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VARIORUM THEORIES ON CONSUMERS' SURPLUS,
RENT, DUOPOLY, ENTREPRENEURS' REMUNERA-
TION

[IN this Paper published under the title "Appreciations of Mathematical Theories," *ECONOMIC JOURNAL* (pp. 221-231 and pp. 524-531), 1907, some points raised by (then) recent writers on mathematical economics are discussed. Attention is called to Professor Pigou's views on utility or "satisfaction" as an object of economic theory, and its relation to utilitarian ethics. There is raised the question how far it is possible and desirable to contemplate simultaneously—as it were in one and the same picture—a series of (short-period) laws of demand (or supply) which change with time and circumstance (*cp.* Review of Cunynghame). The properties and proper designation of Consumers' Surplus are also considered.

Professor Flux suggests a variant of the construction commonly employed to represent the return of agricultural produce to successive "doses" of outlay. He proposes to represent the *total* return by an ordinate corresponding to an abscissa representing outlay. This representation would have the advantage of fixing attention on what we know in general about the relation of the product to the outlay, namely, its increase at a decreasing (or sometimes increasing) rate with the increase of outlay. The concavity (or convexity) of the curve traced by the extremity of the ordinate representing total production exhibits the character of diminishing (or increasing) returns without bringing into view so conspicuously as in the alternative scheme the assumption involved in the shape of the diagram about a coefficient not known to us, the rate at which the rate of decrease (or increase) varies, the third differential of product with respect to outlay.

In connection with the discussion on outlay and return, reference is made to the old theory that improvements in the art of cultivation are apt to be attended with a fall in rent.

Professor Loria's principal contribution to the Symposium consists of an objection to our description of the indecisive fight between monopolists (**E**); on the ground that in concrete fact

they would cease fighting and make a compact. In this connection the reader should bear in mind that the monopolists to whom the theory relates are not necessarily purveyors of the *same* article. The articles which they respectively control may be not identical, but only partial substitutes for each other. They may be not substitutes for, but *complements* of, each other. The difference in the kind of indeterminateness, according as the articles are rival or complementary, is illustrated in the *ECONOMIC JOURNAL*, September 1922 (Review of Amoroso).

The theory propounded by American theorists, that the remuneration of the entrepreneur is exactly equatable to the loss which his removal from industry would occasion, is examined in the light of Professor Chapman's constructions; and it is found that the proposition is neither quite true nor very useful.]

Some theories which have recently been published in the *ECONOMIC JOURNAL* are the object of the following reflections :—

I. Beginning with Mr. Pigou's article in the *ECONOMIC JOURNAL* for 1903,¹ I remark that he has justly described the relations between the economic measure of utility and the philosophic doctrine of utilitarianism. The measure of utility proposed by Dupuit is applicable in its first intention and obvious interpretation to a great number of transactions.² But the cautions with which the second author of the method has qualified its statement³ are often required. His followers are not committed to Mill's doctrine, "that to think of an object as desirable (unless for the sake of its consequences), and to think of it as pleasant, are one and the same thing; and that to desire anything except in proportion as the idea of it is pleasant is a physical and metaphysical impossibility."⁴ Like Browning, as elsewhere portrayed by Mr. Pigou, the philosophical economist may think of himself as "able, if he so chooses, to resist his own pleasure-seeking desires" . . . "not an inert mass chained, as Bentham believed, to the irresistible power of imagined pain and pleasure."⁵ It is not inconsistent with the use of the money-measure to indulge in a disinterested pursuit of the happiness of others, or even aim at some good distinct from the joy of

¹ "Some Remarks on Utility," *ECONOMIC JOURNAL*, Vol. XIII. p. 58.

² *Loc. cit.*, p. 68, last par.

³ Marshall, *Principles of Economics*, Bk. I. ch. iii. § 5, note.

⁴ *Utilitarianism*, p. 58.

⁵ *Browning as a Religious Teacher*, p. 103.

sentient beings. Nor can the technical apparatus for the measurement of economic utility be employed for "the summation of total happiness."¹

Agreeing with the general tenor of Mr. Pigou's remarks on these sublime topics, I still think that the economic measure of utility may have some tendency to establish the end proposed by utilitarianism, and considerable efficacy in supplying means thereto. Economics cannot indeed prove that to procure the greatest possible sum of satisfaction for all is the criterion of what is right for each. But the authority of an accredited science is lent to show that this object is not absurd² as some metaphysicians have suggested: T. H. Green, for example, when he says that "the aggregate of possible enjoyments" of which modern utilitarians tell us "simply represents the vain attempt to get a definite by the addition of indefinites. It has no more meaning than 'the greatest quantity of time,' would have."³ A scruple worthy of the Eleatic school is solved by walking in the way of the modern economist. To some it is not simply the addition of pleasures, but the addition of pleasures belonging to different persons, which forms a stumbling-block.* The familiar and successful application of the conception collective total utility, *Gemeinnutzen*,⁴ is calculated, I think, to dispel this prejudice. Moreover, the technical apparatus may conceivably be employed as a means towards ascertaining the greatest possible sum of satisfaction in a utilitarian sense, in two large departments of economics which border on ethics and politics. A utilitarian end, the least possible sacrifice on the part of all concerned, may be accepted as the criterion of taxation.⁵ Against the principle of arbitration between combinations there has been suggested the maximum total utility, not so much that of society as a whole, as that of the parties to the agreement.⁶

However we define the "satisfactions" which are measured by the method under consideration, the question may arise: With what unit are they measured? Professor Irving Fisher's unit, "the desire of an arbitrarily chosen individual A for a small increment of a given commodity under given circum-

¹ Pigou, *Economic Journal*, *loc. cit.*, p. 63.

² *Cp. ibid.*, 1895, p. 587.

³ Introduction to the moral part of Hume's Treatise, § 7. [*Cp. Sidgwick Lectures on Ethics of T. H. Green.*]

* To Jevons even, *Theory of Political Economy*, p. 14.

⁴ Auspitz and Lieben, *Theorie des Preises*, p. 23 *et passim*.

⁵ *Pure Theory of Taxation*, above, §.

⁶ Index, s.v. *Arbitration*.

stances," is preferred by Mr. Pigou to a "just perceivable increment of pleasure." * Perhaps it is better to say, with Professor A. Voigt,¹ that no unit is required: quantities like utility are to be measured only by *ordinal* numbers. In confirmation of this conception Professor Voigt refers to the view, now prevalent among mathematicians,² "which sees in ordinal number rather than in cardinal the primary conception of number." †

In a first view of the measure under consideration utility may be considered as related to money in the way of a continuous function. Thus if travel is the commodity enjoyed, and a lowering of fares (which may be supposed to be reckoned on the "zone" system) enables the consumer to procure more of this article, he may simply take new tours, in addition to those which he would take even at the higher fare. He may, however, be induced by the change of fares to rearrange his whole scheme of travel. The difficulties connected with such change of function appear to be minimised by the plan which Auspitz and Lieben have adopted for representing total utility.

A more serious disturbance of the relations between Demand and Total Utility arises when the amount that any one person demands is affected by the amount which other persons demand.⁴

* This unit was proposed in my essay on *Mathematical Psychics*. I have suggested that its use has some affinity to physical measurement.—*Mind*, 1922, p. 274.

¹ "Zahl und Mass in der Oekonomie," *Zeitschrift für die Gesamte Staatswissenschaft*, 1893, No. 3; referred to in the *Economist Journal*, Vol. III., p. 202.

² Cp. Professor Love in the article on "Functions of Real Variables" in the *Encyclopædia Britannica*, Ed. X. Vol. 28.—"The capacity of numbers to answer questions of how many and how much—in other words, to express the results of operations of counting and measuring—may be regarded as a secondary property derived from the more fundamental one of expressing order. Natural numbers form a series with a definite order, and the expression 'greater than' and 'less than' mean 'more advanced' and 'less advanced' in this order."

† Cp. Poincaré's pronouncement on the measurement of utility; cited below (θ), p. 472, and published by Walras in the brochure *Économique et Mécanique*, Lausanne, 1909.

³ *Theorie des Prixes*, pp. 9, 78, 87, 141, et passim.

⁴ The conception may be illustrated by the common supposition that at social gatherings which are choered by alcoholic beverages the consumption of liquor per head is likely to be greater the more numerous the company. An opportunity of testing this belief is afforded by the varying size of the dinners at a certain Oxford College whose members are thought to be susceptible to the influences of good fellowship. The dinners on the Sundays during that part of the Terms which is described as "full" seem well suited for the purpose of verification, the number varying considerably—from seven to forty-two in the course of the period which I have observed—while the character of the entertainment is not otherwise materially altered. The amount of wine (of different kinds) drunk on each occasion, and the cost thereof, have been recorded. I have utilised the records for four years, 1903–1906 inclusive, employing the following assumptions. I take the cost of the wine consumed as the measure of the quantity; and I take the

Mr. Pigou, improving on Mr. Henry Cunynghame's speculations

variations in the quantity consumed per head as an index of variation in effective demand. The demand is, indeed, not measured as usual: for the total cost of the wine drunk on each occasion is divided equally among all who have taken part in the potation. Accordingly the final utility of the last dose which an individual consumes is not, as in ordinary cases, balanced by the cost of that dose; the set-off is only an n th part of that cost, where n is the number of the company. I assume, however, that this peculiarity does not appreciably stimulate the consumption of wine at a College dinner, and that the amount of wine drunk at these social gatherings may be taken as an index of effective demand, just as well as if each individual, as happens at Club dinners, paid for what he individually consumed. I may add that if the circumstance which I neglect were really of account, the conclusion to which I am coming would be *a fortiori*. To test whether the size of the party has any influence upon the depth of the potations, I first find the average size of a party, and then I find the average cost per head, on the one hand, for parties that are below the average size, and on the other hand for parties that are above the average size; for each year, and for the whole period. The results are exhibited in the subjoined table. For example, in the year 1906 the records of 25 Sunday dinners were examined. The average attendance was 21 (the smallest attendance being 10, the largest 42). The average cost per head of the wine drunk at all the 25 dinners in 1906 was a sum which comes to 95.4 per cent. of the average cost per head of the wine drunk at all the 102 parties in the four years 1903-6. The average cost of the wine drunk at those dinners numbering 12 (or more exactly construed, $12\frac{1}{2}$), at which the number of diners was below the average for the year, was 97 per cent. of the aforesaid general average for the whole period. The average cost for the parties at which the numbers were above the average of the year was 93.8 per cent. (of the general average for the whole period). I have thought it sufficient to give the costs as percentages. The statement of the actual figures is forbidden by a scruple such as that which deterred Gibbon from disclosing the amount of his income; lest he should excite the envy of some and the contempt of others.

	Number of parties.	Average size of party.	Cost per head per cent. of average cost for period 1903-6.		
			Small parties.	Parties of all sizes.	Large parties.
1903	25	18.4	99.8	103.4	106.2
1904	26	17.8	97.6	99.8	101.7
1905	26	19.2	104.5	101.4	98.3
1906	21	21.0	97.0	95.4	93.8
1903-6	98	19.1	100.5	100	99.4

It appears from an inspection of this table that no constant or considerable excess of consumption is shown by the larger as compared with the smaller parties. When it is added that the "probable error" incident to the difference between the average consumption of large parties and that of small parties in any one year is about 3 per cent. (per cent. of the general average cost as before), it will be apparent that in this example the influence of the amount demanded by all on the average demand of each is inappreciable.

on this subject,¹ points out that a "consideration of the distribution of the commodity, as well as of the mere quantity of it," should be taken into account.² His elaborate formulæ far transcend the resources of statistics; but they subserve the useful purpose which Dupuit, referring to the impossibility of determining even the simpler law by statistical observation, has thus indicated: "quand on ne peut pas savoir les choses c'est déjà beaucoup que de savoir qu'on ne sait rien."³

The theory improved by Mr. Pigou has useful analogies with the extended theory of supply. The latter, indeed, does not, I think, equally require the consideration of distribution. But both theories profit by the method of representing the element of time which Mr. Pigou has exhibited in a subsequent article.⁴ As I understand his construction, "the ordinary demand curve," at a time which is represented by a point on the axis of X, is in a plane passing through that point, parallel to the plane of YZ; Y denoting the scale of consumption, and Z the price. The diagrams, which are presented at successive epochs, do not occur like the slides which an exhibitor inserts in a regular sequence into his lantern. What diagram at the time $x + \Delta x$ will succeed the diagram presented at the time x will not depend simply on the time. To continue the metaphor, according as a certain index [the intersection of the supply- and demand-curves] is directed to one point or another in the diagram pertaining to the time x , the diagram at the time $x + \Delta x$ will take on different shapes.⁵ The curve which expresses the movement of the scale of production in time is like the path of a particle in a Galtonian error-machine, a path determined partly by a prearranged constitution of things, partly by accidents affecting each step. The pre-arrangement forms the portion of truth in Mr. Cunyngame's doctrine that "a group of successive curves is the expression of a state of facts existing at one time, and is not a group of successive time phenomena."

¹ ECONOMIC JOURNAL, Vol. II. p. 37.

² The new conceptions of collective in relation to individual demand are analogous to the departure in the theory of probabilities according to which the elements or components which generate the law of error are no longer regarded as perfectly independent (see *Journal of the Statistical Society*, 1906, Vol. LXIX.). When each element is correlated with the sum of all, we have a simple case like that conceived by Mr. Cunyngame (*op. review* reprinted Vol. III. p. 138); the case in which each element is correlated with some only of the other elements resembles Mr. Pigou's more exact conception.

³ *Annales des Pontes et Chaussées*, 1844.

⁴ *Monopoly and Consumers' Surplus*, ECONOMIC JOURNAL, Vol. XIV. p. 388.

⁵ I have stated my meaning more fully in a criticism of Mr. Cunyngame's doctrines in the ECONOMIC JOURNAL, Vol. XIV. p. 63 *et seq.*

Referring to Mr. Pigou's article on "Monopoly and Consumers' Surplus," I select, among many points worthy of notice, his account of the pressure whereby the monopolist is able to exploit the "consumers' surplus" of his consumers. It may be suggested that the position of the monopolist is strengthened when the total amount which he controls is, or is supposed to be, limited.¹ The consumers of mineral water controlled by a monopolist, as in Cournot's illustration, are likely to agree to his terms more quickly when it is known that he has only a fixed quantity of the commodity per day at his disposal.

I cannot dismiss this article without commending Mr. Pigou for having followed the highest authority on the subject in substituting "consumers' surplus" for "consumers' rent." I submit that it is better to limit the term "rent" to the income derived from things the supply of which is limited, and cannot quickly be increased,² or some neighbouring definition, rather than to extend the term, as suggested by Professor Clark,³ to all kinds of differential gains. Rent, as defined above, is a species of surplus which differs from other kinds of surplus in important respects.⁴ These nice distinctions may be contemplated more clearly if different words are used for the species, rent, and that which is predicated thereof, the genus surplus.

II. An easy transition leads to Professor Flux's article in the *ECONOMIC JOURNAL* for 1905, of which a leading feature is the use of a line to represent the surplus constituting economic rent. Professor Flux's construction may be regarded as a variant of that which is employed by Messrs. Auspitz and Lieben.⁵ The use of the ordinate rather than an area seems to have, on the side of supply, the same sort of advantage which we have already attributed to it on the side of demand;⁶ it smooths over discontinuities of function.⁷ A similar construction is suitable to represent producers' surplus considered as depending on several

¹ The description given by Mr. and Mrs. Webb of the bargain between the individual workman and the capitalist employer is very instructive: *Industrial Democracy*, Part III. ch. ii. p. 654, ed. 1902.

² See Marshall, *Principles*, Bk. II. ch. iv. § 7.

³ See his article in the *Quarterly Journal of Economics*, 1891, and his book, *Distribution*. The claims of the wider definition are ably stated by Dr. J. Schumpeter in his paper on *Das Rentenprinzip* in the *Jahrbuch für Gesetzgebung*, 1907.

⁴ As shown by Professor Marshall's *Principles*, pp. 479, 626 *et passim*.

⁵ So far, at least, as Professor Flux uses the ordinates to represent money. Reference should also be made to J. D. Everett's *Geometrical Illustrations of the Theory of Rent* read before the British Association, and published in the *Journal of the Statistical Society*, 1899.

⁶ Above, p. 323.

⁷ Cp. Flux, *loc. cit.*, p. 232, referring to his Fig. IV.

variables. Thus if the outlay of borrowed capital is measured on the axis of X, the amount of land rented on the axis of Y, the surplus accruing to a particular farmer, or to farmers collectively, may be represented as the intercept between a certain plane corresponding to Professor Flux's line O E, and a certain surface corresponding to his curve O P.¹ Of course, this is a very abstract conception, neglecting, *e.g.*, sales at particular times and in particular markets on terms which would not pay if adopted generally, and other incidents of a somewhat monopolistic character.²

But I must keep to the subject "improvements and rentability." Among the "controversies of a past age" to which Professor Flux recalls attention is the thesis that the "sudden and general introduction of agricultural improvements" would tend to lower rent.³ I gather that Malthus took the opposite side from Ricardo and the Mills on this question, since he held that the tendency of rents to decrease might be "counterbalanced by extraordinary improvements in the modes of cultivation."⁴ I do not understand that Professor Flux takes either side in this controversy. His construction may indeed be employed to show that neither the Ricardian thesis nor its contradictory is universally true. The mathematical weapon to which Professor Flux has imparted now refinement may be directed against the Ricardians with at least as much effect as Mr. Cannan has turned against them their own primitive instrument of arithmetical illustration.⁵

But I submit that the Ricardian thesis is not refuted by the proof that it is not universally true. For, interpreted generously, the theory purports to be only *probable*. The probability with which we have here to do is not merely the sort of credibility short of certitude which characterises empirical knowledge generally, but rather a species of presumption not founded on specific experience, which is peculiar to the Calculus of Probabilities. Such is the postulate that when an event must occur in one of two ways, and is not known to occur more frequently in one way than another,⁶ then the event may be considered

¹ Cp. *Quarterly Journal of Economics*, 1904, Vol. XVIII. p. 165.

² Cp. *Economic Journal*, Vol. VII. p. 238.

³ J. S. Mill, *Political Economy*, Bk. IV. ch. iii. § 4.

⁴ Quoted by Professor Flux, *loc. cit.*, p. 277.

⁵ *Theories of Production and Distribution*, p. 322 *et seq.*

⁶ While, if there was a difference of frequency, it would have come to our knowledge—I am disposed to add, in accordance with the view which I have expressed respecting "The Philosophy of Chance."—*Mind*, 1884.

as equally likely to occur in one way as in the other. With this may be connected the postulate that—in the absence of specific knowledge to the contrary—the coefficients with which we have to deal have not extreme values; in particular that the differential coefficients are not infinite; and accordingly the functions not discontinuous; and so forth. These probabilities might be conveniently called “*a priori*,” from their use in the Calculus. I have elsewhere¹ dwelt on this kind of probability as required in the human sciences, and content myself now with an additional example. Investigating the incidence of our import duties Mr. Pigou properly begins an argument thus: “Presuming, as in the absence of knowledge is reasonable, that the elasticity of production is the same at home and abroad.”² So Mr. Bickerdike, in a paper to which we are coming, properly presumes that the coefficients of elasticity with which he has to do have not extreme values.³

Now let us apply this principle to the problem in hand, one datum of which, it should be remembered, is that the total quantity of corn⁴ consumed before and after the improvement is the same. We suppose with Mill “population stationary, and a sudden improvement made in the arts of production”;⁵ and we also follow Mill in treating it as a matter of general knowledge that of an article, “such as the habitual food of the people of England, wheaten meal,” “there is probably as much consumed, at the present cost price as there would be with the present population at a price considerably lower.” With these presuppositions, and—to begin with—treating the amount of land used as constant, let us look first at the construction which Professor Marshall has made familiar, in which the degrees of the abscissa represent successive doses of outlay and the ordinates the corresponding increments of “corn.”⁶ The total area of produce being constant, the consequence of an improvement is to shorten the abscissa representing the amount of capital which

¹ Explicitly and in general in the article in *Mind*, which has been referred to, and in *Metretike* a pamphlet published in 1887 by the now defunct Temple Company; summarily, and with reference to Economics, in *Giornale degli Economisti*, 1897, Vol. XV. p. 318; and in *ECONOMIC JOURNAL*, *passim*. See Index, s.v. *A priori probabilities*. [See also *Mind*, 1922, p. 261.]

² *Protective and Preferential Import Duties*, p. 29.

³ *ECONOMIC JOURNAL*, Vol. XVI. p. 632. *Op.* Vol. XVII. p. 100 *et seq.*

⁴ J. S. Mill, *Political Economy*, Bk. IV. ch. iii. § 4, par. 1. *Ibid.*, par. 5, “if no greater produce is required.”

⁵ *Ibid.*, Bk. III. ch. iii. § 2, par. 2; Bk. III. ch. ii. § 4, par. 2.

⁶ Used in the general sense defined by Professor Marshall, *Principles of Economics*, Bk. V. ch. viii. § 3, par. 1, with reference to the classical theory of rent.

is laid out on the given land; and to lengthen the ordinates representing the returns to doses of capital—if not for every dose, every value of the abscissa, at least in general, and on the whole so that an equal area may stand on that smaller value of the abscissa which represents the capital outlay after the improvement. We must give up the pretension to formulate the variation in the law connecting the ordinate with the abscissa. We must neither affirm that the improvements always add an equal absolute amount to the produce of each of the successive doses of capital, nor yet that they always add an equal percentage.¹ But we may argue, I think, that there is no reason for expecting the average slope of the new curve to be either larger or smaller than that of the old curve; that as to its general trend the new curve will *probably* resemble the old one. It follows that corn rents will probably go down; and money rents very probably.

The argument is easily translated into the form proper to Professor Flux's construction. The argument is, I think, not affected by taking into account the circumstance that the amount of land rented by an entrepreneur-farmer is theoretically in general variable.²

III. I go on to the year 1906, which was fruitful in mathematical contributions. There is first the article³ in which Professor Loria has honoured me by his criticism. Referring to an article of mine in the *Giornale degli Economisti* of 1897,⁴ some of which is embodied in the *ECONOMICAL JOURNAL* of the same year,⁵ Professor Loria, as I understand, does not traverse the abstract reasoning which I have applied to the case of dual monopoly. Rather he appeals to what may be called scientific common sense when he intimates that the case is not worth treating, since it could not long exist: "the two monopolists, instead of carrying on an unprofitable war . . . will settle the value between them." Now if each of the monopolists were producing the same article, this consummation might, perhaps, with propriety be assumed as the general rule. As Mill says, in a passage which is surely remarkable when the date at which it is written is taken into consideration, which is, perhaps, even

¹ *Cp.* Cannan, *loc. cit.*

² The relation between rent and the productivity of the marginal dose of land is elegantly exhibited by Mr. J. D. Everett in the paper which has been referred to, *Journal of the Statistical Society*, 1899, p. 707.

³ Marshall and Edgeworth on Value. *ECONOMIC JOURNAL*, Vol. XVI. p. 365.

⁴ *Giornale degli Economisti*, II. pp. 23-24 (misprinted "*ECONOMIC JOURNAL*," in the reference given in a note to Professor Loria's article).

⁵ Vol. VII. pp. 237-8.

truer now than it was then, "Where competitors are so few they always end by agreeing not to compete."¹ But the two monopolists whom I am considering are not always producers of the same commodity. That case might be of no practical importance, and yet its analysis might be useful, for the sake of the analogous concrete cases in which the two monopolists control *correlated* articles, either "complementary" or "rival." These two cases are hardly comprised under Professor Loria's description of my theory: "it is assumed that the production of a given commodity is monopolised by two producers only."² I gave as instances of complementary articles owned by different monopolists the ground and the water-power required by millers.³ The following comment on this case was given in the article of the same date in the *ECONOMIC JOURNAL* :—

"The theorem may have some bearing on a system which is regarded by some as the ideal of the economic future, that each industry should be consolidated into a trust or combination. Such a system would be characterised by instability, by fluctuations of prices such as now occur in railway wars, but more prolonged; for in so far as the combatants, like the two landlords in the example given, are not direct competitors, the combat seems less likely to be terminated by either the ruin of one party or the amalgamation of the two."⁴

I admitted that when the two monopolists supply rival commodities amalgamation is more apt to occur.

"But," I added, and still submit, "even in this case the proposition that value is between certain limits—over a certain range of prices—indeterminate, may well be of theoretical importance."⁵

The problem raised in Cournot's Chapter VII., as well as the problem of his Chapter IX.,⁶ may repay attention. If he was not right in his solution of either problem, he may have been right in thinking them both worth solving.⁷

As to the remainder of Professor Loria's article directed against Professor Marshall's theory of value it would be presumption on my part to speak on behalf of one so capable of

¹ *Political Economy*, Bk. I. ch. ix. § 3, par. 3.

² *ECONOMIC JOURNAL*, Vol. XVI. p. 306.

³ *Giornale*, *loc. cit.*, p. 20 *et seq.*

⁴ *ECONOMIC JOURNAL*, Vol. VII., above, §, p. 99.

⁵ *Loc. cit.*

⁶ It is discussed in the *Giornale degli Economisti*, 1897. Above, E.

⁷ Some considerations supporting this judgment will be found in the article on "Paradoxes of Competition," by Mr. Henry L. Moore, in the *Quarterly Journal for Economics*, Feb. 1906.

defending himself as Professor Marshall. I am concerned to notice only one point in the position which Professor Loria attacks. It appears, from the context of the passage in the *Principles of Economics*¹ to which Professor Loria adverts, that Professor Marshall is there dealing with market value in the sense in which it is contrasted with normal value. He adduces² a mathematical construction adapted to his purpose; which I notice here only because I am about to adapt a similar construction to a somewhat different purpose, contemplating not market value but normal value. I shall not suppose two sets of dealers to meet and settle their bargains at one go-off, so to speak. On the contrary, I shall suppose them to meet again and again under like initial conditions continually renewed, and to resume bargains until a system of contracts, not likely to be varied by recontract, has been set up. A conception of this sort³ is required for the full discussion of the topic to which I proceed, Professor Chapman's remarks on the "Remuneration of Employers."⁴

IV. Continuing my examination of mathematical theories contributed to the *ECONOMIC JOURNAL*, I come to Professor S. J. Chapman's article on the "Remuneration of Employers."⁵ Professor Chapman takes up a question which has been agitated in recent economic literature,⁶ whether the remuneration of an employer is to be regarded as just equal to his final productivity in the same sense as the remuneration of the workman, or more generally the payment for a unit of any agent of production. He brushes away⁷ the more extreme statements of this analogy which would oblige us to suppose the entrepreneur's work capable of being bought by the piece, like an ordinary commodity. As I understand, Professor Chapman rightly addresses himself to this issue: whether, if an additional employer is taken on, the total product of a society tends to be increased by an amount that is just equal to the normal remuneration of an employer. It is thus that the thesis has been conceived by one of its ablest supporters,

¹ *Principles*, Bk. V. ch. ii. Note on Barter.

² In his Mathematical Appendix.

³ I have endeavoured in the article on "The Theory of Distribution" in the *Quarterly Journal of Economics*, 1904, Vol. XVIII. p. 187 *et seq.*, to win conceptions appropriate to normal exchange value. [Vol. I., B.]

⁴ *ECONOMIC JOURNAL*, Vol. XVI. p. 523.

⁵ See *ibid.*, December, 1906.

⁶ For a *résumé* of authorities and arguments see the article on "The Theory of Distribution," Vol. I., B.

⁷ Chapman, *loc. cit.*, p. 528.

Professor T. N. Carver. "The law of marginal productivity," he says,¹ "can be applied to the earnings of business management as well as to the wages of other labour. The amount which any individual business man can get by means of his superior management (not through his superior bargaining capacity²) depends upon the amount which he can add to the product of the community over and above the amount which it would produce without his help."

Assuming that there exists only one industry, and making other simplifications which are legitimate with reference to the extremely abstract proposition under consideration, Professor Chapman supposes z employers, each managing a firm in which there are x employés, the total number of employés in each of the firms being constant, say c . The wage of an employé in any firm is the addition to the product of the firm which is made by taking on an additional employé; multiplying that final productivity by x , the number of employés in any firm, we obtain the total wages paid in any firm; subtracting the total wages from the product of the firm, we have (making a legitimate abstraction of other agents of production) the profits of an employer. Professor Chapman investigates the question, whether if the number of employers be increased from z to $z + 1$, the addition thus made to the total produce of all the firms will be just equal to the profits of an employer. He finds the new and remarkable result that this equation holds good, so long as we abstract the effect on the organisation of industry which may be produced by the introduction of an entrepreneur. It is a nice question how far, through what range of instances, it may be legitimate to neglect this effect. Professor Chapman is no doubt right in treating the effect as not negligible in general. Distinguishing the cases in which an increase of entrepreneurs "raises or lowers the curve of marginal value of labour to employers,"³ he finds that the

¹ *The Distribution of Wealth*, p. 263. Cp. p. 262:—"That is the amount which the community is able to produce with his [the business man's] help over and above what it could produce without his help, and this is the only sense in which any factor can be said to be productive."

² The parenthesis is explained by the remarks at p. 261.

³ He uses the hard-worked terms "increasing and diminishing" returns to distinguish these cases. The use of these terms is apt to breed confusion, because, as I have elsewhere pointed out (see Index, s.v. *Increasing Returns*), there are two essentially different meanings, according as that which diminishes (or increases) is (1) the rate at which product increases with the increase of means, or (2) the product divided by the means, the share of a unit factor of production in the product. A further sub-division is formed by the distinction between (a) the cases in which there is only one species of means, and (b) the general case of several kinds of factors. The definition of 1b is not obtained by mere

statement in question exceeds or falls short of the truth, according as one or other of these cases prevails.

The proposition, then, is not in general true. It might still, however, be useful if it were true in a typical case. It might then, in our ignorance whether it exceeds or falls short of the truth, be treated as the most *probable* general statement; upon the principle of a *priori*—or unverified—probability¹ which was adduced in a preceding section.

This sort of usefulness proves to be less than it appears to be at first sight, when, pushing the investigation up to first principles, we consider the labour market as a species of the general theory of exchange. The mathematical method of presenting this theory may, it is hoped, become more popular now that M. De Foville has recognised it by employing curves of Demand and Supply to explain the “mechanism of prices.”² M. De Foville would certainly not have diverged from the literary method in which he excels, unless, in his authoritative judgment, the advantages of the technical expression had justified the departure from classical usage.

The Corn-market, M. De Foville’s illustration, is not the only type of market to which curves of Supply and Demand are appropriate. They apply also to transactions in factors of production, such as the labour-market; and not only to “market value” in the sense of the term which refers to short periods, but also to “natural” or normal value, provided that the periods considered are not so long but that the dispositions, and “disponibilities” in M. De Foville’s phrase, may be supposed constant.³ We are to conceive two groups of dealers encountering each other, not once only, but from time to time, and ascertaining by repeated tentatives a rate of exchange at which a steady flow of trade is maintained.⁴ With respect to this kind of exchange we may say of the mathematical representation, in M. De Foville’s words,

composition from the simple case of 1a; the character of a *maximum* which distinguishes diminishing returns now involves an additional condition. Likewise 2b differs from 2a in requiring a principle of distribution among the different factors.

Professor Chapman’s use of the terms may be referred to the heading 2b.

¹ The use of the term *a priori* is unhappy so far as it is employed to mean, not only, as here intended, probabilities established by general presumption, without specific experience, but also “antecedent” probabilities which enter into the investigation of causes (*cp.* Mill, *Logic*, Book III. ch. xviii.) Probabilities which are *a priori* in the second of these senses are often, but not always, *a priori* in the first sense. [See Index, s.v. *A priori probabilities*.]

² In his last book, *La Monnaie*, p. 150.

³ A conception favoured by the stability of averages.

⁴ See Index, s.v. *Normal Equilibrium*.

“ This image, purely symbolical though it is, is good to keep in mind, because it tells us clearly and roundly (*nettement*) that in a free market there is nothing arbitrary in the formation of the prices.” That under stable conditions things tend to a definite level, is not the only lesson to be derived from the mathematical method. It is also employed to answer questions of this sort: If the conditions are disturbed in an assigned manner, in what direction will the level be altered? One example of such problems is afforded by the imposition of a tax; another example, by the addition of a new dealer on one side of the market—in the case before us a new entrepreneur put on the labour-market.

To start from first principles, let us suppose the market to consist of a set of dealers X_1, X_2 , etc., on one side, and another set Y_1, Y_2 , etc., on the other side; each X supplying commodity x in return for commodity y , and likewise each Y supplying commodity y in return for x . The attainment of a determinate level at which exchange is maintained is explicable by two principles: (a) contract, and (b) competition.¹

(a) In virtue of the first principle, if any X deals exclusively with only one Y , they will agree to vary the terms on which they deal up to a limit at which further variation would cease to be advantageous to one or other of the parties.² Even if one X is not restricted to dealing with only one Y , a similar statement still remains true with respect to the *final increments* disposed of by each X and Y .

(b) Now let competition be introduced. Then, as M. De Foville has it, “ we won't see (*on ne verra guère*) transactions concluded simultaneously on different bases. Why should Peter give up for twenty francs what Paul has just sold for twenty-five?” If each X is restricted to dealing with only one Y , and, conversely, the number of X 's and Y 's being equal, then the position of one X will be as good as that of another X , and likewise the positions of the Y 's will tend to equality. Yet the positions are not determinate.³

Now let us render competition perfect by removing this

¹ Cp. *Mathematical Psychics*, by the present writer.

² A point on the *contract-curve* relating to the two parties considered (*loc. cit.*).

³ If, for the sake of illustration, we suppose all the X 's to be of one type in respect of their dispositions or disponibilities, and likewise all the Y 's to be of one type (not the same as that of the X 's), then the system, which, under the supposition of the preceding paragraph, consisted of a set of points on the *contract-curve* between an X and a Y , is now reduced to a single point on the *contract-curve*. But that point is not determined without the condition of perfect competition, which is about to be introduced in the text.

restriction. Then the conditions of equilibrium will no longer be satisfied by an indefinite number of arrangements. For in general it will be possible for a dealer of one type or the other, *e.g.*, an *X*, say *Xr*, to offer his commodity *x* in small parcels to several *Y*'s on such terms that not only each of these *Y*'s disposes of a parcel of his *y* to greater advantage than before, but also the position of *Xr*, as defined by the total quantity of *x* which he gives, and the total quantity of *y* which he receives in exchange, is bettered.¹ This sort of disturbance will continue until an arrangement is reached in which every portion of *x* is exchanged for a portion of *y* at one and the same rate; a rate such that every *X* gets as much *y* as he is willing to purchase, and not more than he is willing to purchase, at that rate; and every *Y* is similarly satisfied.

This "symbolical image" is no doubt an artificially simplified representation of the actual processes by which a uniform rate of exchange comes to be determined. It is thus that, in a first view of the molecular theory of gases, the physicist is allowed to imagine a system of equal perfectly elastic spheres. If we are to compare our theory with the hypotheses of Mathematical Physics, we must admit that in the economic molecular theory there is wanting the cogency which is conferred by a nice adaptation of premises to conclusions. But it is worth considering whether our premises, human motives evidenced by consciousness and sympathy, do not possess the character of a *vera causa* in a higher degree than the foundations of some received hypotheses as to the constitution of matter.

The conditions which determine the equilibrium of the market are indicated by the curves *OP* and *OQ* in the accompanying figure. These are Demand and Supply curves, but not of the kind proposed by Cournot and accredited by M. De Foville. They are, rather, of the kind proposed by Professor Marshall in an unpublished pamphlet referred to in his *Principles* and in Professor Pantaleoni's *Principii*.^{*} In this, as in the more familiar system, one of the above elements represents the amount of a commodity supplied. But the other co-ordinate does not now represent a rate of exchange, but the amount supplied of another commodity. The explanation is most easily enunciated in the simple case in which one of the commodities, say *y*, is money.² Then any point

¹ The proof primarily applicable to the simple case may be extended to the general case.

^{*} The substance of the papers referred to has been published by Marshall in his *Money Credit and Commerce*, 1923.

² According to the construction of Messrs. Auspitz and Lieben.

P , on the curve OPA indicates that Op , where p is the foot of the perpendicular let fall from P on OX , is the amount of x supplied by the X 's at a price which is equal to Pp divided by OP ; a price which is assigned by the angle POX .¹ The amount of x demanded by the Y 's at the same price is in Or , where Or is the foot of the perpendicular let fall from R on OX . Rr may be described as the amount of money supplied by the Y 's at the given price; and Pp as the amount of money demanded by the X 's at that price. The intersection of the curves indicates the terms at which Supply equals Demand.

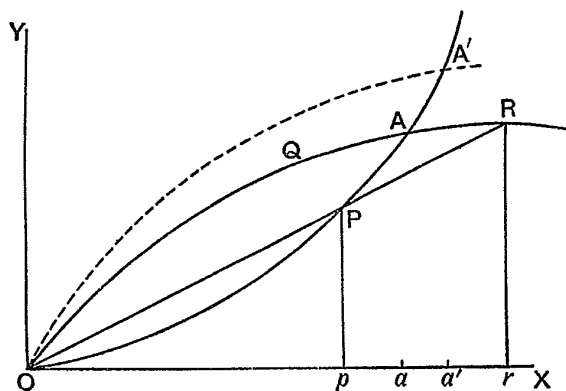


FIG. 1.

Now let us introduce the character of *Distribution*; let us suppose that the x offered by one party is work, and the y offered by the other party is wages paid out of product.² The most general conception would include the case in which not only each employer employs several employés, but each employé is free to work for several employers. But with reference to modern industry, it seems proper to suppose that while each master employs several men, no man can serve two, or more, masters. The limitation does not impair the essential characteristics of a market, provided that the number of the employers continues to be large. But the limitation is not entirely inoperative. For, as I have elsewhere argued,³ it is apt to obstruct the process by which

¹ Equal to the tangent of POX .

² *Theory of Distribution* (*loc. cit.*), p. 161.

³ *Giornale degli Economisti*, 1891. (Above, β .)

workmen competing against each other lower wages.¹ The incident seems to be of little practical importance, and to have no direct bearing on the question now at issue. But it may serve to render more conceivable the view which will presently be suggested: that between the position of the employé and the employer there may be an asymmetry to the disadvantage of the class usually supposed to be the most favoured—the employers.

Let us now disturb the equilibrium which we have contemplated, by introducing an additional employer. And at first let us abstract that effect on the organisation which Professor Chapman has particularly considered. The abstract proposition which is found by his reasoning when that effect is neglected is now to be reconsidered. That reasoning appears to presuppose that not only the total number of workmen, but also the total quantity of work done, is constant. But in general this is not to be supposed. The raised offer on one side of the market is apt to be attended with an increased offer on the other side. In our figure, let OA' denote the curve pertaining to the employers, varied by the introduction of an additional employer. Then A' is the new position of equilibrium. Oa' where a' is the foot of a perpendicular (not shown in the figure) let fall from A' on OX is the new quantity of work supplied; greater than the old quantity Oa . To be sure, the curve pertaining to the employés might be shaped so that Oa' would be less than Oa . But there is some reason to think that the shape represented in the figure is the more probable and typical. At any rate there is no presumption that Oa' is equal to Oa .*

The same result may be shown by way of symbols.** The product of each firm is now to be regarded as a function, not only of x the number of employés in the firm, but also of e the average amount of work which they put forth, an average which may be supposed the same for each firm. With e should be included, theoretically, another variable B , the work of the entrepreneur.

¹ Consider the analysis above given (p. 334). But observe that the number of X 's is there supposed to be equal to that of the Y 's. When each Y employs several X 's, the competition of these X 's against each other, the quasi-monopolistic position of the Y with whom they deal, no doubt neutralises the incident noticed in the text.

* Still less is there any presumption, if P was the total product before the disturbance and ΔP is the increment due to the taking on of an additional entrepreneur, that (as the disputed theory requires) $\Delta P = \frac{P + \Delta P - A'a}{n + 1}$, or

approximately $\frac{P - A'a}{n}$, where n is the original number of entrepreneurs.

** I have abridged the argument, which is somewhat confused as originally stated.

The work put forth by each employé is subject to the condition that the final disutility to him of the work is equal to the final utility to him of the remuneration. Whence it follows—account being taken of the maximum condition on the part of the entrepreneur to which production is subject—that the total output of a firm is determined as a function of x , the number of employés in the firm, and w , the rate of remuneration per unit of e . Now the product of x and z is by hypothesis constant. Whence it is deducible that the total product involves z , not only explicitly, but also implicitly in that x and w are functions of z . Accordingly, when the number of employers is varied from z to $z + 1$, the quantities which we have to compare receive variations depending on differential coefficients about which we have not in general sufficient knowledge to sustain the theorem in question. If it is still the most probable statement, yet it is less probable than appeared; less useful as a typical mean of possible cases.

It may be added that even if the theorem were accurately true, it would not have the importance attached to it by some writers; among whom, however, Professor Chapman is not to be included. This may be shown by considering a case in which Professor Chapman's abstract reasoning holds good without qualification. Let the entrepreneurs be cottiers, renting a homogeneous tract of land limited in extent. If an additional entrepreneur is introduced, the addition to the total product is found by parity of reasoning to be exactly ¹ equal to the remuneration of an entrepreneur.* But what of that? Where is the consolation to the cottiers whose complaint is that their share of the product is so small, that "this principle of remuneration is in itself an injustice."²

So it is no *eirenicon* between employers and employed to affirm that, according to the definition of Professor J. B. Clark, "every workman gets the product of his work." He is not thereby deterred from desiring more than what is his product according to that definition. "You may call it what you please provided you hand it over," the Socialists would say, as Mr. Cannan happily observes.³

On the whole I see no reason to modify the opinion that the theorem in question is neither quite true nor very important.

¹ That is, to within quantities which are negligible.

* See note at the end of this article.

² Mill, *Political Economy*, Book II, ch. i, § 4.

³ *Quarterly Journal of Economics*, "The Division of Income," May, 1905

If it could have been saved, it would have been by the more abstract part of Professor Chapman's brilliant reasoning.

(Note referring to p. 338.)

[Let $f(l)$ be the amount of produce which the cottier will produce (per unit of time) if he rents l units of land. This expression may be considered as derived from data connecting the amount of produce with several variable data, the amount of energy (measured objectively) put forth by the worker—which will depend partly on the rent which he has to pay—and any number of "gratuitous factors," arrangements which it is in the power of the worker to vary (*cp.* I, Vol. I. p. 298). Or is it simpler to suppose that there is determined by the Calculus of Variations the *form* which most advantageously connects the produce with the extent of land, the amount of work done on it and the rent per acre? Say we thus obtain for the worker's net advantage $U = F(l, e) - \rho l - \phi(e)$; where ρ is the rent per unit of land, $\phi(e)$ is the money-measure of the disutility attending the exertion e , F is the money-value of the produce (the price of which may be supposed constant). Differentiating U with respect to e and eliminating e with the aid of the equation $\left(\frac{dU}{de}\right) = 0$ from U , we obtain for U an expression of the form $U = f(l) - \rho l$. We may suppose a population homogeneous in respect of circumstances and disposition so that the above formula applies adequately to each member.

A perfect land-market being assumed—cottier-entrepreneurs in competition with each other dealing with competitive landlords—the rate of rent ρ will be so related to l , the holding of each cultivator, that for each (at any assigned rate) U shall be a maximum; that $f'(l) = \rho$. Suppose that the total extent of land available is limited ($= L$, say) and all of the same quality. Then if there be n cultivators, the portion cultivated by each will be L/n ; the value of the crop raised by each, $f(L/n)$; the rent (per unit of land), $f'(L/n)$; the gain of the cultivator, his producers' surplus, $f(L/n) - (L/n)f'(L/n)$. The total value produced will be $nf(L/n)$. Now let one entrepreneur be abstracted. The value of the total produce becomes then $(n-1)f(L/n) = nf(L/n) - \left\{ f(L/n) - \left(\frac{L}{n}\right)f'(L/n) \right\}$; fractions of the order $1/N$ being neglected. Thus the amount lost to the community by the abstraction of an entrepreneur is just equal to his remuneration.]