*, what degree of divergence in principle between authorities would be fatal to the collation of their judgments. Jurymen who differed materially as to the law or facts of a case could not with reason or advantage take a mean between their individual assessments. Similarly our monetary jury must be supposed to be agreed as to the general scope of the inquiry. Minor differences of opinion might be waived. The discrepancy between the various received formulæ for the Consumption Standard [3] would not be fatal, or rather would be favourable, to the combination of all the estimates into a mean result likely to be less fallible than any one of the measurements thus averaged. The methods of Messrs. Sauerbeck, Mulhall, Sidgwick, Marshall, Palgrave, Giffen, Lehr, and perhaps it may be added, Drobisch, and the one which is specially recommended in this Memorandum,[4] may be advantageously mixed. But, on the other hand, those who hold with the present writer that, in the construction of a standard for general purposes, a unique importance should attach to the items of National Expenditure—the average budget—the numerous adherents of this *Consumption-Standard*, might not consent to

[1] See W. K. Hunter's description of this practice.

[2] M. Dubos, in his *Etalon*, is perhaps the only writer who has frankly asserted that the value of gold is a metaphysical matter to be decided by cultivated intelligence.

[3] Above, p. 213.

[4] Above, p. 215.

merge an estimate so formed with the results of those who adopt a fundamentally different principle; for instance, Dr. Geyer's method, or another mentioned by him, which may thus be described. Take the price of each ware, just as it has been quoted. Add together these figures. The ratio between this aggregate at one epoch and the aggregate at another is put for the measure of the variation in the purchasing power of money.

The doctrine of the Mean, or principle of collated authority, admits of a certain analogical extension beyond mere arithmetical results to the determination of a function or form of combination. Accordingly that solution of our last problem, which is offered in the Report herewith printed, derives a certain confirmation, and the only sort of proof of which it is capable, from the general assent which it has received from the Committee of experts who have been appointed to consider this subject. A short analysis of that Report may fittingly conclude this Memorandum.*

The first part of the Report points out the necessity of distinguishing in theory several ends and methods [such as those which have been analysed in the preceding sections], the expediency of in practice giving precedence to some one mode [such as it is the main object of this section to discover.]

Part II., A, of the Report sets forth this mode, "the principal standard." It is a compromise between the principles of the Consumption-Standard, A B C D, and the more objective Mean, a f; an unequal compromise, inclined in favour of the first principle.[1] Agreeably to the first principle, yet without prejudice to the second,[2] the "weights" of the price-variations are the quantities of commodities. The form of combination, the "arithmetical" mean (or linear function), is prescribed by the first principle. In deference to the second principle, if not entirely on account of statistical exigencies, the prices used are wholesale prices, and the items of domestic service and residential rent have been excluded.

Part II., B, of the Report propounds six "subsidiary" index-numbers. Of these, three, *Wages*, *Workmen's Budgets*, and

* The Committee consisted of Mr. S. Bourne, Professor F. Y. Edgeworth (*Secretary*), Professor H. S. Foxwell, Mr. Robert Giffen, Professor Alfred Marshall, Mr. J. B. Martin, Professor J. S. Nicholson, Mr R. H. Inglis Palgrave, and Professor H. Sidgwick. The report, drawn up by the Secretary, and accepted with some amendments by the Committee, was published in the Report of the British Association for 1887.

[1] In giving these reasons the writer speaks only for himself.

[2] See Section IX. p. 247.

Exports and Imports, may be regarded as corresponding to those "partial interests," which were noticed at the end of the *Introductory Analysis* as of especial importance. Of the remaining three, the index-number based on *Wholesale Goods in General* may be perhaps put for the Producer's Standard, here designated A B c d E.[1] There remain the Consumption-Standard, A B C D,[1] and the Capital-Standard, A B c d e; the former pure and simple, the latter shorn of the item of labour, to which it may have some claim.[2] *

In concluding this paper, the writer desires to acknowledge gratefully that he is indebted for many important suggestions and corrections to his colleagues, the fellow-members of this Committee, especially Professor Foxwell.

THIRD MEMORANDUM.

ANALYSIS OF CONTENTS.

SECTION I.

Professor Newcomb's Method.

One additional definition of the *quæsitum* which has come under the writer's notice since the completion of that Memorandum is that which has been propounded by the eminent mathematician Professor Simon Newcomb, of the Johns Hopkins University. He proposes to measure the variation in the value of the Monetary Standard by the change in the volume of value which is produced by the labour of an average individual in a unit of time.[3] He writes: "One possible hypothesis would be this. We might assume that the absolute value of everything produced by the population of the country remains unchanged, except that as a

[1] For convenience of reference the symbol B has been retained here; but the meaning would be more exactly expressed by omitting it, or substituting (B + b). We are not here concerned to distinguish whether the index-number is to be used as a *Standard for deferred payments*, or with some other view.

[2] See above, p. 230.

* There are omitted here, as rather hypercritical, some further remarks on the capital standard.

[3] *Principles of Political Economy*, Book III., ch. ii. § 10.

population increases the total value produced increases in the same ratio. In other words, we may suppose the average productiveness of each individual to remain the same from year to year."

Now this hypothesis may appear doubtful in the light of the statistics furnished by Mr. Edward Atkinson and others. There is reason to think that in an improving country the productivity of labour increases. But an intelligible rationale can still be assigned to Professor Newcomb's scheme considered as a standard for deferred payments. It may be regarded as just that the debtor should pay, the creditor receive, a constant proportion of the goods produced by an average man's labour. If the productivity of the average man increases, the creditor gains without the debtor losing. The principle may be illustrated by the present writer's proposal (in the former Memorandum) that the standard might be a constant proportion of the average income.[1] It is a principle which appears to be countenanced by some high authorities. Thus Sir Thomas Farrer, in his able Memorandum on *Gold and Credit* prepared for the Commission on the Precious Metals, asks: "If prices fall, not by reason of any change in the measure of value, but by increased abundance of the things sold, what considerations of justice or of convenience are there which call for an alteration in the measure of value?"

There is, however, a more important difficulty in the way of adopting Professor Newcomb's plan as a standard for deferred payments. Apparently there would be no distinction between articles of immediate consumption and those which are only agents of production; articles of each class would figure equally in the "value of everything produced" per year. Suppose that the national consumption might be divided into two classes of articles, one consumed nearly raw, the other elaborated through several stages of production, at each of which the transformed material changes hands by a mercantile transaction. Suppose the prices of the former category to rise on an average, while the prices of the latter category—both the long series of materials and with them the finished articles—fall on an average. It might

[1] Compare the definition of variation in the Monetary Standard which Mr. Giffen implies in the following passage of his important paper "On Recent Changes in Theories and Prices" (*Journal Royal Statistical Society*, Dec. 1888): "There may be a case of what may properly be described as depreciation of money where prices do not rise. . . . Measured by incomes, though not by the prices of commodities, there may unquestionably in such case be depreciation." *Cf.* Professor Walras's conception of a general diminution of the *rarity* (or final utility) of commodities. *Élements d'Économie politique pure*, Leçon xxxix. § 390.

happen that the value-in-use of the same monetary income,* say £100, would remain nearly constant for the average citizen. Yet, according to the new index-number, money might seem to be appreciated. Thus the annuitant or creditor might suffer, as he would receive, say, only £90 or £80 for every £100, if this scheme were adopted as a standard for deferred payments.

Section II.

Professor Foxwell's Method.

The conception of quantity produced, or rather sold, per unit of time has been embodied by Professor Foxwell in a distinct definition, which it was an omission on the part of the present writer not to have presented more clearly in the former Memorandum. Professor Foxwell is understood[1] to regard as the ideal measure of the variation in general prices an index-number which is based upon all vendible commodities whatever. He would make no distinction between articles of consumption and agents of production. In averaging the respective price-variations he would assign to each an importance proportioned to the corresponding value, or rather to that value multiplied by the number of times it changed hands (in a day, month, or year) by way of a monetary transaction. This plan is regarded as *par excellence* the measure of appreciation or depreciation.

If pressed with the objection which has just been addressed to Professor Newcomb, namely, that the index-number thus obtained is not the exactest possible measure of the change in the purchasing power of money experienced by the consumer, Professor Foxwell would reply that the consumer is not everyone. The interest of the producer, damnified by appreciation of money, is also to be regarded. The question set to us is a pure currency-question; and the answer to be sought primarily is, not by how much are debts to be scaled up or down, but by how much the metallic currency is to be multiplied in order that the monetary *status in quo* may be restored.

An extreme example may serve to bring out the character of the method. Suppose that the national consumption were divisible into two categories of commodities, the one involving

* This is hardly an objection to the method considered as a Production Standard.

[1] The present writer is responsible for the exposition and illustration of the views which he has obtained in the course of repeated conversations with Professor Foxwell.

only two mercantile transactions in their production, the other sold or re-sold some twenty times at different stages of its production. Suppose the prices of the former class drop on an average five per cent., while those of the latter drop as much as fifteen per cent., other things, and in particular the national taste, remaining constant. Then, according to the Consumption Standard, the index-number will be of the form $\frac{\frac{1}{2} \times 95 + \frac{1}{2} \times 85}{\frac{1}{2} + \frac{1}{2}}$; that is 90. But the new index-number may be written $\frac{\frac{1}{2} \times 2 \times 95 + \frac{1}{2} \times 20 \times 85}{\frac{1}{2} + \frac{1}{2} \times 20}$; that is approximately 86. This is not a Tabular Standard adapted to the interest of creditors and annuitants. It is the measure of the seriousness of appreciation for the community.

It will be observed that the example derives its force from the occurrence of a displacement in the rates of exchange between two classes of consumable articles; for without such displacement, if the drop of price in both categories were the same, there would be no difference between the results of the contrasted methods. Now (it may be said) such displacement is not one of the evils which " laws and kings can cause or cure." Let debtors and creditors regulate their private affairs by a special index-number if they like. That is not the affair of statesmen and financiers. But currency *is* within the province of government. It is competent to governments so to augment the currency, that the appreciation accused by the proper index-number may be reduced.

It should be explained that this scheme does not commit its propounder to any of the extreme views which in the former Memorandum [1] were connected with the conception of amount of sales and the work which gold has to do. He is not bound to refer to the quantity of gold actually existing in currency, or relative to an initial epoch. He need not pretend to calculate the amount of gold in use as money at present, or at the initial epoch. He need not pretend to calculate the ratio in which the quantity of gold at the initial period requires to be multiplied in order to equate the present with the original level of prices. He need not state the amount which for that purpose should be added to the present currency. What he professes to obtain is that ratio in which, if the quantity of currency were increased, other things remaining constant during the increase, the level of prices would be restored. But the amount of coin to be actually added is not

[1] Section IX.

necessarily deducible from the ratio thus conceived; because (1) the quantity of precious metal in use as money may not be ascertainable with any degree of precision, and (2) other things, in particular the condition of credit, may alter during the process of augmentation. In short our Professor is not to be confounded with the currency-quack who pretends to calculate the exact dose of currency which ought to be administered in order to keep the circulation in a healthy condition. Professor Foxwell's Index is rather of the nature of a diagnosis than a prescription; or at least it only enables him to prescribe the general character of the treatment—whether increased aliment or depletion—but not the exact quantity to be taken.

The *Currency Standard*, as Professor Foxwell's special *protégé* may be designated, is to be distinguished as follows from the *Consumption Standard*, which the Committee, in their collective capacity, have favoured. According to each method, the variation in the value of money is measured by a change in the monetary value of a certain quantity of commodity, supposed to be constant. But the standard quantity is not the same for the two methods. The choice is between the sum of valuables consisting of all the finished goods which pass into the hands of the consumer yearly, and that consisting of all goods whatever which change hands yearly. The basis of the one standard is, to use a bold phrase, the mass of final utility; the basis of the other standard is, to use a bolder phrase, the momentum of final utility.*

Upon the whole, it appears that the Currency Standard deserves more attention than it has received. The stone unaccountably set aside by former builders of index-numbers may become the corner-stone of future constructions.

It is not to be thought because the proposed method is likely not to be so revolutionary in practice as it is distinctive in speculation,[1] that therefore it is unbefitting a separate and high place here. For we are concerned here with distinctions of method rather than differences of result. There is attempted here—to

* There is here omitted a lengthy illustration which was introduced in the original with the design of making the conception of the Currency Standard clearer. But further explanation seems superfluous now that Professor Irving has made familiar the cognate conception which he designates MV. The reference may remind us that for the purpose of ascertaining the change in the value of money the Currency Standard as above formulated requires to be divided by a denominator corresponding to Professor Fisher's "T."

[1] Thus the example which we have imagined is probably an extreme one; yet it presents a difference between the compared index-numbers of only seven per cent.; which, in view of the "probable error," say two or three per cent., to which *any* index-number is liable, cannot be considered as colossal.

illustrate small things by great—for a particular province of industry, the sort of analysis which an eminent member of our Committee has performed upon the "Methods" of conduct in general. In the sphere of Finance, as well as Ethics, theoretical distinctions are important, although they may not correspond in practice to such marked discrepancies as might have been expected.

Nor is it a fatal objection to the scheme that it would be impossible to ascertain with precision the proportions in which each commodity absorbs, or exercises a pull upon, the currency; that here the number of resales, and there the exceptional use of credit, would defy calculation. For, regarding the proposed index-number as a Weighted Mean of numerous given variations of price, we see that the objection amounts to saying that the weights are liable to a considerable error. But, as shown in a former Memorandum and to be insisted on again in the present one, the erroneousness of the weights is likely to produce much less error in the computed mean than might have been expected.

Nor is it to be objected that, in the present state of statistics, it would be impossible to obtain returns under several of the headings, that many important articles would have to be omitted altogether. For the plan still may present an ideal in the direction of which it may be thought advisable to move as far as possible. It may supply a rationale to some practical method. Thus, any large aggregate of miscellaneous articles, finished and unfinished, may be regarded as a sample taken at random from the immense incalculable series which forms the data of the ideal index-number. For instance, such a sample may be afforded by the statistics of foreign trade, which we now proceed to consider.

Section III.

Mr. Giffen's Methods.

The next solution of our problem which calls for some additional remarks is that which is deduced from the Statistics of Foreign Trade. It is proposed first to examine the principles upon which Mr. Giffen's masterly calculations are based.[1]

The primary object of the whole investigation appears to have been to compare the volume of trade in different years.[2] The purpose is, in the language of this Committee's first Report, to

[1] *Parl. Papers*, 1878–9, C 2247; 1880, C 2484; 1881, C 3079; 1884–5, C 4456.

[2] Consider the title and introductory sentences of the Reports.

enable us, "given the increase of value [of exports or imports in one year as compared with another], to estimate the increase in quantity of the class of commodities under consideration." *

But there is room for casuistical discrimination when we inquire what is the meaning and measure of increase in the volume of trade or quantity of commodities.

At first sight the following method of comparing the volume at different epochs might seem plausible. Compare the (given) quantity of one article, say a, in one year, say year x, with the quantity of the same commodity in the compared year y. We thus obtain a ratio

$$\frac{\text{Quantity of commodity } a \text{ in year } y}{\text{Quantity of commodity } a \text{ in year } x}.$$

Form now a similar ratio for article b, and again for c, and so on. The circumstance that the unit of a is avoirdupois, that of b, it may be, liquid measure, and so on, need not clog these calculations of ratio. We shall thus obtain as many ratios as there are articles, say fifty, as approximately in some of Mr. Giffen's computations. Now take the mean of these fifty ratios. That mean ratio represents the variation in the volume of trade between the years x and y.

This solution of the problem is by no means to be despised as naïve. It presupposes no doubt a certain sympathy and conformity to a common type on the part of several augmentations of which an average is taken. But it will be shown that this hypothesis is adequately verified.

As to the first point, consider the figures on p. 266, which are obtained by dividing the quantity of every export in 1883 by the corresponding quantity in 1880. The quantities are taken from Mr. Giffen's Table V., Part I.; [1] and the quotients are given in the order in which those quantities occur. Thus for *Alkali* the quantity (of cwts.) is for 1880, 6,888, and for 1883, 6,947; the figures after the first four being neglected. Accordingly, the quotient is 1·01, or 1·0. The figures are true to the first place of decimals.

The grouping of these ratios is exhibited in the diagram on p. 267; where each upright line, surmounted by a figure

* The implication between change in general prices and change in total quantity which is involved in the construction of an index-number such as that considered in this section is well exhibited by Professor Irving Fisher in his *Making of Index-numbers*.

[1] "Reports on Recent Changes in the Value of Foreign Trade, *Parl. Papers*, 1885, C. 4456; and 1888, C. 5386, Part III. Table 2.

expressing its length, represents the number of times that a certain ratio occurs. Thus the ratio 1·1 is presented eleven times; the ratio 1·2 ten times. The median is 1·1; a result which agrees with that obtained by Mr. Giffen's more elaborate and accurate method. He, in effect, weighting each of these ratios with the values of the corresponding article for 1883, finds for the ratio of the volume of 1883 to the volume of 1880 the quotient 146,371,015 ÷ 138,032,674 = 1·06, or approximately 1·1.

Ratios of Quantities in 1883 to Quantities in 1880.	Continued.	Continued.	Continued.
1·0	0·6	1·1	1·2
1·4	0·8	1·0	0·9
1·0	1·2	1·1	1·4
1·1	1·0	0·4	1·0
0·9	1·0	1·0	1·0
2·2	1·1	0·7	1·3
1·3	1·4	1·3	1·2
1·0	1·1	1·1	1·1
1·1	0·9	0·9	1·3
1·4	1·3	1·1	0·9
1·2	1·2	1·1	1·0
1·2	1·2	1·3	1·0
1·4	1·1	1·2	1·2

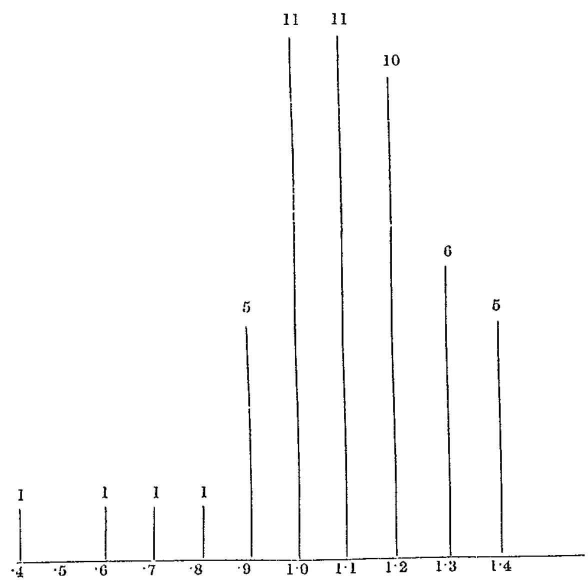
Again, comparing 1886 with 1883, and taking each of the fifty-two ratios to two decimal places, I find for the median 1·00; while Mr. Giffen's Weighted Mean is ·98.

This consilience might have been predicted by the Calculus of Probabilities, if cotton and perhaps one or two other articles whose values constitute abnormally large weights had been omitted.[1] The fact that even without that omission the results coincide shows an even greater symmetry in the movement of trade than might have been expected.

This problem, as we have seen, presents a variety of phases. But for the particular purpose in hand it will be sufficient to make two divisions. First, we may suppose the variation in the Monetary Standard ascertained by examining a wider sphere of industry than foreign trade, or we may confine ourselves to the statistics of exports and imports. The first alternative has not been entertained by Mr. Giffen in his Reports; and it will be dismissed here as leading back to varieties of our problem which have been already considered. Again, the following distinction may be taken. In combining the comparative prices or ratios between the prices of each article at two compared epochs we

[1] The verification holds good when one, or more than one, of the returns for *cotton* are omitted.

may assign a certain weight to each ratio proportioned to the importance of the corresponding commodity, or else we may suppose a change in the level of prices propagated over a whole zone of trade in such wise that we may take a simple average of the given ratios without attending to the corresponding masses of commodity. For a further enunciation of this hypothesis the reader is referred to the former Memorandum, and to the sixth section of the present one. Of these alternatives Mr. Giffen has adopted the former.



It is submitted that this course commits us to some such hypothesis as the following. If, in order to compare the volume of trade for a series of years, we assign a weight to the price of each article proportioned to the importance of that article, we must regard the relative importance of each article as constant for that series of years. If, then, the relative importance of each article is to be measured by the pecuniary value of the quantity bought or sold, the proportions which the value of each article bears to the value of any other article or to the total value of all the articles, ought to be pretty constant during the whole series of years. This assumption is strikingly verified by Mr. Giffen's Table II. Again, if the proportionate amount expended on each article is pretty constant from year to year, we may conceive a purchasing public (whether the community in whose interest the computation is being made, or the foreigners with whom they deal) constant as to the nature of their wants [though it may be increasing in numbers in the course of years]. Accordingly the rates of exchange between the different articles ought to be constant. In other words, the ratio between the prices of the different articles ought to be constant during the series of years. This assumption is verified as well as could be expected by Mr. Giffen's Table I. and Table III., A.[1]

The values and prices being constant, it is implied that the proportionate quantities also, the number of tons, or it may be gallons, of each article exported or imported, have a degree of constancy. This proposition also may be verified by glancing at the quantity columns in Mr. Giffen's Table IV., or the same figures in the statistical abstract.

These assumptions as to the steadiness of the course of foreign trade being admitted, a definite interpretation may be assigned to the otherwise vague idea of increase in the volume of exports and imports. Or rather two or three definitions become possible. The primary significance of an increase in the volume of foreign trade is as a measure of the benefit which the community desires from foreign trade.[2] This conception is particularly germane to the case where the articles on which the computation is based are

[1] There are reasons why Mr. Giffen's table of price variations (Table III., A) should present the appearance of stability in a less degree than his table of proportionate values (Table II.). First, each entry in the former table is obtained by comparing one item with another item, viz. the price of an article in any year with the price of the same article in 1861; whereas each entry in the latter table is obtained by comparing an item (the value of an article) with an *aggregate* (the total value), which of course is apt to be more stable than an item. If the suggestion made below of referring each price to the mean price of the article for adjacent years were adopted this contrast would doubtless be diminished.

[2] The variation in the volume of trade as thus conceived is very similar to Cournot's definition of "real gain," or loss, of social revenue (*Recherches sur les Principes Mathématiques de la Théorie des Richesses*, ch. x.; and later redactions). But Cournot, who seems not to have seized the idea of "final utility," strains the monetary measuring-rod beyond its legitimate application when he propounds his paradox that freeing a commodity from a prohibition results in a loss of real gain to the country which becomes an importer thereof (*ibid.* Art. 89). For this case implies a change in the *quality* of trade, a diversion of the streams of commodity into new channels with which our methods are unable to deal, through failure of the hypothesis enunciated in the text.

commodities imported for the consumption of the community. "In some countries," writes Mr. Giffen, "the whole imports less the re-exports may be treated as imports for final consumption." If the imports are materials as distinguished from finished products, still the unfinished articles may be taken as more or less perfect representatives of consumable commodities.

The case of exports may be thus fitted to this interpretation. It is to be assumed that, given the steadiness in the course of trade which we have postulated, an increase in the volume of exports normally corresponds to an increase in that of imports. Thus exports afford a measure of the advantage derived from foreign trade of the same sort as that which imports afford.[1]

There is a special difficulty in the case of those articles which are imported like cotton in order to be re-exported at a subsequent stage of manufacture. Take the extreme case, mentioned by Mr. Giffen, of tea, which figures as part of the domestic produce exported from France. The French of course derive some advantage from the handling of this article. But the interest which they have in the tea thus transmitted is not proportioned to the value of the article in the same sense as the value of a genuinely native export measures its importance to the nation.

With regard to this special difficulty, and indeed the whole computation, it is to be remarked that we are concerned—not so much with the absolute volume of trade—as the relative volume in one year as compared with another. The relative volume as already stated may be regarded as a sort of mean of the ratios between the quantity (in tons, gallons, etc.) of each commodity in one year and the corresponding quantity. It is a weighted mean, the weights being the respective values of the commodities.[2]

[1] It may be objected that the volume of trade is *per se*, and apart from hypothesis, an interesting datum, or rather *quæsitum*, as affording the measure of profits accruing to the country, or for some such reason. This remark seems just if the corrections of the Monetary Standard which are made for the purpose of estimating the volume of trade are based upon some principle extraneous to the trade, or at least some other principle than that of assigning to each article an importance proportioned to the value exported or imported. All that is contended here is that the received method of measuring the trade by itself, so to speak, postulates a certain analogy between this species of index-number and the more general one which is based on national consumption. Indeed, it is partly on account of this analogy that the subject appears to deserve such full treatment here.

[2] In the symbols to be presently introduced the ratios of quantity are of the form

$$\frac{q_{ay}}{q_{ax}}, \frac{q_{by}}{q_{bx}}, \text{etc.}$$

The corresponding weights are $q_{ax} p_{ax}$, $q_{bx} p_{bx}$, etc. Thus the weighted mean is

$$\frac{q_{ay} p_{ax} + q_{by} p_{bx} + \text{etc.}}{q_{ax} p_{ax} + q_{bx} p_{bx} + \text{etc.}}$$

Grant, now, that in the proposed case of tea transhipped from France the weight is exaggerated. Yet, as pointed out by the writer in a former Memorandum, some inaccuracy of the weights is not likely to affect the result much. It is only in the case of the larger values, notably cotton imported into the United Kingdom as the material for future manufacture, that the difficulty is serious. Such items ought no doubt to be placed in a separate category, and considered on their own merits; not merely on account of their possible inaccuracy, but also on account of their mere magnitude. The domineering pre-eminence of one or two items is fatal to the application of the Calculus of Probabilities which flourishes, so to speak, only in a republic of numerous independent not very unequal constituents.

Implicated with this definition of the volume of trade there is a definite method of measuring the variation in the value of money.* This method is of the same general character as that proposed by the Committee, but more partial and imperfect, as concerned only with a fraction of the national consumption, and that fraction often very indirectly represented.

A slightly different conception of the method may be distinguished by an exhaustive casuistry. The measure of the variation in the value of money, which is afforded by the statistics of foreign trade, may be of the species which was defined in the third section of the former Memorandum. This is a standard, adapted indeed to deferred payments, yet for which the items entering into the index-number " are not copied from the statistics of national expenditure, but are selected on some other principle." It is presumed in virtue of the general sympathetic movement of prices that the change in value of the articles of national consumption is adequately represented by the change of value in certain other articles selected on what may be called a random principle from the whole mass of trade. Whichever of these two slightly distinct views we take, we may perhaps describe the principle of measurement as a *quasi*-Consumption Standard.

This reference to first principles is by no means otiose. It assists in deciding what differences of method are fundamental, how far our choice may be governed by regard for mere elegance and ease, and so we may say of rival methods—

" Whate'er is best administered is right."

Thus it will be maintained that Mr. Bourne's dissent from

* See the new note at the beginning of this section.

Mr. Giffen's practice is not justified by first principles. On the other hand, reasons will be given for differing from the opinion which Mr. Giffen seems to entertain, that his second method, set forth in the fourth table of his earlier Reports, is less serviceable than the method to which his first three tables refer.[1]

The discussion of the questions raised may be facilitated by the use of symbols. Let a, b, c, etc., denote the commodities of which we are given the quantities and prices for a series of n years. Let the successive years be designated by the numerals 1, 2, 3, etc. Let q_{a1} be the quantity (imported or exported) of the commodity a in the year 1; q_{b1} the quantity of commodity b in the same year, and so on. And let q_{a2}, q_{b2}, etc., represent the quantities in the year 2, and so on. The absolute magnitude of the quantities of commodity increases in general from year to year; but the *proportions* between the respective masses of commodity are, by hypothesis stated at page 267, constant. Thus we are to imagine each set of ratios $q_{a1} : q_{b1}$, $q_{a2}, : q_{b2}$, etc., $q_{an} : q_{bn}$, as quantities of the same order hovering about a mean or diverging from a type which we may denote by $\gamma_a : \gamma_b$. Similar suppositions are made with respect to the other articles.* The annexed arrangement of the symbols brings these relations clearly under view :—

	Article a	Article b		Article r
Year 1	q_{a1}	q_{b1}	. . . . .	q_{r1}
Year 2	q_{a2}	q_{b2}	. . . . .	q_{r2}
.	.			
.	.			
.	.			
Year n	q_{an}	q_{bn}	. . . . .	q_{rn}
	γ_a	γ_b		γ_r

Here each γ stands for any of the q's in the column above it; or, rather, the ratio of any one γ to another; *e.g.*, $\gamma_a : \gamma_b$ stands for the ratio of the corresponding q's for any year.

Similarly let p_{a1} denote the price of the article a in the year 1, p_{a2} the price of the same article in the year 2, and so on. Here the absolute magnitude of the prices varies from year to year with the appreciation or depreciation of money; but the proportions between the prices of the respective commodities are regarded as fairly constant. Then we have a scheme for the p's like that of the q's :—

[1] *Parl. Papers*, 1878-9, C. 2247, p. 4, par. 4.

* The intrusion of assumptions proper to the theory of *Sampling* should be noted.

	Article a	Article b		Article r
Year 1	p_{a1}	p_{b1}	. . . . .	p_{r1}
Year 2	p_{a2}	p_{b2}	. . . . .	p_{r2}
.	.			
.	.			
.	.			
Year n	p_{an}	p_{bn}	. . . . .	p_{rn}
	π_a	π_b		π_r

Here each of the π's is typical of the column above it; or, rather, the ratio of any one π to another is typical of the ratio between the corresponding p's for any year.

Upon these hypotheses the following appears to be the most general, or at least a sufficiently general, representation of the proportionate volumes of trade for the series of years.

Volume of imports or exports in year x is proportional to value of imports or exports in year x $\div$ index-number indicating the ratio of the price-level in the year x to the level of prices which is taken as standard.

$\therefore$ Volume of imports [or exports] in year x is proportional to value of imports [or exports] in year x

$$\div \frac{\gamma_a p_{ax} + \gamma_b p_{bx} + \text{etc.}}{\gamma_a \pi_a + \gamma_b \pi_b + \text{etc.}},$$

where, as already explained, γ_a, γ_b, etc., π_a, π_b, etc., are the typical quantities and prices.

So far we have supposed both the quantities and prices of the articles imported or exported to be given. In the concrete case where these data are wanting for a considerable set of articles of which the value only is given, the formula is still the same, viz., Volume in year $x \propto$ Value in year x $\div$ index-number indicating ratio of the level of prices in the year x to the standard level. The only difference is that we must now base our index-number upon a part only of the total trade whose volume is required, assuming that what is true of a part is true of the whole.

How now are we to determine the *types* which enter into our formula? First as to the quantities. The most obvious course is to take the q's of a particular year for the typical γ's, *e.g.*, q_{ax} for γ_a, and so on. In the absence of special reasons in favour of or against certain years, we may select any one of the n years to furnish the typical quantities. We have thus at once n different schemes. But it need not be postulated that the same system of quantities should be adopted for each of the series of years. In fact, in the scheme of Mr. Giffen's Table IV. different factors are employed for each comparison, namely, the factors furnished by each year which is being compared in respect of its level of prices with the standard year (1861). If this additional liberty is used to its full extent for every one of the n schemes already enumerated, we have now n variants; that is, in all we have n^2 different formulæ. However, it may be admitted that these additional schemes, with the important exception of the particular one used by Mr. Giffen in his Table IV., are, if not less accurate, at least less elegant than those which were mentioned first. We shall therefore dismiss these variants with the exception of that one which seems peculiarly appropriate. So far, then, we have $(n + 1)$ schemes presented by the varieties of the quantity-types, the price-type being supposed fixed.

But the price-types also are manifold. A system of such types is furnished by the actual prices of every year—in the absence of special reasons against some particular year. Thus Mr. Giffen has chosen 1861 as the year of standard prices, Mr. Bourne 1883. We have thus n additional cases, which, compounded with the $(n + 1)$ above ground give $n\ (n + 1)$ distinct schemes or formulæ for comparing the series of volumes.

Out of this whole number there are $2n$ which deserve particular attention, namely, those in which the quantities or factors employed in each comparison are supplied by one of the compared years. One system of such schemes in number n is obtained by using in every comparison the factors supplied by the year of standard price; in other words, by taking the types of price and quantity from the same years. The other system, also numbering n, is that which was noticed in the last paragraph but one as having been used by Mr. Giffen in his Table IV. The quantities in this system are supplied by the year which is being compared in respect of its level of prices with the year of standard price. Of course, if we were concerned with only one comparison at a time, if each comparison were an independent operation, these selected schemes would be entitled to a decided preference. But where the object is to find a series of numbers, representing by the ratio of any one to any other the proportion between the volumes of trade for the corresponding years, there seems to be no advantage in constructing our measuring-rod with the factors of one year rather than another. The whole computation presupposes some such hypothesis as that which has been enunciated above; and on that hypothesis one year has no claim to be preferred before another.

What may be said in favour of the selected schemes is that they are very slightly more convenient than the other ones. In general, it may be observed that we have n operations, each of a kind illustrated by the formation and addition of the columns in Mr. Giffen's Table III., B. One such operation is required to construct the denominator of the index-numbers which express the ratio between the level of prices in the standard year and each other year. The denominator is in general terms, as we have seen, $\gamma_a \pi_a + \gamma_b \pi_b +$ etc; or, if we take the γ's from one year, say x, and the π's from another year, say y, the denominator becomes

$$q_{ax} p_{ay} + q_{bx} p_{by} + \text{etc.}$$

To be compared with this denominator there are $(n - 1)$ numerators, one for each of the years except the standard one, each numerator of the form

$$q_{ax} p_{az} + q_{bx} p_{bz} + \text{etc.}$$

where z is a year compared with y in respect of the level of price.

There are, in general, then, n such operations: $(n - 1)$ for the numerators, and one for the denominator. But in the particular case where the types of price and quantity are taken from the same year, where $x = y$, the denominator reduces to

$$q_{ax} p_{ax} + q_{bx} p_{bx} + \text{etc.} = \text{Value for year } x,$$

a given figure which requires no computation. Accordingly one operation—that of calculating the denominator—is spared. Again, if we take the factors from the particular year, say z, which is being compared in respect of the level of prices with the standard year, that is, if in the last paragraph we put $x = z$, the numerator reduces to

$$q_{az} p_{az} + q_{bz} p_{bz} + \text{etc.}$$

forming the total value for the year z, which is a given figure. But meanwhile, in employing a different scheme of factors for each numerator, we have necessitated the use of $(n - 1)$ different denominators, each of the form

$$q_{az} p_{az} + q_{bz} p_{bz} + \text{etc.}$$

The valuation of each of these forms will require $(n - 1)$ operations of the kind described.

In addition to this slight advantage in respect of ease there may also be ascribed a peculiar elegance to the selected formulæ. But they have no claim to the highest degree of accuracy. That distinction belongs to a more complicated system, which is now to be described. We have so far taken for granted that each typical quantity γ is furnished by some q which is the actual quantity for a particular year. But it is more agreeable to the

Calculus of Probabilities to take some *Mean* of the given q's for the type γ. This principle has been recognised in the First Report of the Committee, where it is proposed that for the purpose of comparing the level of prices at two different epochs the factors employed should be the mean of the respective quantities. In the variety of the problem with which we are at present concerned we may suppose a whole series of corresponding quantities, *e.g.*, for the article a q_{a1}, q_{a2}, etc., q_{an}. The mean of all or any number of these quantities may be taken for our type. Now, out of n quantities $(2^n - 1)$ distinct combinations may be formed. Instead, therefore, of the n different arrangements of factors which we at first found we have now $2^n - 1$; which, being combined with the one peculiar scheme employed in Mr. Giffen's Table IV., makes 2^n.

Similar remarks apply to the types of price. We have so far taken actual prices for our types. But it may be better to imagine a sort of mean year with normal or typical prices formed by arranging the actual prices of several years. This principle has been employed to some extent by Jevons, Dr. Soetbeer, Mr. Palgrave, and Mr. Sauerbeck. In virtue of this principle the n different bases of price which we found before are swelled to $2^n - 1$. Altogether, therefore, we have $2^n \times (2^n - 1)$ distinct schemes of index-number.

This account may be further multiplied if we have a choice, as would often be proper, between the Arithmetic Mean and a certain other species of average which is noticed below. However, it may be well to leave some margin for the occurrence of abnormal years (like 1873) whose data cannot be used freely. So let us be content with the modest estimate just furnished, as resulting from that degree of liberty of choice which we have so far contemplated.

That is, however, a very narrow view. For each q which we have employed may be replaced by an expression which is by hypothesis *of the same order*. For instance, we are entitled to put for q_{ay}, q_{by}, etc., the expressions $q_{ay}\frac{p_{ay}}{p_{ax}}$, $q_{by}\frac{p_{by}}{p_{bx}}$, etc.; say q'_{ay}, q'_{by}, etc. This is, in effect, what Mr. Giffen has done in the classical computations comprised in the first three tables of his reports. His formula for the volume of any year, z, may be written in our notation :—

$$\text{Volume of year } z \propto \text{Value of year } z \div \frac{\left(q_{a75}\frac{p_{a75}}{p_{a61}}\right)p_{az} + \left(q_{b75}\frac{p_{b75}}{p_{b61}}\right)p_{bz} + \text{etc.}}{\left(q_{a75}\frac{p_{a75}}{p_{a61}}\right)p_{a61} + \left(q_{b75}\frac{p_{b75}}{p_{b61}}\right)p_{b61} + \text{etc.}}$$

The reader will easily see the equivalence of this formula to that which Mr. Giffen has made familiar, if the symbols $p_{a_{61}}$, $p_{b_{61}}$, etc., are brought outside the brackets both in the numerator and denominator. The denominator, for instance, will become $(q_{a_{75}} p_{a_{75}}) + (q_{b_{75}} p_{b_{75}}) +$ etc.; corresponding to the column headed 1875 in Mr. Giffen's Table II.

By parity it may be shown that the index-number constructed by Mr. Palgrave implies the following formula for volume :—

$$\text{Volume of year } z \propto \text{Value of year } z \div \frac{\left(q_{az}\frac{p_{az}}{\pi_a}\right) \times p_{az} + \left(q_{bz}\frac{p_{bz}}{\pi_b}\right) p_{bz} + \text{etc.}}{\left(q_{az}\frac{p_{az}}{\pi_a}\right) \times \pi_a + \left(q_{bz}\frac{p_{bz}}{\pi_b}\right) \pi_b + \text{etc.}}$$

where π_a, π_b, etc., are types of price obtained by taking an average over certain years.[1] In fact, Mr. Palgrave's scheme may be regarded as a variant of the plan employed in Mr. Giffen's Table IV., which was above commended to particular attention.

These q''s may be combined with each other in the same way as the q's; and, indeed, the q''s and the q's may be mixed. However, these operations would be laborious and inelegant. We shall, therefore, cull from the infinite field which has just been opened up only just such a number as to double the estimate already reached. It may be useful to show how this additional contingent is reached, taking as a conspicuous instance the materials of Mr. Giffen's work. It was open to him to have taken for the basis of prices some year other than 1861. In fact, in his fourth table he has so used both 1873 and 1883. Or the base line might have been composed by taking the average prices of each article for several years, after the manner of Mr. Palgrave or Mr. Sauerbeck, except that the years entering into the average need not be consecutive. It may be asked, What reason could there be for taking half-a-dozen years—some at the beginning, it might be, and some in the middle, or at the end, of the period under review? The reason might be the very absence of a reason. Suppose it were thought desirable, in order to avoid accidents, to take a mean of half-a-dozen years, and not worth the trouble of including more than half-a-dozen. In the absence of special objections to certain years any one half-dozen is as good as any other. There are, therefore, as many half-dozens as there are combinations of six to be formed out of the n years. To avoid the suspicion of cookery it might be best to make a selection at

[1] See, on Mr. Giffen's and Mr. Palgrave's index-numbers, Section II. of the present writer's first Memorandum, *Brit. Assoc. Report*, 1887, p. 265.

random—by spinning a teetotum, or by some equally arbitrary process. It should be observed that the labour of taking averages over several years need not be so formidable as might be supposed, if *Medians* instead of Arithmetic Means be employed. In view of abnormalities like the irregular rise of prices in 1873, there would be a peculiar propriety in the use of the Median.[1]

Exactly similar considerations apply to the factors or proportions which form Mr. Giffen's second table. It was open to him, as he points out, to take these proportions from some other year than 1875. In fact, he tried several years with substantially identical results.[2] There are, therefore, at once as many factors as there are years in the series. Moreover any mean of these factors may be taken. Here, again, then, we have $2^n - 1$ schemes to choose from.

Well, then, any one of these $(2^n - 1)$ measuring-rods may be used in connection with any one of the $(2^n - 1)$ price-scales above mentioned. Thus arise $(2^n - 1)(2^n - 1)$ arrangements for comparing the years in respect of the level of prices. To these may be added Mr. Palgrave's system of factors combined with any one of the $(2^n - 1)$ price-scales. This addition swells the contingent to $2^n \times (2^n - 1)$. This number is to be added to the previous estimate, viz., $2^n \times (2^n - 1)$, which is thereby doubled, becoming $2^{n+1}(2^n - 1)$.

The question may now arise, How large is n to be? It may be suggested that it should be as small as possible, namely, 2. We should proceed according to the method recommended by Professor Marshall,[3] and exhibited at length in the former Memorandum.[4] We should compare the present year with last year only, next year with the present, and so on. The fact that Professor Marshall refers to the general problem of a measure based on articles of consumption, whereas we are now particularly concerned with the volume of trade, does not appear to affect the reasons on which his recommendation is based. However, it may be well to combine that principle with the practice of averages over several years. At any rate, the latter procedure is countenanced by the most eminent statisticians. Extending their review over a considerable tract of time, they have, in effect, taken for granted that sort of solidarity between the years which we have all along supposed. Ten years, twenty years, nay, even forty years, have thus been compared *inter se*. Let us take the

[1] See below, Section V., and the papers to which reference is there made.
[2] *Parl. Papers*, 1878–9, C. 2247.
[3] *Contemporary Review*, March 1887. [4] *Brit. Assoc. Report*, 1887, p. 269.

period of twenty years as quite permissible; then by the formula above reached we find the total number of available arrangements to be more than a *billion.** All these billion schemes are on the whole about equally good, some having a slight advantage in respect of safety, and others of ease.

SECTION IV.

Mr. Bourne's Method.

These elucidations assist us in discerning the character of a method which was proposed by Mr. Bourne so long ago as 1873,** and more recently has been submitted by him to the British Association together with some criticism of Mr. Giffen's celebrated computations.[1] It will be found that Mr. Bourne has discovered, not *the* method, but only *a* method—a very good method, no doubt, but not much better than many others, not more serviceable than hundreds, not more accurate than millions that are available.

A little attention will show that Mr. Bourne's reasoning is virtually identical with that which Mr. Giffen employs in his fourth table when he compares the quantities in any year at the prices of that year with the same quantities at the prices of 1883; and goes on, as, for instance, in his first Report, page v, to compare the measure (for the level of prices) so obtained in order to deduce the comparative volume of any year from its value. It is not to be denied, indeed, that this method, under the neat handling of Mr. Bourne, has acquired great elegance. But we must take care not to exaggerate its pre-eminence over other methods.

In the first place it does not seem to have any advantage over the twin-method which was noticed along with it above. This method is, in brief, to take as the measure of changed level of prices

$$\frac{\text{Quantities of 1883 at prices 1887}}{\text{Quantities of 1883 at prices 1883}}.$$

There is no reason to think that this method would be less accurate than its converse. And it would enjoy the distinction of not having been worked out in detail by Mr. Giffen (in his latter tables).

* Compare Irving Fisher's *Making of Index-numbers.*

** The fact that the priority in the method of calculating the volume of trade belongs to Bourne may justify the attention here given to his opinions.

[1] *Brit. Assoc. Report,* 1885 and 1888.

A certain precedence, perhaps, attaches to these twin-methods in virtue of a slight superiority in ease and elegance.[1] But this slight distinction must not be mistaken for a serious difference in worth or power. Nor is Mr. Bourne's position defensible when he disapproves the method set forth in Mr. Giffen's first three tables. The gist of Mr. Bourne's objections is contained in the following passage, of which the context should be studied :—[2]

"The proportions of [quantities of] cotton yarn for 1865, 1875, 1883 stood as 104 : 216 : 265, but by value as 10 : 13 : 14, and the percentages of increase or decrease from the standard of 1861 were as + 91·23 : + 16·91 [misprinted in the Report 41·63] : — 2·3. It is difficult to see how any combination of these factors, so widely differing in their ratios, can bring about the result that the index-numbers for cotton yarn should be altered as + 5·38 : + 1·00 : — 0·14 as shown in the Board of Trade tables."

This passage, with its context, presents great difficulties. As Mr. Giffen's "index-numbers" do not purport to be measures of volume, but of changed level of prices, there is no reason for surprise that the "factors" of quantity and value should have no visible effect on the "result that the index-numbers for cotton yarn should be altered" by certain additions. The additions to the index-number are proportional to the percentages of increase or decrease of price (+ 5·38, + 1, — 0·14; proportional to + 91·23, + 16·91, — 2·31), and that is all that is to be expected. It seems as if the original writer had stated the relation between a yard and a metre as a preliminary to comparing the height of an Englishman and a Frenchman, the former height having been given in yards, the latter in metres. The critic gives the relative height of the Englishman and Frenchman, and then complains that this factor has no correspondence with the relation between a yard and a metre.

Such appears at first sight to be the drift of the passage above cited. It will be found, however, from the context that the critic has not overlooked the fact that the object of the "index-number" in question is, to continue our metaphor, the comparison of the two scales, yard and metre. But he seems under the mistaken impression that this comparison can best be effected by giving the Frenchman's height in metres and also in feet, and comparing these figures. Now, it is here contended that the two scales may equally well be compared by taking the Englishman's height

[1] Above, pp. 231, 273.
[2] *Brit. Assoc. Report*, 1885, p. 868.

both in metres and feet.[1] Nay, a German will do equally well for the purpose of comparing the two scales of measurement.[2] But, in order to bring out the truth which is here implied, it will be well to employ a metaphor which is more nearly an analogy.

The following apologue may put the whole matter in a clear light. Suppose there were given the increase per cent. in the number of births in a certain district, the increase per cent. in the number of the population, and in the number of persons to a birth (or the inverse birth-rate) for several years. There would, of course, be a visible connection between these figures; and any one set, in particular the proportionate population, could be deduced from the other two. Now, if a statistician had assigned an index-number purporting to represent the alteration in the numbers of the population, and the alterations so assigned were not deducible from the first and third sets of data, and not coincident with the second, it would, no doubt, be reasonable to complain that it was difficult to see how the given factors brought about that result.

But our problem is by no means so simple. It is like those problems in vital statistics which Laplace, in the absence of a complete census, proposed to solve by the aid of the Calculus of Probabilities. He supposes that the total number of births in a country has been ascertained from registers of baptisms, and that the birth-rate, or its reciprocal, the number of persons to one birth, has been observed at two or more epochs in several districts, which are taken as fairly representative of the whole country. If the birth-rate were constant from year to year, we might reason thus :—

Population in year y : Population in year x :: Total No. of births in y
: Total No. of births in x (x being the standard year).

But if the birth-rate is considered as varying between the two epochs compared a correction must be made for this circumstance. We have then :—

$$\text{Population in } y = \text{population in } x \times \frac{\text{Total No. of births in } y}{\text{Total No. of births in } x} \div \frac{\text{Average birth-rate in } y^1}{\text{Average birth-rate in } x}.$$

Now, the last-written fraction may on certain suppositions

[1] The twin-method alluded to on our page 278.
[2] Mr. Giffen's first method.

be determined by taking a measure of the variations in birth-rate (at one epoch compared with another) in each of the observed districts, with due attention to the varying size of the districts, the different importance (for the purpose in hand) of these rates. In other words, if the districts are named a, b, etc., we may write.

$$\frac{\text{Average birth-rate in } y}{\text{Average birth-rate in } x}$$

$$= \frac{\text{Population of } a \text{ in } y \times \text{birth-rate of } a \text{ in } y + \text{population of } b \text{ in } y \times \text{birth-rate of } b \text{ in } y + \text{etc.}}{\text{Population of } a \text{ in } y \times \text{birth-rate of } a \text{ in } x + \text{population of } b \text{ in } y \times \text{birth-rate of } b \text{ in } x + \text{etc.}}$$ [1]

This is the analogue of Mr. Bourne's method, in which it will be seen that *there is postulated a certain constancy in the proportions, between the population of each district, to that of the others and of the whole country*.

Suppose a writer had employed the proportions furnished by the year 1883 in order to determine the relation between the birth-rate of that year and of the year 1865.[2] He, in effect, postulates the constancy of proportions (between the different districts and the whole country) to prevail over that period. It is not open, then, to him to complain of another writer who employs the proportions furnished by the year 1875 in order to compare the population for a series of years between 1865 and 1883. But, if the use of those proportions is admissible, then the sort of verification which the writer of the vexed passage under review appears to expect was not to be expected.

In short, given the hypothesis which has been hinted metaphorically here, and stated explicitly above, the method which Mr. Bourne has propounded has no great advantage over the other methods. That hypothesis not being given, Mr. Bourne's method, equally with the others, falls. Of the varied ramifications of the problem he has occupied a particular, and no doubt an eminent, branch. He cannot hope that this particular branch should stand when the others have fallen. One can only bring them down by striking at the root of the whole reasoning.

From this class of methods we shall now proceed to a substitute for them, which has recently been proposed by Sir Rawson Rawson.

[1] Compare the general formulæ given above, p. 275.

[2] Cf. *Brit. Assoc. Report*, 1885, p. 865 *et seq.*

SECTION V.

Sir Rawson Rawson's Method.

Sir Rawson Rawson's original method may be contemplated under two aspects, according as the primary object is to measure variations in the volume of trade or—our peculiar care—in the value of the monetary standard. Sir Rawson's solution of the problem in its former phase is simple: to put the tonnage of "ships cleared or entered with cargoes"[1] as representing the volume of exports and imports.

Now, we have seen above that volume of trade must be understood in some such sense as equivalent, or rather proportional, to volume of value estimated in a corrected monetary standard, or, if the expression is not too harsh, volume of utility as measured by money. Therefore, in order that the new method should be available for the comparison of volumes in different years, say x and y, the following equation ought to hold approximately:—

$$\frac{\text{Tonnage in year } y}{\text{Tonnage in year } x} = \frac{\text{Corrected value in year } y}{\text{Corrected value in year } x}\text{'}$$

where "corrected value" is used as a short title for the figure which is obtained by reducing the total value for each year to a standard or normal level. In other words,

$$\frac{\text{Tonnage in year } y}{\text{Tonnage in year } x}$$

$$=\frac{\text{Quantity of } a \text{ in } y \times \text{normal price of } a + \text{quantity of } b \text{ in } y \times \text{normal price of } b + \text{etc.}}{\text{Quantity of } a \text{ in } x \times \text{normal price of } a + \text{quantity of } b \text{ in } x \times \text{normal price of } b + \text{etc.}}$$

Now, "tonnage" is the measure of a ship's capacity for cargo. Tonnage is, or is proportioned to, the cubical capacity of that part of a ship which is available for cargo.[2] Accordingly

[1] Given in the *Statistical Abstract.*

[2] Accordingly Sir Rawson Rawson's priority is not affected by Drobisch's suggestion (noticed in the former Memorandum) to put the number of tons or hundredweights in the total mass of commodities as the measure of their volume. *Mutatis mutandis*, the tests here applied to Sir Rawson's method are applicable to that of Drobisch. The validity of the latter is confirmed by the statistics of the German foreign trade for 1885 and 1886, which have recently been published by the Board of Trade, along with an estimate of the change in volume between 1885 and 1886, based upon the method employed in Mr. Giffen's Table IV. (*Parl. Papers*, 1888, C. 5597). The "quantities" of the German exports and imports are all expressed in (German) tons, so that Drobisch's method is readily applicable. The following tables exhibit the results of that method in contrast with the theoretically more perfect computation. The results are expressed as index-numbers for the volume and the level of prices in 1886 as compared

the first step towards establishing the relation above stated is to show that the capacity for cargo bears from year to year a constant ratio to the space actually occupied by cargo. In other words, we require to be assured that an average ship (entering or clearing with cargo) is as fully loaded in one year as another. Sir Rawson Rawson, whose sagacity and candour have anticipated every objection, is satisfied that we may dismiss this scruple.

We may therefore write the postulated equation :—

$$\frac{\text{Bulk of } a \text{ in } y + \text{bulk of } b \text{ in } y + \text{etc.}}{\text{Bulk of } a \text{ in } x + \text{bulk of } b \text{ in } x + \text{etc.}}$$

$$= \frac{\text{Quantity of } a \text{ in } y \times \text{normal price of } a + \text{etc.}}{\text{Quantity of } a \text{ in } x \times \text{normal price of } a + \text{etc.}}$$

where " bulk of a in y " is short for the total space, the volume in cubic yards, occupied by the whole mass of commodity a which is exported, or as the case may be imported, in the course of the year y. The relation of these two fractions may be better seen by putting each of them in the form of what may be called

with 1885. The imports and exports of the precious metals have not been included in the data :—

German Imports of 1886 comparative with those of 1885.	Drobisch's Method.	Mr. Giffen's Method.
Index-number for Volume . . .	·95	·98
Index-number for Price-level . .	1·03	·99

German Exports of 1886 comparative with those of 1885.	Drobisch's Method.	Mr. Giffen's Method.
Index-number for Volume . . .	1·01	1·04
Index-number for Price-level. . .	1·04	·976

This complete consilience affords an indirect verification of Sir Rawson Rawson's method, in so far as it is on the same footing with Drobisch's; each admitting of being regarded as an arbitrarily weighted mean of certain ratios, such as

$$\frac{\text{tons of commodity } a \text{ in } 1886}{\text{tons of same commodity in } 1885}$$

(the ratios of quantity described above). Whereas the theoretically correct expression is the value of commodity a at normal (or corrected) prices; Drobisch puts tons avoirdupois of a, and Sir Rawson Rawson puts (in effect) tonnage (or cubical volume) of a.

From this point of view it will appear that both methods derive confirmation from the experiment tried above, at p. 266, of taking an altogether unweighted mean of the ratios between quantities.

a "weighted mean" of the ratios of bulk; (or, as implied in the last note, we might take as the ratios to be operated on: $\frac{\text{Quantity (in tons or gallons) of } a \text{ in } y}{\text{Quantity of } a \text{ in } x}$, etc.). To effect this in the left-hand member of the equation, we should leave the denominator as it is, and we should alter each term of the numerator thus: For Bulk of a in y write Bulk of a in $x \times \frac{\text{Bulk of } a \text{ in } y}{\text{Bulk of } a \text{ in } x}$, and so on. The left-hand side of the equation is now in the form of a weighted mean of the ratios, $\frac{\text{Bulk of } a \text{ in } y}{\text{Bulk of } a \text{ in } x}$, etc., the weights being bulk of a in x, bulk of b in x, etc. Treating the right-hand member in the same spirit, we obtain a weighted mean of the same ratios, each weight being of the form, Bulk of a in $x \times$ No. of tons [gallons, pieces, etc.] in unit of bulk $\times$ normal price of ton [gallon, piece, etc.], or, as it may be more shortly written, value of Bulk of a in x at standard prices—that is, assuming that the number of tons, etc., in a unit of bulk is constant from year to year. But if this cannot be assumed we must add a remainder, of which the numerator is made up of terms like the following:—

Bulk of a in y (No. of tons in unit bulk of a in y — No. of tons in unit bulk of a in x) $\times$ normal price of a; and the denominator is the total value in x.

Omitting this remainder for the present we have now to compare two weighted means of the same set of quantities (the ratios above specified), the weights being in the one expression each of the form, bulk of a in x; in the other expression of the form, value of a in x. Now, it has been shown by the present writer in a Memorandum on the Accuracy of Index-numbers, published in the Report of the British Association for 1888, that, in forming a mean of any given set of quantities, the difference between the results obtained by adopting different systems of weights is apt to be inconsiderable. This proposition has been established both by reasoning from the theory of probabilities and by pretty copious examples. It is shown that the divergence between the two results tends to diminish as the number of (supposed independent) items entering into the average increases; the probable deviation being proportioned to the inverse square root of the number of items. This tendency to evanescence is resisted by three circumstances: the inequality of the given items which are to be averaged, the inequality of the weights which constitute the set or system which is regarded as true, and the largeness of the difference between each weight in that one system and the corresponding weight in the other system. It can be shown that the last two circumstances are equivalent to, or, rather, are contained under, one attribute, namely, the inequality of the weights in either system.[1]

These criteria are now to be applied to the case before us. In the first place we have a very large number of elements to deal with—much larger than the number of enumerated articles which enter into Mr. Giffen's index-numbers. For Sir Rawson Rawson's index-number includes the unenumerated as well as the specified articles. There is, therefore, a strong *prima facie* presumption that the divergence between the two compared expressions will prove to be unimportant; even smaller than in the case of the index-numbers compared in the paper referred to, the number of items being larger here than there.

Then, as to the counter tendencies. There is no reason to apprehend any fatal inequality in the ratios of the form $\frac{\text{Bulk of } a \text{ in } y}{\text{Bulk of } a \text{ in } x}$. At least it would only be in cases of articles where the bulks were very small that such an influence need be apprehended, the ratio in such a case tending to infinity. It is easy to see, however, that this tendency would be corrected by the "weights"; that such an article would not be likely to have much effect on the whole expression. There seems no reason to apprehend any much more marked inequality in comparative bulks than in comparative quantities, which, as we know from Mr. Giffen's tables, are not fatally unequal.

There remains the twofold condition that the weights of either system should not *inter se* be very unequal. The most serious violation of this condition seems to be coal in the case of exports. It appears from Sir Rawson Rawson's statistics that the bulk of coal takes up an inordinate proportion of the total bulk of all commodities. Accordingly he has very properly excluded coal from his index-number. It is interesting to observe that, as shown in Tables I. and VII., the inclusion of coal does not, as a matter of fact, distort the result so much as might

[1] The measure of "inequality" is the square root of the sum of squares of all the weights in a system ÷ their sum. The divergence between the results is directly proportionate to this expression. If the weights are perfectly equal the factor reduces to *unity* ÷ $\sqrt{n}$. But suppose one weight preponderates over its fellows to such an extent as to constitute half of the total mass, the remainder of which we may imagine split up among a number of small weights; the resulting expression is no longer of the order 1 ÷ $\sqrt{n}$, but equals, at least, $\frac{1}{2}$.

have been expected, or indeed in any considerable degree. The tables referred to should be compared with the Appendix at p. 336, as strikingly illustrating how different principles of averaging bring out the same mean result; in short, that in our sort of work it is not very easy to go wrong.

Among imports, grain and timber are suspicious. But with regard to timber Sir Rawson Rawson shows that, though the "weight" (determined by its bulk) is large, yet it is not materially different from what it ought to be as determined by value. However, he is no doubt judicious in excluding such-like items from his final index-number.

With regard to the inequality of *values*, as this has not proved fatal to Mr. Giffen's and the cognate methods, there is *a fortiori* less reason to apprehend it in the case of an argument which is based on a greater number of independent items. However, it might be well to examine specially the influence of cotton.

There remains to be considered the remainder, which is made up of differences between the density of packing in different years. It is natural to suppose that these should compensate each other except so far as in the course of years a general tendency to increased economy of room makes itself felt. Sir Rawson Rawson sets off against this tendency the increase of passenger traffic; a quantity which he has abundantly shown to be of an order which may be neglected. For short periods, at any rate, the new method appears to constitute an important adjunct to, if not a complete substitute for, the received methods.

Sir Rawson Rawson's method may be regarded in another aspect as affording a measure of the level of prices in different years. If the hypotheses made in the earlier part of this paper are conceded, no additional remark is called for here. We have simply to write Index-number for level of prices in year y as compared with x = average price in y ÷ average price in x = $\frac{\text{Value in } y}{\text{Volume in } y} \div \frac{\text{Value in } x}{\text{Volume in } x} \left(\text{or } \frac{\text{Value in } y}{\text{Value in } x} \div \frac{\text{Volume in } y}{\text{Volume in } x}\right)$; where the values are given figures and the volumes are proportioned to the respective tonnages. We thus obtain a new and remarkably easy solution of our problem.

SECTION VI.

The present Writer's Method.

We have so far (in this third Memorandum) been supposing that the importance attached to each variation in price is, or ought to be, proportioned to the value of the corresponding article. But we have now to entertain a different supposition and distinct method. We are now to imagine a general change coming over the monetary world—or some zone of it like wholesale prices--like a general variation in temperature or atmospheric pressure over a physical region which is not perfectly level and uniform in its conditions. In reading a barometer or thermometer in any particular place with a view of ascertaining the fact and amount of a general change it would not be appropriate to attach importance to the mere size of the tube and quantity of the rising or falling liquid. In fact the *smaller* thermometer has so far the preference, as it takes on more quickly changes of temperature in the surrounding medium. Sensitiveness, not size, is the criterion of these indicators. So also, in virtue of well-known analogies between the unity of price in the same market and the equilibrium of fluids in the same vessel, the change of price in a large market is not more indicative of the sought mean variation than a change of price in a small market. *Prima facie*, for the purpose in hand, each observation should count for one. Or, if more weight attaches to a change of price in one article rather than another, it is not on account of the importance of that article to the consumer or to the shopkeeper, but on account of its importance to the calculator of probabilities, as affording an observation which is peculiarly likely to be correct—peculiarly likely to coincide with that *type* which he is seeking to elicit.

This type of mean variation may be generally defined as that figure which would be presented most frequently if we were to continue indefinitely the long series of price-ratios, or at least that return in whose neighbourhood the greatest number of these statistics cluster. It is, in other words, the Greatest Ordinate of the complete curve, or the highest column of the rectilinear diagram, which represents by its abscissa ratio between the prices of two compared epochs, and by its ordinate the frequency with which that ratio would be returned if the statistics were extended over every region of industry which is subject to independent fluctuations. It is even allowable to imagine series of statistics still longer,[1] namely, those which would ideally occur if we could go on and on multiplying observations under unchanged conditions. As Dr. Venn says :—

" We say that a certain proportion begins to prevail among

[1] Compare Dr. Venn, *Logic of Chance*, chap. i. § 14.

the events in the long run; but then, on looking closer at the facts, we find that we have to express ourselves hypothetically, and to say that, if present circumstances remain as they are, the long run will show its characteristics without disturbance."

The grounds for thus defining our *quæsitum* were stated in that part of the former paper which referred to semi-objective averages or types. There should be added a reference to the sections on the Greatest Ordinate in Dr. Venn's *Logic of Chance.*[1] Compare also the following weighty words in the masterly study on "Cambridge Anthropometry" which he has recently contributed to the Anthropological Institute: "The ordinary mean here is obviously an imperfect guide. . . . What we ought to do, owing to the obvious asymmetry of the curve of frequency, is to take, not the arithmetic mean, but what is called 'the point of maximum frequency,' as this is a far truer index of what may be considered the normal length of vision." Dr. Venn is discussing a problem analogous to ours, namely, how to extricate from an *unsymmetrical* group of observations that mean value which may be taken as a representative type.*

Such being the question, it might seem appropriate to put as answer that return which occurs most frequently in the statistics actually given. But it must ever be remembered, though it is often forgotten by statisticians, that the statistics of prices with which we have to do are of the nature of *samples:* specimens taken at random from a much larger, if not an indefinitely large series. In interpreting these evidences, in inferring the type from a limited number of individuals, we must be guided by the methodical rules which the Calculus of Probabilities prescribes. The theory of errors of observation is here as high above ordinary induction as in the general field of modern science the received inductive methods transcend the simple enumeration of the ancients. Now, the Calculus of Probabilities teaches that the best answer to our question will

[1] The third edition of this unique work, especially the first two chapters and the last two chapters, should be studied by all who wish to contemplate that phase of our problem which is now under consideration.

* The statistics with which Venn was dealing were imperfect, one extremity of the frequency curve not being given; otherwise it may be doubted whether the preference expressed in our text for the greatest ordinate is in general justifiable for a purpose like that of Venn, which was to determine whether the average eyesight of Cambridge "honour-men" differed significantly from that of "poll-men." For—in addition to the difficulties acknowledged in our text—the Mode labours under the disadvantage that there is not available a measure of its accuracy, it does not afford a test of significant difference.

not be obtained by taking that which on the face of the evidence seems to be manifested.[1]

The need of this caution is illustrated by the annexed statistics. Looking at these three groups of statistics you might conclude that the first one, designated A, emanated from and, if prolonged, would converge to 30, as that number is the one most frequently repeated. It might similarly be inferred that B in the same sense represents 38. With regard to C, there might be more hesitation, since no one place or figure preponderates. If, however, we double the size of our compartments and consider which is the fullest of these enlarged places, that distinction will be found to belong to 57–58. Accordingly 57·5 might seem the type represented by this group.

But in fact all these groups appertain to the same series, each figure in all of them being formed in the same way, namely, by the addition of ten digits taken at random from mathematical tables. If this series were indefinitely prolonged the figure most frequently repeated would be 45, a figure which in two of the groups does not even occur once. A much better approximation to the greatest ordinate of the complete series is obtained by taking an average other than the greatest ordinate of each set of samples. For instance, the *median*—or figure which has as many of the given observations above it as below it—is for A 42, that being the fourteenth figure in the group of twenty-seven. Similarly the median of B is 44; of C 50. The median of the whole set, numbering eighty-one, is 45; whereas the greatest ordinate is *prima facie* 38, or perhaps 57·5.[2]

[1] Well does Dr. Venn say in the context of the passage cited from his *Cambridge Anthropometry*: "Any successful appeal to this [the point of maximum frequency] requires far more extended statistics than those at our disposal." Yet he has 520 returns before him!

[2] See *Journal Royal Statistical Society*, June 1888, where it is attempted to meet the difficulty presented by such ambiguity. The method there recommended is to rearrange the statistics in larger groups defined by a new "degree" or "unit" which is some multiple of the given one (that is, of unity in our example). The unit to be adopted is the *smallest* interval which will bring out the one-headed character of the curve; in the cases above instanced generally 6 or 7. Now, we may begin this operation not only from either extremity of the given discontinuous curve (as stated in the paper referred to), but also with equal plausibility from any intermediate point. There are thus about as many systems as the new degree is greater than the old one; in the cases before us usually six or seven. The apex of *any* of these arrangements giving an equally plausible solution, it is proper to take the Mean of them all. I have performed this operation on each batch of twenty-seven figures (given in the text), and on the united eighty-one, with results in each case differing very little from the Arithmetic Mean, which is the best answer that can be extracted from these data.

Professor Unwin, to whom this problem has been submitted, recommends

It appears, then, that, though our end is the greatest ordinate of the complete series, the best *mean*—if we may be excused a pun which it is not easy to avoid—is not necessarily the greatest ordinate of the sample group. The position of greatest frequency is an object, like happiness, best reached by not aiming at it too directly.

The indirect and ancillary average need not be the one which we have taken for the sake of illustration in the last paragraph. In fact, in the case there instanced the arithmetic mean would be the preferable method. But in the case of prices there is reason to believe that the median is peculiarly appropriate. The nature and varieties of this mean have been fully discussed by the present writer both in his former Memorandum on the same subject as the present one, and also in the Memorandum of 1888 On the Accuracy of Index-numbers.

However, it may not be out of place here to give an additional example taken from the statistics of exports. In the annexed table each figure in the first column (on the left hand) expresses a proportion between the price of an article in 1887 and the price of the same article in 1883. Thus, the price of gunpowder per lb. being in 1887 6·46*d.* and in 1883 5·83*d.*, we have the proportion, or comparative price, $111 = 100 \times 6{\cdot}46 - 5{\cdot}83$.

Opposite each comparative price are written in the second column or space the values of the corresponding articles for 1887, the proportionate values or actual values divided by a certain figure which is the same for all the entries, viz. 240,000. For

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forming a derived curve by joining the tops of each pair of adjacent ordinates in the given discontinuous curve; and continuing this process of graphical derivation until we reach a smooth (one-headed) curve. He has been so kind as to subject to this treatment the eighty-one figures above given, and after *eight* repetitions of the process finds for the eighth derived curve one whose greatest ordinate is 43—a very respectable approximation, when we consider that what may be called the real point is 45; that the result given by the Arithmetic Mean, which is here the best solution, is 45·2; and that the probable error to which even that best solution is liable is 1·4.

These processes are, however, very troublesome. Still, in doubtful cases, it may be well to check the Median by recurring to first principles and ascertaining the whereabouts at least of the Greatest Ordinate.

A	27 30 31 32 33 34 36 37 40 41 42 43 46 47 48 49 50 51 52 59 64
	30 41 42 46 52
	30
B	29 31 32 33 34 36 38 39 41 43 44 45 46 49 50 52 53 56 57 59 62
	38 45 46
	38 45
	38
C	38 40 41 42 44 46 47 49 50 51 52 54 57 58 61
	40 41 44 46 50 57 58
	40 50 57 58

instance, the value of gunpowder is 1, being, in round numbers, its actual value 260,000 divided by 240,000. With the reason for adopting this divisor we are not here concerned. Any other basis would serve our purpose as well. It often happens that the same proportion of price is enjoyed by two articles. Thus the comparative price 127 appertains both to arms (fire) and to silk, of which articles the proportionate values are respectively 1 and 6. Accordingly against the entry 127 are written (it does not matter in what order) the figures 1 and 6. Both the prices and the proportions of value are taken from the table given by Mr. Bourne in the paper on index-numbers contributed by him to the Report of the British Association for 1888.

Well, then, the simple or unweighted median is thus found. There being in all 64 proportions (some of them coincident), we are to select that one which has as many returns above it as below; in short, a point between the thirty-second and thirty-third in the order of magnitude. This is easily effected by counting up the numbers of the " proportionate values " in the right-hand space. The thirty-second and thirty-third, counting from the highest, are 12 and 4, both corresponding to the ratio 89. The simple median is thus 89.

127	1, 6	92	7, 5
		91	11
		90	42, 2, 5, 3, 3,[2] 7
		89	47, 12, 4
		88	79, 4
		87	137, 16, 2
		86	5
		85	17
113	4	84	4, 1
112	2	83	29
111	1	82	3
110		81	1, 6
109		80	1
108	1	79	3
107		78	8, 1, 6, 3
106		77	11,[2] 20
105		76	1
104	2, 41	75	
103		74	
102	2	73	19
101	17	72	
100	2,[1] 1, 2	71	
99		70	1, 5, 9,[2] 2, 3
98	2	69	
97		68	
96	7, 7, 4	67	4
95	1	66	5
94	0, 6, 1	62	2
93			

[1] Comparative price of *stockings* per dozen; not explicitly given by Mr. Bourne but inferable from the entries in his *value* and *volume* columns.

[2] Not explicitly given by Mr. Bourne, but inferable from his data.

It was pointed out in the former Memorandum that there is a plausible hypothesis on which, even for the present purpose, it is proper to attach some importance to the values of the commodities, though not necessarily that degree of importance which is prescribed for the standard based on national consumption. The simplest method of attaching importance to the values is to take the simple median of the ratios on the supposition that each of them occurs as often as the number which indicates the corresponding value, or the sum of such numbers where there are more than one of them. Upon this understanding there are in all 666 constructive observations—as near as may be, half above and half below 88. That figure then is the weighted median.

It is pretty certain that this complex median assigns too much importance to the values. And it is probable that the simple median assigns too little. Accordingly a good solution is afforded by combining or comparing the two results, in the example before us taking 88·5 for the answer. Should the two results be markedly different, inquiry may be made as to the cause of the difference, and a preference should be given in general to the simpler combination.

A more elaborate method of weighting the median by taking the square roots of the values was recommended in the former Memorandum. But on second thoughts it appears that the special advantages which this plan may confer hardly compensate for the additional trouble which it involves.

For further illustrations and suggestions the reader is referred to the writer's paper, "On some New Methods of Ascertaining Variation in General Prices," in the *Journal of the Royal Statistical Society* for June 1888. It is hoped that the familiarity of the arithmetic mean will not prevent statisticians from attending to the reasons for preferring in certain circumstances the Median.

SECTION VII.

Ricardo's Method.

Ricardo suggests a method of measuring variation in the value of money, when he lays down that a commodity "which at all times requires the same sacrifices of toil and labour to produce it" is invariable in value.[1] From this point of view the Labour Standard is to be regarded as independent and substantive, not subsidiary to the "Consumption" (or any other) standard, as represented in the first report of the Committee. The Labour

[1] *Principles*, III. chapter xx. (On Value and Riches).

Standard thus conceived and the Consumption Standard are to each other as "value" and "riches" in Ricardo's terminology. "The labour of a million of men in manufactures will always produce the same value, but will not always produce the same riches. . . . A million of men may produce double or treble the amount of riches of 'necessaries, conveniences, and amusements,' in one state of society that they could produce in another, but they will not on that account add anything to value." [1] The Consumption Standard measures the change of money with respect to "riches"; the Labour Standard with respect to "real value." The former relates to the utility of consumption; the latter to the disutility of toil.

Ricardo only proposes the idea of an invariable commodity, of which "we have no knowledge, but may hypothetically argue and speak [2] about it as if we had." He does not assist us to ascertain the change in the pecuniary worth of that hypothetical commodity. A more definite scheme is suggested by the remarkable passage of Professor Marshall's evidence before the Royal Commission on Gold and Silver, where he says, speaking of appreciation of gold: "When it is used as denoting a rise in the real value of gold, I then regard it as measured by the increase * in the power which gold has of purchasing labour of all kinds—that is, not only manual labour, but the labour of business men and all others engaged in industry of any kind."

It may be remarked on this that the Labour Standard and the Consumption Standard present a certain analogy, the former standing in much the same relation to the fundamental laws of Supply as the latter to those of Demand. As before we posited as normal certain quantities of purchasable commodities, and compared the pecuniary worth at different epochs of that constant sum of commodities; so now we should posit certain amounts of work of various sorts, and compare the pecuniary remuneration required at different epochs for the same quantity of work. Or, in other words, we should form the ratio of "new" to "old" rate of pay in each department of industry, and take the mean of this set of ratios, each *weighted* by the amount usually paid in the corresponding department.

[1] *Principles*, III. chapter xx. (On Value and Riches).

[2] "And," he adds, in the exclusive spirit which has characterised almost every propounder of an original method, "may improve our knowledge of the science, by showing distinctly the absolute inapplicability of all the standards which have been hitherto adopted."

* The word "increase" has here been substituted for the obviously misprinted "diminution" in the original Report (Question 9626).

Moreover, since upon Ricardian principles the value in exchange of commodities is proportioned to the "comparative quantity of labour expended on each," there may be expected some correspondence between the two expressions, not only as to their general form, but also as to the constants which they involve, the *weights* with which the variations of wages and prices are respectively to be affected. But the idea of such a correspondence is marred by the fact that the denominations of finished products do not coincide with the classification of wages. Also the suggested analogy is vitiated by a circumstance which is of great theoretical importance: that values in exchange—and accordingly the proportions which form the weights of the Consumption Standard —depend not only on quantity of labour, but also on interest, according to the different degrees of durability of the capital employed in producing them. This circumstance, as it creates a difficulty [1] with regard to Ricardo's first principles, so it suggests a scruple about the method which is here connected with those principles. When we "hypothetically argue and speak" of an invariable commodity "which at all times requires the same sacrifice of toil and labour to produce it," [2] should we include in the idea of "sacrifice" not only bodily and mental labour, but also *abstinence?* * Shall we introduce into our Index-number the variation in the rate of Interest, weighted by the total amount paid in the way of Interest? Or shall we follow the example of the great theorist himself, and omit the consideration of Interest as often as convenience and rotundity of statement and the purpose of a rough approximation may require? The management of these and other difficulties connected with the Labour Standard must be resigned to the abler hand which has already touched this part of the subject.

[1] *Cp.* Sidgwick, *Political Economy*, Book I. ch. ii. "It is rather a perplexing question how Ricardo and M'Culloch could deliberately adhere to the statements above quoted [that labour is the measure of the real value of things, etc.], while they at the same time drew attention to the differences in the value of different products, due to the different degrees of durability of the capital employed in producing them."

[2] Ricardo, *loc. cit.*

* I should now answer to this question "yes;" having regard to Marshall's above cited definition of "appreciation," with which may be compared the conception introduced in his later work with reference to international trade, of representative "bales," "each of which represents uniform aggregate investments of her [country's] labour (of various qualities) and of her capital." I have altered some passages in the context, so as to exclude the term "wages" which, designating a *share* in the total product, is not germane to the Ricardo-Marshall conception.

CONCLUSION.

In conclusion it may be useful to enumerate and summarily characterise the principal definitions of the problem, or "Standards," [1] which have been discussed in this and the preceding Memorandum. An alphabetical order will be adopted, the order of merit being not only invidious, but also impossible in so far as different methods are the best for different purposes.

1. The *Capital Standard* takes for the measure of appreciation or depreciation the change in the monetary value of a certain set of articles. This set of articles consists of all purchasable things in existence in the community, either at the earlier epoch or at the later epoch, or some mean between those sets. This standard is due to Professor Nicholson. It is stated by him (in terms a little less general than those here adopted) in his book on *Money*. It is discussed in the sixth and the tenth sections of the former Memorandum.

2. The *Consumption Standard* takes for the measure of appreciation or depreciation the change in the monetary value on a certain set of articles. This set of articles consists of all the commodities consumed yearly by the community either at the earlier or the later epoch, or some mean between those two sets. This standard has been recommended by many eminent writers, in particular by Professor Marshall in the *Contemporary Review* of 1887. It is proposed by the Committee as the principal standard. It is discussed in the second section of the former Memorandum.

3. The *Currency Standard* takes as the measure of appreciation or depreciation the change in the monetary value which changes hands in a certain set of sales. These sales comprise all the commodities bought and sold yearly at the earlier epoch or at the later epoch, or some mean between those quantities. This standard appears to be implicit in much that has been written on the subject, but to have been most clearly stated by Professor Foxwell. It is discussed in the second section of this Memorandum.

4. The *Income Standard* takes as the measure of the appreciation or depreciation the change in the monetary value of the average consumption, or in the income per head, of the community. This standard is proposed in the fourth section of the former Memorandum.

[1] The methods discussed in connection with the names of Mr. Giffen, Mr. Bourne, and Sir Rawson Rawson are rather solutions than statements of the problem.

5. The *Indefinite Standard* takes as the measure of appreciation or depreciation a simple unweighted average of the ratios formed by dividing the price of each commodity at the later period by the price of the same commodity at the earlier period. The average employed may be the Arithmetic Mean used by Soetbeer and many others, or the Geometric Mean used by Jevons, or the Median recommended by the present writer. This standard is recommended by the practice of Jevons [1] and the theory of Cournot.[2] It is discussed in the eighth and ninth sections of the former Memorandum, and the fifth section of the present one.

6. The *Production Standard* takes as the measure of appreciation or depreciation the change in the pecuniary remuneration of a certain set of services, namely, all (or the principal) which are rendered in the course of production, throughout the community, during a year, either at the initial or the final epoch; or some expression intermediate between the two

[1] Most of Jevons' celebrated calculations (*Currency and Finance*, II., III., and IV.), and in particular his calculation of the Probable Error incident to his result (*ibid.*, p. 157), involve this conception.

[2] Cournot has considered our problem in each of the five volumes in which he has treated of, or touched on, Political Economy (*Dictionary of Political Economy*, Art. Cournot). It is sufficient here to refer to the first and the last of those works, the *Recherches* of 1838 and the *Revue Sommaire* of 1876—the Alpha and almost the Omega of economic wisdom. From these it is clear that variation in the " absolute " or " intrinsic " value of money, in Cournot's view, corresponds to the " Indefinite Standard " as defined in Section viii. of the predecessor to this Memorandum. Cournot illustrates the variation due to a change on the part of money, by that change in the position of the earth with respect to the stars, which is due to the motion of the earth. In this analogy the stars are treated as " points " (*Recherches*, Art. 9). No account is taken of their mass. The context shows that Cournot contemplates a simple average of distances between the earth and each star; not a *weighted* average, or the distance between the earth and the *centre of gravity* of the stars. In his later works he expressly declares against, or at least thinks unbefitting highest place, the measure of what he calls the " power of money " (*Review Sommaire*, Sect. 3), that is, in our terms, the Consumption Standard; the analogy of which is the distance of the earth from the *centre of gravity* of the stars, or rather of certain select stars—say those which are nearest to our human sphere. The Currency Standard, of which the analogy is the distance of the earth from the *centre of gravity* of all stars whatever, does not seem to have been entertained by Cournot.

Cournot, alluding to Jevons' treatment of the problem in *Money*, not unjustly takes him to task for not having distinguished " assez nettement " variations in the " intrinsic value of money " [of which the measure is our Indefinite Standard] from variations in the " power of money " [of which the measure is our Consumption Standard] (*Revue Sommaire*, p. 121). Referring to Jevons' proposal to construct a *Tabular Standard of Value*, Cournot expresses his approbation in words which may fittingly conclude the present study :—" Ce sont là des idées qu'il faut laisser mûrir. Quand le moment sera venu de construire effectivement *l'étalon monétaire* les géomètres pourront y trouver une application intéressante de leur *Théorie des Moyennes*, telles qu'ils l'ont déjà construite pour les besoins de l'astronomie et de la physique."

specified. The theoretical basis and practical construction of such a standard are indicated in Ricardo's *Principles of Political Economy* (ch. xx. and elsewhere), in Professor Marshall's evidence before the Gold and Silver Commission (*Parl. Papers* 1888, C. 5,512, Question 9,625), and in the papers contributed by Mr. Giffen to the second volume of the bulletin of the International Statistical Institute. The standard is discussed in the last section of this Memorandum. It is akin to the method suggested in the fifth section of the First Memorandum, and to the standard proposed by Professor Simon Newcomb which was discussed in the first section of this (third) Memorandum.