

ON SOME THEORIES DUE TO PROFESSOR PIGOU

[THIS examination of some new theories due to Professor Pigou appeared in the *ECONOMIC JOURNAL* (June 1913) as a Contribution to the Theory of Railway Rates; in virtue of a connection which is explained in the opening sentences. The introductory contrast between Professor Pigou's terminology and that which is commonly employed in the present Collection brings into view the peculiarity that in our treatment of Value and Production there is contemplated more explicitly than is now usual the entrepreneur aiming at his maximum advantage, exercising something of the freedom of a monopolist; "external economies" not figuring conspicuously. (They are not ignored, because not presented in one comprehensive picture. Rather, they are, so to speak, shown on a "film," a moving series of pictures each of a differently constituted market, the conditions of supply—perhaps also of demand—changing with the time. See Index, sub voce *External Economies*.) Professor Pigou's conception of the Supply-curve, of which the ordinate includes payment for rent, is exhibited in relation to his new construction of the "curve of marginal supply prices." It should be observed that in reproducing this construction (June 1913) the writer had not the advantage of having seen Professor Allyn Young's criticism of the construction, nor the modifications introduced by Professor Pigou in deference to that criticism. However modified, the new curve, with the corresponding "curve of marginal demand prices," serves to sanction reasoned departures from the rule of *laissez-faire*, to extend Marshall's "limitations of the abstract doctrine of maximum satisfaction."

In subsequent paragraphs there is offered a defence of Professor Pigou's theory of Joint Cost as applied to Railway Policy (see Index, sub voce *Joint Production*).]

Graphical Representation of Cost.—The relations between cost of production and quantity produced present such a variety of

aspects as almost to defy the subtlety of speech, even when rendered precise by mathematical conceptions. Similar shapes designated by the same name, a supply-curve, are often employed in a misleading manner with reference to quite different circumstances. For instance, it may make all the difference whether we are considering (α) long periods, or (α) short ones; (b) the presence, or (β) the absence, of what Dr. Marshall calls "external economies"; (c) collective cost, or (γ) that which pertains to a single individual (or constituent group); (d) the regime of competition, or (δ) that of monopoly; (e) the remuneration of the entrepreneur as included in the cost of production, or (ϵ) as a residue distinguished from the entrepreneur's expenditure. There is a less mistakable division between (z) the use of one of the co-ordinates to represent *price*, the construction which Dr. Marshall has made familiar, and (ζ) the use of one co-ordinate to represent the total amount of money demanded in exchange for the amount of product represented by the other co-ordinate.¹ There is a certain correlation between (corresponding members of) several of these dichotomies; and it is therefore the less surprising that throughout Professor Pigou should have adopted the positive and I the negative attribute. The supply-curves which he employs are mostly of the type $abcdez$; while mine are primarily of the type $\alpha\beta\gamma\delta\epsilon\zeta$. It is not to be expected, therefore, that there should be a close similarity between our representations. But I am concerned to show that there is no essential discrepancy.

For the purpose of instituting a comparison I construct in Fig. 1 a supply-curve of the type above distinguished as z ; and I transfer here as Fig. 2 a curve of the type ζ which I have employed.²

For the present purpose it is not necessary to consider the simplest and most elementary transactions to which such curves may pertain: such transactions as international trade between two imaginary islands, or the bargain between ideal hunters of two different types—say white employers and black employees—as to the distribution of their joint quarry.³ The curves of type ϵ proper to such conceptions are not considered here. Let us rather suppose SS_1 as representing, agreeably to common usage, the

¹ The ζ system is used by Mr. Flux in the *Economic Journal*, Vol. XV., and commented on by the present writer, XVII (γ). It is not quite identical with, being less general than, Dr. Marshall's curves of International Trade referred to in the *Economic Journal*, Vol. III., pp. 69 and 359.

² See III., Fig. 1.

³ Compare *Quarterly Journal of Economics* on the "Theory of Distribution," 1904 (B, I., p. 14).

amount x of, say, agricultural produce offered by a set of farmers

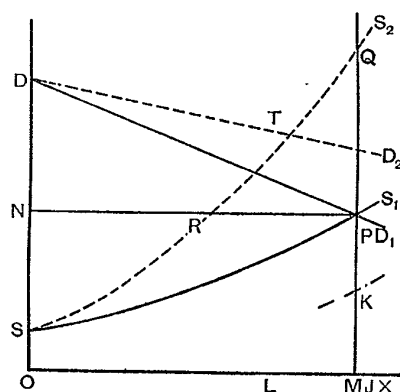


FIG. 1.

In Fig. 1 SS_2 is a parabola of which the equation is

$$Y = \frac{1}{4}(x + a)^2$$

whence

$$yx = \int_0^x Y dx = \frac{1}{4}(a^2x + ax^2 + \frac{1}{3}x^3); \quad y = \frac{1}{4}(a^2 + ax + \frac{1}{3}x^2).$$

The unit in which x is measured is a (= one inch). Accordingly, $MQ = 2.25$ inches; $MP = 1.75$ inch. The demand-curve DD_1 passes through D at the vertical distance of 2 inches from O , and intersects the supply-curve at P . DD_1 is a right line of which the slope relatively to the vertical is $2/1\frac{1}{2}$. Accordingly the slope of the curve of marginal supply prices DD_2 is *twice* that slope. DD_2 intersects SS_2 at T ; nearer the origin than P as might be expected, the law of diminishing returns acting.

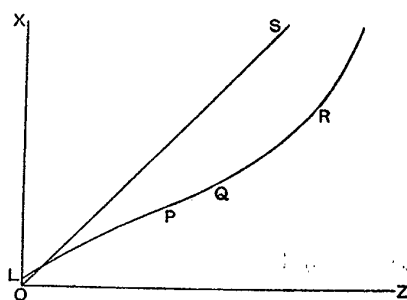


FIG. 2.

using implements, employing workmen, and paying rent. If the increase of produce did not involve any sensible (additional)

pressure on the soil this supply-curve would be, suppose, a horizontal line. But say that there is such a pressure; and let it first be of such a kind as merely to necessitate deeper ploughing and digging on the part of each cultivator, without otherwise modifying the conditions of cultivation—in short, without *negative* “external economies.” The collective supply-curve SS_1 would then be obtained by simply adding up the amounts of product offered by each individual farmer at any assigned price. The “general” expenses consisting, suppose, entirely of rent, would be represented by the area SNP ; while prime costs are represented by $OSPM$.

Here it may be well to remind the reader that there is something arbitrary or dependent on unessential circumstances in the distinction between “prime” and “general” as we have used the terms. Suppose that labourers could be changed only after long notice, and that labour and machinery were readjusted less frequently than the amount of land variable in small parcels with imaginary facility; on such a supposition the rent might be regarded as prime cost, the other expenses as general. The same ordinate MP might now represent the price of that increment of land which corresponds to an increment of produce. More generally it is proper to regard the price PM as made up of two (in general, more) portions, MK and KP , corresponding respectively to the values (at the prevailing prices of the factors of production) of that increment of capital and that increment of land which the entrepreneur would take on if free to distribute an assigned increment of resources between the two uses.¹ Even where this freedom does not exist in reality, one factor of production as compared with the other varying *per saltum*, there is some theoretical advantage with a view to the problems which are before us in realising that the price of the product is theoretically not affected by the circumstance that the agents of production are or are not varied continuously.

Curve of Marginal Supply Prices.—Professor Pigou improves the familiar construction of the supply-curve by the addition of a new curve, that of “marginal supply prices,” SS_2 in our Fig. 1. This curve is thus related to SS_1 . If the ordinate at any point of the abscissa, M , intersects SS_1 at P and SS_2 at Q , the area $OMQS$

¹ See Paroto, *Cours d'Économie Politique*, § 718 (referring to § 100—1); and compare Marshall's analysis of the “supply price of a knife as the sum of the supply prices of its blade and handle.”—*Principles of Economics*, Book V. ch. vi. § 1; and his note on marginal product (with reference to J. A. Hobson's theories, *op. cit.*, p. 393, ed. 6).

is equal to the area $OMPN$.¹ But the area $OMPN$ represents the total expenses incident to the production of the quantity OM ; inclusive of rent (the area PSN) and of entrepreneur's remuneration, which by our convention is included in the area $OSPM$. Accordingly, the line MQ —or, more exactly, the little rectangle of which that line is the height and a (small) unit of produce is the base—represents the addition to the total cost incident to the production of an additional unit.

But, it will be asked, Is not this the very definition of "marginal cost"? And have we not just seen that—however we manipulate the distinction between prime and general cost—the marginal cost incident to an increment of produce is OP ? How then can that incremental cost be OQ ? The answer is that both statements are true. The same predicate "marginal increment of cost" is truly coupled both with MP and with MQ ; if in the one proposition it is understood *simpliciter*, in the other proposition as the logicians say, *secundum quid*. For MP is the increment of cost consequent on an increment of production, the cost of production of the units of commodity other than this increment being supposed constant. That is, MP is the marginal cost from the point of view of the entrepreneur producing a small part of the aggregate output in a regime of competition. But MQ is the increment of cost consequent on an increment of production, the price of the produced commodity not being supposed constant.² Rather, account being taken of the circumstance that the price is such that if a sale could be effected at that price the expenses of production would just be covered, that covering price changes (in the case supposed, increases) with the amount produced. Accordingly, MQ might be described as the marginal increment of cost from the point of view of a monopolist.

In this connection mention may be made of another piece of mechanism due to Professor Pigou: the curve of "marginal demand prices."³ The relation of this curve to the ordinary demand-curve may be shown as follows: Let DD_1 be the demand-

¹ In symbols, if y is the ordinate of SS_1 , Y that of SS_2 ,

$$\int_0^x Y dx = xy.$$

² Let x ($= OM$) be the produce, p ($= MP$) the supply-price, or cost of production per unit, xp ($= OMPN = OMQS$) the total cost. Then $MP = \left(\frac{d}{dx}\right)xp$ (p being treated as constant) $= p$; $MQ = \frac{d}{dx}xp$ (the complete differential) $= p + x\frac{dp}{dx}$.

³ See *ECONOMIC JOURNAL*, Vol. X., "Producers' and Consumers' Surplus." VOL. II. F F

curve in our figure intersecting the supply-curve at P ; DD_2 the curve of marginal demand prices. If the ordinate, not drawn on the figure, at any point on the abscissa, J , intersects DD_1 at D , and DD_2 at D_2 , the area OJD_1D is equal to the area of the rectangle of which the base is OJ and the height OD_2 .¹ But the area OJD_1D represents the total utility, or, in Professor Pareto's less equivocal phrase, "ophelimity" accruing to the customers from the quantity of commodity OJ (on the supposition of their obtaining it *gratis*). The corresponding money value is that which would be realised by a monopolist who practised discrimination of the kind defined by Professor Pigou as "ideal"—a conception which he has happily illustrated by the suggestion of a method whereby a monopolist of this particularly grasping type might conceivably touch the total value in question.² Accordingly, a monopolist of this type would push production up to, but not beyond, the point at which the increment to the said total value is just equal to the increment of total cost, that is, the point at which the curves DD_2 and SS_2 intersect, the point T in the figure, or the point L on the abscissa corresponding thereto. A nobler use of the two new curves will presently appear.

The moderately mathematical reader will have no difficulty in translating these constructions into the form which I have employed, above labelled ζ . The curve S_1 in Fig. 1 might be supposed to correspond to the curve PQR in Fig. 2, if we do not attend to the initial convex part of the latter curve, rather suppose it to start from O and be convex to (OZ) throughout. The abscissa OZ in Fig. 2 corresponding to OX in Fig. 1, the ordinate in Fig. 2 (e.g. a perpendicular let fall from R on OZ —not drawn in the figure) would correspond to the area $OMPS$ in Fig. 1. What line then in Fig. 2 corresponds to the area $OMQS$ in Fig. 1? It might be the ordinate of a certain curve derived from $OPQR$ in Fig. 2 which I have indicated as pertaining to the regime of competition,³ the collective supply-curve (*Gesammt-*

¹ In symbols (corresponding to those used above with reference to supply) let y' be the ordinate of DD_1 & that of DD_2 . Then

$$\int_0^x y' dx = Y'x.$$

In the figure, DD_1 is intended to be a straight line inclined to the axis of Y at an angle with tangent 1. Accordingly DD_2 is inclined to the vertical at an angle with twice that tangent.

² *ECONOMIC JOURNAL*, Vol. XIV. p. 391; *Wealth and Welfare*, p. 203. See also, with reference to this kind of monopoly, *ECONOMIC JOURNAL*, Vol. XX. p. 453 (above, p. 417).

³ Defined by me, *ECONOMIC JOURNAL*, Vol. XXI., E, and more fully by Auspitz and Lieben in their *Theorie des Preises*, p. 13.

angebotscurve) of Auspitz and Lieben.¹ Or rather, as we are not here explicitly representing the profits of the entrepreneur as varying with the amount of product, it is proper to take our curve *PQR* as the cost-curve (*Gesammtkostencurve*) with the interpretation (not that of Auspitz and Lieben) that the intersection of any right line drawn through the origin with that curve designates the amount offered at the price represented by the inclination of the line.² With this interpretation the *vertical distance* defined by Auspitz and Lieben² as the measure of Collective Utility (*Gemeinnutzen*) corresponds to the *area DTS* in our Fig. 1.

I have given another construction in which the factors of production—in the case before us “capital” (= labour + implements × waiting) and land—appear as co-ordinates.³ Say the amount of the former factor is measured along the bottom of the page from the left corner, while the other factor is measured from the same point along the left side of the page. The cost κ of any two quantities of the factors (at prices supposed to be given) is measured downwards on an ordinate perpendicular to the plane of the paper. The corresponding amount of produce multiplied by its price (which the monopolist is free to vary), say ζ less by the cost κ , gives z , the quantity which it is the object of the monopolist to maximise. The construction is such that z is measured upwards from the plane of the paper. In seeking this maximum the monopolist entrepreneur will describe a path on the plane of xy ; which will be a broken sort of path in case one of the factors, such as land, comparatively with the other is varied *per saltum*. This construction is applicable to a regime of competition with a little modification. We may suppose different entrepreneurs to move by different paths in seeking each the maximum of the z pertaining to him. The height of the average z may be regarded as small or null; rather in deference to fact than as required by theory.⁴ Each entrepreneur ever strives to make his z as great as possible. So each golfer in every match strives to make the difference between his score and “bogey,” augmented

¹ The construction ζ thus interpreted will, I think, correspond to that which Mr. Flux has employed in his paper on “Improvements and Rentability” (*ECONOMIC JOURNAL*, Vol. XV.); it being observed that he takes cost for the abscissa and product for the ordinate, as in our Fig. 1 b in O, p. 65.

² *Op. cit.*, p. 370.

³ See *ECONOMIC JOURNAL*, Vol. XI. p. 365.

⁴ On the theoretical point, see Index, s.v. *Entrepreneur*. As to the facts, almost all that is known, I believe, is well presented by Ashley in the *ECONOMIC JOURNAL*, Vol. XX. p. 350.

by his handicap, as small as possible; though on an average, in well regulated golf links, probably the difference between the score and (bogey+handicap) is zero, or rather—having regard to very bad players—on the wrong side of zero.

So far we have supposed the curve of marginal supply prices to be ascending. Now let us consider a descending curve of the sort such as SB in Fig. 3 (see p. 437). If we retain the supposition that the collective supply-curve is formed by simple addition from the dispositions of the individual entrepreneurs, the supply-curve SS' derived from SB (according to the rule above given) will be insignificant in a regime of competition. For it represents only that amount of production which at any assigned price affords to entrepreneurs a *minimum* of profit—a position of unstable equilibrium. But in a regime of monopoly it might well happen in the case represented that production might be stable at any point between O and O' .¹

In order that the descending supply-curve may be significant in a regime of competition it must receive a different interpretation. The height MP now denotes as before the price at which the quantity OM is evoked² in a state of industry adapted to that scale of production. But what corresponds to the curve SS_1 of Fig. 1 in our first example, considered as representing the sum of the amounts offered by each entrepreneur at any (one) assigned price, is a quite different curve from SS_1 of Fig. 3, an ascending curve, the "short-period" supply-curve. It is here represented by a right line—in the neighbourhood at least of any point P on the supply-curve, for it may be supposed lower down to twist and cut the axis OY near O . The construction is explained in my review of Mr. Cunyngame's *Geometrical Political Economy* in the *ECONOMIC JOURNAL* for 1905 (pp. 66–68). For the sake of convenience I virtually made the assumption which Professor Pigou has made on perhaps other grounds, that "the price at which anybody supplies a given quantity of commodity is made up by the addition of two parts, one depending on the quantity that the person himself supplies, and the other upon the quantity that the whole market collectively supplies."³ The ascending part of the supply-curve SS_1 is similarly to be interpreted, and not as the curve SS_1 in Fig. 1.

¹ As noticed (with reference to the curve there employed), *C*, p. 73. *ECONOMIC JOURNAL*, Vol. XXI, p. 361.

² Defined more exactly by Professor Pigou, *ECONOMIC JOURNAL*, Vol. X, p. 358.

³ Pigou, *ECONOMIC JOURNAL*, Vol. XIII, p. 21; the reference to "demand" there made being omitted to suit my context.

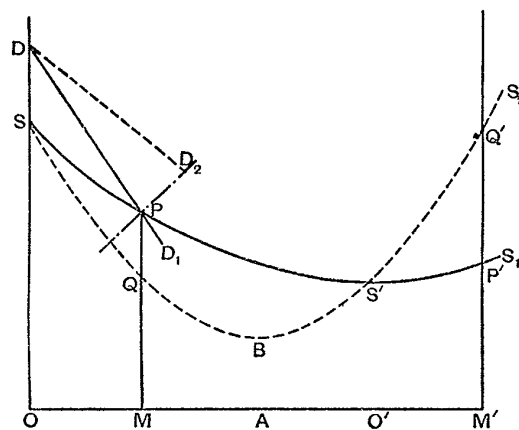


FIG. 3.

In Fig. 3 the curve of marginal supply prices $SS'S_1$ is a parabola with vertical axis and apse at B . If $AB = b$, $OA = 2a$, $OS = 2a^2 + b$; the equation of the curve referred to O as origin is

$$Y = b + \frac{1}{4}(x - 2a)^2.$$

From this the expression for y the ordinate of the supply-curve is obtained by putting

$$yx = \int_0^x Y dx = bx + \frac{1}{4}x^3 - ax^2 + 2a^2x.$$

Whence $y = b + 2a^2 - ax + \frac{1}{4}x^2$. There is a minimum of y at the point of intersection S' between the two curves. This property is general; since

$$Y = \frac{d}{dx}xy = y + x\frac{dy}{dx};$$

and accordingly when $Y = y$, $\frac{dy}{dx} = 0$. In order to construct a simple system of short period supply-curves, formed by right lines with a positive slope of 45° (*Op. Economic Journal*, Vol. XV. p. 68), put

$$y \equiv x + \psi(x);$$

where $\psi(x) = b + 2a^2 - ax + \frac{1}{4}x^2 - x$. Then for the equation to any line of the family, we have

$$y = x + \psi(x'),$$

where x' is the abscissa of any point on the curve $SS'S_1$. For instance, when $x' = a (= OM)$, $\psi(x') = b + \frac{1}{4}a^2 - a$; and accordingly the equation of the corresponding line is

$$y - (b + \frac{1}{4}a^2) = x - a;$$

the equation of a line passing through P , the broken line in Fig. 3. In the figure the unit a is taken as three-quarters of an inch, and b is taken to be half an inch. Accordingly, $OS (= M'Q') = 2$ inches. The demand-curve is a right DD_1 starting from the point D which is at the height 2.5 above the origin. This demand-curve intersects the supply-curve at P of which the height is $1\frac{3}{4}$ inches. The slope of DD_1 with reference to the vertical is $\frac{3}{4}$. Accordingly that of DD_2 is $\frac{1}{4}$. The intersection of DD_2 with SS_1 (not shown in the figure) is at a greater horizontal distance than P from the origin; as might be expected, the law of increasing returns acting.

As in the case represented by Fig. 1, we may here derive from the demand-curve DD_1 the curve of marginal demand prices DD_2 . As there, the intersection of DD_2 with SS_2 determines the maximum of Producers' + Consumers' Surplus, not subject to the condition that prices are assigned by competition. It might be described as the aim of a monopolist, but now a monopolist of a very peculiar kind, a monarch of enlightened benevolence who, surveying the vast plexus of transactions throughout the community, would wish the terms to be altered in such wise as to increase (the money-measure of) the aggregate of satisfactions. This maximum of satisfaction thus aimed at transcends that which is attained by *laissez-faire*, the $H - V$ of Dr. Marshall's deep mathematical note xiv. The latter might be compared to the state of health and efficiency resulting from the practice of what is natural and habitual in diet and therapeutics. An *arbitrary* departure from that practice, based on a mere association of ideas, like the mediæval *similia similibus*, may be compared to crude Protectionism, as likely to do harm. But we are not thereby forbidden to depart from what is called natural, in a direction pointed out by science. One of the directions in which it may prove possible to improve on *laissez-faire* is afforded by Professor Pigou's doctrine supplementing that of Dr. Marshall with respect to the "limitations of the abstract doctrine of maximum satisfaction."¹ The new and less abstract maximum, $H' - V'$, as we may call it, transcends the state of unrestricted competition of which it is sometimes said by mathematical hedonists, and implied by practical free traders, that "this regime realises the maximum of satisfaction and the minimum of sacrifice for each of the co-exchangists."² To advance some way in the direction of $H' - V'$ may be better than to have attained $H - V$;³ just as you are higher when half-way up Mont Blanc than on the top of Snowdon.

Increasing Returns.—I cannot claim to have anticipated this sublime use of the new curves.⁴ It is relevant here as bearing on

¹ *Principles of Economics*, Book V. ch. xiii. p. 467 *et seq.*; referred to by Professor Pigou in the *Economic Journal*, Vol. XI. p. 366.

² I quote from *Histoire des Doctrines Économiques* (p. 636, ed. 2) of Gide and Rist, who are transcribing faithfully enough the doctrines of the mathematical economists.

³ Compare Pigou, *Wealth and Welfare*, p. 106.

⁴ The system of co-ordinates here called ζ (above, p. 430) may be adapted as I have indicated (Index, s.v. *Increasing Returns*) to the system of long-period supply curve with intersecting short-period curves, proper to increasing returns in a competitive regime. The area DTS in Fig. 1 would then correspond to a line in a modified form of Fig. 2, the greatest vertical distance between two curves which are modifications of Auspitz and Lieben's Collective Cost and

a question which I have particularly considered, the signification of the term "increasing returns." In view of much tedious discussion in recent literature I sought to fix the meaning of that evasive term. I distinguished as "primary" and "secondary" two definitions respectively importing that an added dose of productive power increases (1) the marginal, or (2) the average produce. Among other considerations in favour of the primary definition, I remarked: "When we contemplate the working of a competitive regime as bearing on the interest of the community, from the point of view of the philosophic statesman, then we welcome the phenomenon of Increasing Return (or deprecate its contrary) as tending to (or from) some quantity which it is proposed to maximise. But the criterion of such a maximum is analogous to our *primary* conception."¹ Now the point of view of this philosophic statesman is exactly that of the benevolent monarch whom we have just imagined—except that the view of the latter is assisted by the new implement which has just been described. Accordingly, I claim Professor Pigou's authority for my *primary* definition. To be sure, the denotation is generally the same for the two connotations; but not always, as we may see in Fig. 3, where the tract (of produce) AO' presents diminishing returns according to the first definition, but increasing according to the second. My interpretation is confirmed by Professor Pigou's use of terms in the important passage, too long to quote in full, in *Wealth and Welfare*,² which resumes "the general analysis of distribution developed by Dr. Marshall." The "*law of diminishing returns to individual factors of production*," it is there said, "states that the increment of product due to the increase by a unit of any factor of production in any industrial field will in general be smaller, other things remaining the same, the greater is the supply of that factor already employed there." If I mistake not, a typical instance of this doctrine is afforded by our introductory lemma; when the land being considered as constant the "capital" laid out thereon is increased. As shown by the rise of the curve SS_1 (considered as a short-period supply-curve) in Fig. 1, the increment of product due to the increase by

Collective Utility Curves; the distance measuring the total utility called by them *Gemeinnutzen* (*op. cit.*, p. 370). But neither they, I think, nor I proposed to employ this conception for the purpose of contemplating the ideally best distribution of resources; for instance, that as between two classes of industries of the types pertaining to our Fig. 1 and Fig. 3 (Diminishing and Increasing Returns) it would be theoretically advantageous to diminish the output determined by *laissez-faire* in the former case and to increase it in the latter.

¹ *ECONOMIC JOURNAL*, Vol. XI. p. 359. C, p. 72.

² Part II. ch. ii. § 3.

a unit of "capital" will be smaller (in the neighbourhood at least of the point of equilibrium) the greater the supply of that factor already employed. Assuredly, the law of diminishing returns which such a factor fulfils is diminishing returns in the primary sense. Professor Pigou evidently treats that as *the* sense of the term when he does not even notice that in the same circumstances *increasing* returns in the *secondary* sense *must* prevail—initially. Yet the term "increasing returns" is largely employed in such a case by the leading American writers on railway and general economics. One whom I shall often quote as not only the latest but also one of the greatest of them, thus expresses himself: "The law originates primarily in the fixed conditions attaching to the heavy capital investment—the fact, namely, that fixed charges up to a given point of saturation tend to remain constant absolutely: but become proportionately less as the volume of business expands. From this fact, therefore, rather than because of any marked economies of large-scale production, may it be affirmed that railroads offer a notable example of the law of increasing returns."¹ It is in virtue of this fact that "a railroad theoretically presents a clear example of an industry subject to the law of increasing returns."²

Joint Cost.—In the case of another important term, Joint Supply (and its synonyms), I am disposed to accept Professor Pigou's usage for the primary definition, while admitting as secondary the definitions sanctioned by the authority of railway experts. In the first section of this study I have given a general definition covering the cases included by the American writers; but I place in a special category the cases excluded by Professor Pigou; for instance, "where the Joint Cost depends upon a quantity such as total weight or volume which is the sum of two or more items each pertaining to one of the joint products."³ My typical example, clover and honey, fulfils, I think, Professor Pigou's definition that "two products are supplied jointly when a unit of investment expended upon increasing the normal output of one *necessarily* increases that of the other also."⁴ This is not evident at first sight; for, of course, apiculture without clover seed would not result in an output of clover. Professor Taussig makes a very natural criticism when referring to Professor Pigou's example of joint cost, back loading, he remarks: "Now

¹ W. Z. Ripley, *Rail-roads* (1913), p. 99.

² *Op cit.*, p. 71 *et seq.* *Op. C.*, p. 84.

³ *ECONOMIC JOURNAL*, Vol. XI. p. 560. *C.*, p. 88. See Index, s.v. *Joint Cost*.

⁴ *Wealth and Welfare*, p. 215.

in back loading, as in other cases where 'discriminating' rates are made, it cannot be said that a railroad '*necessarily*' (I follow Professor Pigou's example in italicising the word) puts on the market a supply of one kind of service when it supplies another kind. There are always some separable expenses: for example, in the case of back loading there are the terminal expenses and the extra cost of hauling a loaded train over that of an empty one."¹ Ninety-nine out of a hundred critics would probably endorse this criticism. And they would be justified in so far as ninety-nine out of a hundred writers might be supposed to use the phrase "increasing the normal output" as meaning no more than "increasing the output." But it is not to be supposed that the disciple and successor of Dr. Marshall employs the term "normal" as a merely decorative epithet. Professor Pigou has, I think, all along very properly used "output" as the output of something that is demanded, and has implied that the demand is not of an exceptional, perfectly inelastic, character. It follows that the output necessarily tends to be, and we may therefore say normally is, increased by the diminution of its marginal cost. Now a diminution in the marginal cost of producing a commodity such as the transportation of a back load is caused when new trains are put on—not merely to meet a temporary emergency, but as a permanent arrangement—to meet an increased *direct* traffic. But the marginal cost of an article transported by a returning "empty" is not similarly increased by another item in the back load. These statements are not affected by the existence of "terminal expenses" and the like.

It were to be wished, perhaps, that Professor Pigou had expressed himself in terms less liable to misconstruction. But, in fact, it would not be easy to give a more unequivocal definition without making it either very long or very technical. For an explicit description which, I think, nearly covers the instances contemplated by Professor Pigou I again quote Professor Ripley:—

"Railroad expenditures, as Taussig clearly pointed out a number of years ago, afford a prime illustration of the production of several commodities by a single great plant simultaneously at joint and indistinguishable cost. The classical economists illustrated this law by the joint production of wool and mutton and of gas and coke. In both of these instances neither commodity could conceivably be produced alone. . . . The law of joint cost with reference to the production of transportation is somewhat

¹ *Quarterly Journal of Economics*, Vol. XXVII, (1913), p. 380.

different. Compare, for instance, the carriage by a railroad of thousands of passengers and different commodities in every direction, under varying conditions, singly or wholesale, slowly or by express, over a given set of rails every day; with the operation of a great refinery, producing simultaneously kerosene, gasoline, lubricating oils, and greases, as well as various odd chemicals. Both are examples of production at joint cost, but with various important contrasts. In the refinery all the costs are joint. All the processes are interlocked. Every increase in the output of kerosene produces *pari passu* an increase of the other commodities. On the railroad not all, but only a part of the costs are joint, in such manner as has been shown. For, from the joint portion of its plant—roadway rails and locomotives—the railroad may produce transportation of different sorts quite independently. It may choose to especially cultivate its passenger traffic or cotton or coal business.”¹

The “important contrasts” so clearly exhibited by Professor Ripley would not be materially affected if the increase of other commodities *pari passu* with kerosene required some special or separable expense; just as the output of copperas as a joint product with wire, which Professor Pigou by implication instances as a genuine case of joint supply,² requires some special cost for the erection of necessary sheds.

A short but technical definition may be based on the form of the (mathematical) *function* which expresses the relation between assigned quantities of several commodities, x, y, u, v, w , etc., and z , the cost of producing the whole set. Materials for the construction of such a definition may be found on a former page.³ There may be some doubt as to where the line should be drawn which separates the primary from the secondary definition of Joint Cost. But there can be no doubt that it should be drawn well above the case in which the total cost z is related to the quantities of the products simply as a *function* of their *sum*: that is, in the manner below indicated by a quotation from Professor Pigou. I apply the term “sum” to the addition of adjusted units (like those below supposed for pease and beans), ordinary (*e.g.*, *avoirdupois*) units each multiplied by a proper coefficient corresponding to special costs.⁴ The ground of the distinction lies

¹ *Op. cit.*, p. 67.

² *Wealth and Welfare*, p. 299, note 1.

³ *C*, p. 88 *et seq.*

⁴ The bearing of Joint Cost proper on the power of predicting competitive price may be illustrated by supposing that normal equilibrium, after having been reached, is disturbed by a change in demand for each of two commodities; and

herein, I think, that in the case of joint cost proper we cannot, and in other case we can (theoretically), *predict* the relative charges for the different commodities without regard to the relative demand for the commodities.

"*Cost of Service*" Principle.—But the question what is the proper or primary definition of the term Joint Cost is itself of secondary interest. Professor Pigou will perhaps allow the Americans to have their terminology if they will concede to him his propositions. The main issue, of far deeper importance than the definition of a word, is whether Professor Pigou is right in concluding that, in the regulation of railways, discrimination or the "value of service" principle should, after an initial—probably brief—stage, give place to the "cost of service" principle.¹

First appearances, it must be admitted, are against Professor Pigou. Using terms in a strange sense, and accusing distinguished economists of common fallacies, he propounds a thesis contradicting the doctrine of the highest authorities on railway economics. What though in power of mathematical reasoning he wields a bow which few can bend! Does he not aim with it at the clouds? An airship, indeed, would seem to be just the object which he has in view. For his refined reasonings would be admittedly sound if all transportation was effected by flying-machines. For then presumably each flying-machine might be worked to the full for one kind of traffic only. The case would in this respect resemble that of those railways for which discrimination is not claimed, where "each has in the main its own expenses

observing the effect according as Joint Cost proper is absent or present. Let the cost, z , = $F(T)$, where (1) $T = ax + by + cu + \dots$, and x, y, u, \dots are assigned quantities of the commodities designated X, Y, U, \dots . Then the respective prices, in competitive equilibrium, of the commodities X, Y, U are $F'(T)a, F'(T)b, F'(T)c, \dots$. Now let a change in the demand for X and Y occur. Then in general there will be a change in volume affecting cost. "Cost is unknown until volume is ascertained," as Professor Ripley well says (*loc. cit.*). But in the case before us the effect on price may well be small, if there are many commodities; the new set of prices being $F'(T + \Delta T)a, F'(T + \Delta T)b, \dots$. At any rate the *relative* prices, the ratios in which the total charge is distributed among the different commodities, are unchanged. (Compare Marshall, *Principles of Economics*, Mathematical Note xvii., par. 1.) Next (2) let $T = ax + 2hxy + by + cu + \dots$. Now when x and y are "interlocked," to borrow a phrase from Professor Ripley, the prices are no longer as independent of the quantities as before. The new price of X is now $F'(T + \Delta T)[a + 2h(y + \Delta y)]$ and the new price of Y is $F'(T + \Delta T)[a + 2h(x + \Delta x)]$. It is evident that the prediction of the prices from the costs is not such a simple affair as before. Once more (3) let $T = ax^2 + by^2 + \dots$, or more generally $T = \phi(x) + \psi(y) + \chi(u)$. The disturbed price of each commodity will now involve, in a more disturbing manner than in case (1), the quantity of that commodity. But it will not do so in the same way as in (2). It is a nice question whether this case should be described as Joint Cost proper.

¹ *Wealth and Welfare*, p. 234, and context.

of operation as well as its own road-bed and other plant." ¹ But, as it is, "the freight service of a railway comprises the carriage of all kinds of goods simultaneously from the most valuable high-priced commodities, such as silk and satins, down to lumber, coal, cement, and even sand." ²

Such are the first appearances. But on reflection, in the light of the principles which have been above recalled, it will be discerned that if the flying-machines are perfectly competitive, no essential difference is introduced by their having mixed loads; supposing, with Professor Pigou, that "a unit of investment is responsible either for x units of one kind and y units of the other, or for $(x + h)$ units of the first kind and no units of the second, or for no units of the first kind and $(y + k)$ units of the second." ³ Thus, in our introductory Lemma, ⁴ suppose that the produce in wheat is destined for different kinds of cakes and bread. The price of a unit of wheat for different destinations would still be the same. And if the same ground is equally suitable for pease and beans—joint effects in the way of rotation of crops being abstracted—then if the prime costs (in the sense explained) of (properly assigned) units of pease and beans are the same, the same will be the selling price for pease and beans (of units so assigned). The orthodox economist stating this familiar doctrine would not be put off by the affirmation that a great part of the cost was indeterminate, being joint for all the products in large part; that it is impossible to *allocate* the amount proper to each product. This objection might be made to Professor Wieser's doctrine of "imputation"; ⁵ or to the pretension, censured by Mill, of assigning, in a philosophical sense, the amount due to each of two concurrent causes—like the blades of a pair of scissors. But *this* indeterminateness is quite consistent with the determination of value in exchange—proportioned to marginal cost—in a regime of perfect competition. But the prices so determined, according to the received theory, afford a maximum of advantage to producers and consumers. A similar maximum of advantage must be ascribed to the charges for mixed loads which would be adopted by airships conceived as sufficiently

¹ Taussig, *Quarterly Journal of Economics*, Vol. XXVII. p. 379; and *op. p.* 380.

² Ripley, *op. cit.*, p. 169.

³ *Wealth and Welfare*, p. 218.

⁴ Above, p. 431 *et seq.*

⁵ Referred to by Marshall, *Principles of Economics*, p. 393 (6th edition), and unfavourably reviewed by the present writer in the *Economic Journal*, Vol. IV. p. 281 (III, p. 50 *et seq.*).

numerous to realise perfect competition. Railways, indeed, cannot be conceived so numerous as to bring about that scale of charges through the play of competition; but it is to be believed that maximum advantage would be attained if there could be imposed by authority in this case that proportion of charge to marginal cost which is known in other cases to have that desirable result.

I must confess to have countenanced an erroneous view in this matter. Concerned mainly with monopoly, I incidentally misstated a law of competition. I argued that in general a single undiscriminated price might be replaced by two (or more) discriminated prices with advantage both to the (monopolist) producer and the customer. For any value of monopolistic revenue or any value of customers' benefit assigned at random the maximum of advantage to the other party will be realised not by a unique price, but by discrimination. But I omitted to notice that the case in which the initial unique price (or the assigned amount of advantage to one party) is that which occurs in a regime of perfect competition is a particular limiting case of which the statement generally probable is known not to be true. The general reasoning breaks down when we suppose the initial (unique) price of two commodities to be equal to the (equal) marginal cost of each.¹ In this case if any neighbouring system of discriminating prices be assumed, it will be the interest of one or both parties to return to the unique price.

I subscribe, then, to Professor Pigou's thesis; but with two considerable reservations, pointed out by Professor Pigou himself.

¹ It was shown in a previous paper (*ibid.* p. 412) that if b is the undiscriminated monopoly price of two articles (or species of the same article), and $b(1 + \eta_1)b(1 + \eta_2)$ are any two discriminating prices in the neighbourhood of b ; then the curve representing that the Customers' Surplus (considered as a function of η_1 and η_2) is constant (the same as what it was when η_1 and η_2 each = 0) and the (likewise interpreted) curve of Constant Producers' Surplus intersect, in such wise that it is in general possible to adopt a system of discriminating prices which will be better both for the producer and the customer than the undiscriminated price b . It is supposed (in the absence of joint cost) that the cost of production is the sum of two costs each a function of (the amount of) one of the products (*loc. cit.*, p. 423); or more generally a function of the sum (or of a linear function) of the quantities produced (above, p. 443, note). The proposition remains true *in general* when by b we understand not only the monopoly price, but any unique price for the two articles. But in the *particular* case when the marginal cost of producing the amounts saleable at the unique price b is just equal to b the proposition breaks down; the curves do not intersect, but *touch* at the point ($\eta_1 = 0$, $\eta_2 = 0$), in such wise that it is not possible to move off from that point in a direction advantageous to both parties. It should be observed that the existence of a *maximum* at this point is not inconsistent with the possibility that some other point represents *greater* advantage both to producer and customers, as suggested in the text (p. 438). [But see Prefatory note to D, Vol. I, p. 100.]

Firstly, if a railway cannot be made to pay with rates and fares assigned on the principle of cost of service, it is better that it should practise discrimination than that it should not exist. More generally, let it be supposed possible to operate the railways of a country so that the marginal cost of each ton-mile is the same. Then the maximum of the type $H - V$ is attained. But it may be better to pursue the type $H' - V'$ by employing discrimination so as to increase the output of transport for which the demand is very extensible, and where the advantages of increasing returns are thereby secured. It might be one of the exceptions to the general rule that there should be equality of "marginal net products" in order to secure maximum satisfaction.¹ No doubt the conditions are *a priori* improbable.² But there is specific evidence of high authority for their existence; so far as we may thus interpret the dicta of the experts, such as "Much of this business is made possible only by special rates adapted to the case in hand. A higher rate . . . would kill the business." "To compel each of these classes of goods [silk and satin, . . . cement, and even sand] to bear its proportionate share of the cost of carriage would at once preclude the possibility of transporting low-priced goods at all."³ The testimony of high authorities would, no doubt, carry even greater weight if it should be repeated with a full recognition of the *a priori* improbability to which Professor Pigou has called attention.

Secondly, let it be granted that the cost of service principle, the system of charges which would be realised by perfect competition, is ideally the best. Yet with regard to a system so complex, how can we ascertain in the absence of competition what charges would be fixed by competition? The attempts to do so for railway rates have often proved ludicrous. They remind one of the pretension sometimes made by politicians to tell us what some dead chief—Mr. Gladstone or Lord Beaconsfield—would have thought about a measure which was never before them. The defunct authority ought at most to be invoked only to sanction a general line of policy, not to furnish details such as, say, the items of a tariff. As Professor Pigou says: "It is plain that anything in the nature of exact imitation of simple competition is almost impossible to attain." . . . "A considerable gap between the ideal and the actual is likely to remain."⁴

¹ Cp. *Wealth and Welfare*, p. 107.

² *Op. cit.*, p. 211 *et seq.*

³ Ripley, *Railroads*, pp. 152, 168 *et passim*. The dynamic use of discrimination claimed by Professor Ripley would, I think, be admitted by Professor Pigou as pertaining to an initial state (*op. cit.*, p. 234).

⁴ *Op. cit.*, p. 265 *et seq.*

The impracticability of the cost of service principle seems to be largely the ground on which it is dethroned by leading economists from the sovereignty which it might otherwise claim. Professor Ripley begins: "There can be no question that for an indispensable public service like transportation, conducted under monopolistic conditions, the ideal system of charges would be to ascertain the cost of each service rendered and to allow a reasonable margin of profit over and above this amount."¹ But he goes on, in view of the difficulty of ascertaining those charges, to attribute a position of collateral supremacy to the principle of value of service: "Two general theories governing the rates chargeable by railways are entertained, known respectively as cost of service and value of service. . . . Neither of these views [pertaining to the two theories] is entirely sound by itself. Both have large elements of truth in them. Each qualifies the other."² "Our final conclusion then must be this: That both principles are of equal importance, and that both must be continually invoked as a check upon each other."³

These dicta no doubt embody the highest practical wisdom. And it is perhaps vain to desiderate that the limits of these practical principles should be defined more closely by reference to the more general conditions of welfare, the "equality of marginal net products," or the still more ideal principle that the money measure of economic satisfactions should be as great as possible.

Theory of Limited Monopoly.—Nor do I attempt here to formulate the relation between the cost of service principle and the mixed modes of monopoly which I have elsewhere discussed. Suffice it to submit that in the present state of scientific opinion about the subject those discussions seem not otiose. In this part of the work I have obtained support from the adjacency of Professor Pigou's constructions at two points. First, he lends countenance to the use of a right line for the demand-curve as a device for exploring the probabilities of more concrete cases; though he himself seems to use the construction chiefly for the sake of convenience.⁴ I am fortified in the assumption that the right line may be provisionally taken as the type of the demand-curve pertaining to the customers of a railway company.⁵ I am

¹ *Op. cit.*, p. 168.

² *Op. cit.*, pp. 166, 167.

³ *Op. cit.*, p. 184.

⁴ Once at least to show that as there is nothing knowable in this simple case, "our ignorance would not be lightened" by abandoning the assumption of linearity (p. 107).

⁵ But the claim which I have made in favour of the right line that it is intermediate between the convexity predicated by Dupuit and the concavity predicated by Professor Pigou (*Economic Journal*, Vol. XXIII, p. 65) must be retracted. It was based on a misinterpretation of Professor Pigou's doctrine concerning the

therefore confirmed in the deduction that discrimination accompanied with a moderate control is likely to be better, both for the customers and the monopolist, than monopoly forbidden to discriminate.¹

In this and other theories I have largely employed a sort of Probability which has been described in this journal as *a priori*,² and elsewhere perhaps more unequivocally as "unverified." This species of probable inference bears to the more solid parts of statistics and economics a relation something like that which Adam Smith has pointed out between literary and mathematical compositions. The authors of the latter kind, he says, "may have the most perfect assurance both of the truth and importance of their discoveries"; and accordingly they are, much more than the others, "indifferent about the reception which they may meet from the public."³ Now the unverified or non-statistical part of Probabilities, though it is but common sense reduced to formula, yet is not so commonly recognised, not so obviously objective, but that those who employ it should desiderate the approbation of good authorities." This sort of confirmation is largely afforded by Professor Pigou, who employs this sort of inference repeatedly and with respect to the most momentous interests.⁴

The problem in my second section, which comes nearest to one of those which Professor Pigou has handled, is that which relates to the effects of discrimination in a regime of monopoly.⁵ Supposing with him that the law of demand is linear, and that the law of constant return holds, I find with him that the monopolist will produce the same quantity after discrimination as before.⁶ But I have not attended particularly to the alteration

third differential of utility (*Industrial Peace*, p. 70). I forgot that the theorem related not to a particular commodity, such as railway service, but to money income, being in fact an improved version of what I had myself (*Economic Journal*, Vol. VII. p. 559) described as "the circumstance that as the income is increased by equal increments the differences between the successive increments of utility become less." My misapplication of the doctrine was facilitated by a misprint in Professor Pigou's statement of it.

¹ *See* p. 412 *et seq.*

² *See* Index, s.v. *Probability*.

³ *Theory of Moral Sentiments*, Part III. ch. ii.

⁴ *See* Index, s.v. *Pigou*.

⁵ *Wealth and Welfare*, p. 210.

⁶ From the equations indicated at p. 446 *et seq.* in the *Economic Journal*, Vol. XX., it appears that if ξ'_1 and ξ'_2 are the proportional deviations of the output in consequence of discrimination from what it was before discrimination

$$\xi'_1 + \xi'_2 = -(1 + \beta)\eta'_1 - (1 - \beta)\eta'_2 = 0,$$

where $\eta' = -\frac{1}{2}\beta/(1 + \beta)$, $\eta'_2 = +\frac{1}{2}\beta/(1 - \beta)$

of the output; which, as pointed out by Professor Pigou, has not the significance in a regime of monopoly which it has in one of competition.¹ He uses it here only as a stepping-stone towards a *quæsitum* which I have sought more directly.

The only other remark which seems called for in connection with the problems in my second section is that they are not open to the criticism which has lately been directed against Professor Pigou as one "trained in the mathematical school," and accordingly applying a well-rounded theory of monopoly which does not take account of the *incompleteness* characterising monopoly in the concrete.² My conception of a monopolist seeking a maximum of gain, *subject to limitations* imposed by the threat of competition, by public spirit (or State control), admits, I think, of *degrees* much clearer than the expressions commonly employed in a similar connection, such as "equal sacrifice," or "not charging what the traffic will not bear."³ Not that I mean to endorse the criticism as applicable to Professor Pigou. A sense of continuity is not likely to be wanting in the follower of him whose motto is *Natura non facit saltum*.

¹ *Loc. cit.*, § 17.

² *Quarterly Journal of Economics*, Vol. XXVII. (1913), p. 384.

³ See G, pp. 186, 189, *et passim*.