SECTION V

TAXATION

(S)

THE PURE THEORY OF TAXATION

[THIS article, published in the ECONOMIC JOURNAL, 1897, discusses the incidence of taxes, and the criterion of a good tax system, the rule according to which the burden of taxation ought to be distributed among the tax-payers. For the first purpose there is introduced a new classification formed by four dichotomic cross divisions. The leading case defined by the first, or positive, attribute of each division is, contrary to classical tradition, international trade, including dealings among non-competing groups within the same country. The incidence of taxation in this case having been already considered generally (Sect. IV.), there are here discussed only special cases. One such occurs when there is perfect inelasticity on one side of the market, or even on both. Other peculiar cases arise when commodities are *correlated* in respect of Consumption or Production. The genus correlation includes two species, "rivals" or mutual substitutes, when an increase in the possession of one renders the possession of the other less desirable, or an increase in the production of one renders the production of the other more difficult; and "complementary" articles with converse properties. It is shown that (even in a regime of competition) when demand and production are complementary, a tax on one may cause the price of *either* article to fall; with advantage to the consumers as a whole. It might have been added that if the commodities are rivals both in production and consumption, a tax on one may cause the price of *both* to fall. This *curiosum* does not depend on a change in the marginal value of money. The ordinary assumption that the total utility of consumption and cost of production is measured by money of stable value is throughout retained. Corresponding to the last-named paradox is the incident that when both supply and demand are rival, the producers may benefit by the imposition of a tax. *Both* prices may be raised; that of the taxed article to an extent in

excess of the tax. A somewhat different paradox—a tax on both commodities (correlated as partial substitutes for each other) benefiting the producers of one of them—is instanced in the text. The instance is furnished by house rent; a topic which is treated at length as illustrating the effects of taxation where mobility of capital and labour acts. The effects of an impost like the Inhabited House Duty vary greatly with the length of the period under consideration.

Monopoly is treated on much the same lines as, but in less technical terms than, in the article of the same date (1797) in the *Giornale* (above, **E.**).

The *curiosa* of the subjects are treated more fully there and in subsequent Papers (see Index, *sub voce* "Paradox"). Attention is called to a peculiar species of semi-monopoly. As mentioned above (**D**), it is one of the concrete cases in which differential prices may prove advantageous to both consumers and producers.

In the latter part of the article there is advocated as the criterion of good taxation, minimum aggregate sacrifice, distinguished from equal sacrifice in the obvious sense of the term, or proportional sacrifice, as the peculiar conception of Professor Seligman and Cohen-Stuart may be called.

The principle of *minimum* sacrifice is now very generally accepted, praised by Cannan, and used by Marshall (see ECONOMIC JOURNAL, 1921, p. 350, and 1917, p. 407). Professor T. N. Carver, who was the first to propound this doctrine, has exhibited its application with convincing clearness in his last work, *National Economy*.]

The science of taxation comprises two subjects to which the character of pure theory may be ascribed: the laws of incidence, and the principle of equal sacrifice.

The first subject presents a varietyof distinct cases demarcated by several cross divisions. Of these divisions the following four appear to me the most important for the purposes of theory:—

Either (A) all the transactions [1] under consideration are exposed to competition; or (a) among the parties with whom we are concerned there is at least one monopolist.[2]

[1] I suppose in each case parties to an exchange, the play of demand and supply. Taxation in a regime of socialism or of slavery is not considered.

[2] I understand by a monopolist an individual, or a combination, having the sole control of an article of exchange, and dealing with it solely in the interest of the monopolist. I agree with Professor Walras in thinking that much confusion has been caused by extending the term to cases in which a commodity absolutely

Either (B) all the products with which we are concerned obey the law of increasing cost; or (b) some do not.[1]

Either (C) the mobility of capital and labour [2] is not taken account of, or (c) exists and is taken account of.

Either (D) the taxation considered varies with the quantities of articles exchanged (including money, as in the case of a specific or an *ad valorem* tax, or one in kind), and so may be described as a tax on *margin;* or (d) it does not so vary (as in the case of a tax on profits, or a poll-tax), and so may be described as a tax on *surplus.*[3]

I proceed to consider the more important of the cases formed by the combination of these attributes, giving priority to the first member of each division, the one designated by a capital letter. According to the order adopted, the case first to be considered is that which is defined by taking the first member of each division and which may accordingly be designated as A B C D; indicating that (A) the parties considered consist of two or more groups, the members of each group supplying the same article [4] in competition with each other; (B) each additional increment of every product is obtained by a more than proportional increase of outlay; (C) the groups are "non-competing" in Cairnes'

limited, such as land of a certain sort is in the hands of a *plurality* of uncombined possessors (*Éléments d'Économie Politique*, 2nd edition, Art. 408. Cp. *Dictionnaire d'Économie Politique*, Art. "Monopole"). As to the definition of maximum advantage in the case of a *combination*, see the present writer's article on "The Pure Theory of Monopoly" in the *Giornale degli Economisti* for 1897.

[1] I define the laws of increasing and decreasing cost thus. If $\phi(x)$ be the expense—or more generally the equivalent in money of the "real cost"—of producing the quantity x of a certain commodity, the law of increasing cost holds, when $\frac{d_2\phi}{dx^2}$ is *positive;* the law of decreasing cost, when $\frac{d_2\phi}{dx^2}$ is negative (*cp.* Cournot, *Principes Mathématiques*, Art. 29). Generally if $\phi(x, y, z \ldots)$ is the cost of producing the quantities, $x, y, z \ldots$ of several commodities, the law of decreasing returns does or does not hold, according as the second term of variation of ϕ does or does not fulfil the conditions of a *maximum.* "Decreasing and increasing returns" will be here used as synonyms of increasing and decreasing cost. For fuller explanations and variant definitions, see C.

[2] As mobility may exist with respect to some—but not all—of the agents of production (*cp.* article on "International Values" in the ECONOMIC JOURNAL, Vol. IV. p. 35), the more exact distinction might be between (C) a greater and (c) a less degree of mobility.

[3] For certain theoretic purposes it might be better to distinguish the cases in which the tax (D) strikes the variables by the variation of which the parties under consideration seek each his maximum advantage; or (d) strikes the quantity which it is sought to maximise. The distinction between *margin* and *surplus* hovers between this one and the one in the text. (See "Margin," *Palgrave's Dictionary.* *Cp.* below, p. 76.)

[4] Or articles in the case of joint or more generally correlated production (below, p. 72).

sense, "industrial competition" is not supposed to exist; (D) the tax is of the same genus as an export or import tax.

A B C D. The case thus defined is nearly coincident with the case which I have discussed in a former article; that of an export or import tax on an article of international trade; understanding international trade in the generalised sense of "exchange without mobility." [1] Following Mill, we may begin with the simplest variety where there are only two "nations." The case as conceived by us comprises not only international trade (in the proper sense), say between two islands isolated from the rest of the commercial world, but also a simple abstract market, such as the corn market, of which Professor Marshall has described the "temporary equilibrium" [2] or his ideal nut and apple-market; [3] also the dealings by which the shares of the parties in distribution are determined, the labour market, the loan market, the land market, each considered at first abstractedly by itself, and not yet in its true interdependence with the others.[4]

A tax of the kind now under consideration, affecting such a market, will in general prejudice both parties more or less. If, in the metaphor of a distinguished economist, we represent the undisturbed relation of the parties by the equilibrium of two balls resting against each other in a bowl, it may seem, at first sight, that a wedge inserted between the two balls will raise one of them to the full extent of the thickness of the wedge. But on reflection it is evident that this only occurs in the limiting case when the mass of one ball may be neglected in comparison with that of the other. In the absence of data respecting the relative masses of the balls all we can say is that the distance between them will be equal to the thickness of the interposed lamina. Corresponding to the masses of the two balls are the elasticities of demand and supply for the two parties. The general principle is that the tax inflicts more loss on either party, the less the elasticity of that party's demand or supply; other things, including the other party's elasticity, being the same.[5]

[1] Above, R, p. 5.

[2] *Principles*, Book V. ch. ii., § 1.

[3] *Ibid.*, § 1, *note on Barter* (latter part).

[4] One of the best, and I believe the first statements of the simultaneity, in the mathematical sense, of the several equations pertaining to value and distribution is given by Professor Walras in his *Éléments d'Économie Politique Pure*.

[5] When, as in my Articles on International Value, we make abstraction of money, and consider *price* in the generalised sense of M. Walras, *i. e.* rate-of-exchange, then it is unnecessary to distinguish the elasticity of supply from that of demand. The less the extension of the demand attending a fall of price, the

This proposition has been demonstrated at length in former articles.[1] It must suffice here to add some remarks suggested by an examination of certain extreme cases.

An instance of infinite elasticity of supply is afforded by the labour market upon the Ricardian hypothesis that, in Mill's words, " there is everywhere a minimum rate of wages, that they can never be lower beyond the length of time required for a diminished rate of increase [of population] to make itself felt, and can never long continue higher." [2] Upon this assumption, it is " hypothetically true " that a tax on wages would not permanently rest on the working classes; [3] a conclusion which is justly regarded as the opprobrium of pure theory, if it is applied to justify a tax on wages or on the necessities of the wage-earner. We have, however, Mill's authority for saying that " the assumption contains sufficient truth to render it admissible for the purposes of abstract science." [4]

It should be observed that this perfect elasticity of the supply of labour is predicated only of long periods; for short periods to evoke more work there would presumably be required a higher rate of wages. A similar difference in respect of elasticity between long and short periods is to be noticed in other markets. Thus, according to Professor Seligman, " an equal tax on all capital must fall on the lender, that is the capitalist. There would be no way for him to shift the burden." [5] But he admits that further accumulations might be discouraged. *Pro tanto* then the rate of interest in a long period would be increased.[6] Thus, too, we may

less is the extension of supply attending a rise of price. After the point at which demand becomes perfectly inelastic the elasticity may be said to become *negative*. This is the case alluded to in the criticism of Messrs. Auspitz and Lieben (above, R, p. 59) as not adapted to a curve which represents the variations of supply with money-price.

[1] The general principle is well stated by Professor Carver in his article on " Shifting of Taxation " in the *Yale Review* for November 1890.

[2] *Political Economy*, Bk. II. ch. xi. § 2.

[3] See Mill's application of the principle, *Political Economy*, Book V. ch. iii. § 4, par. 4.

[4] *Cp.* Adam Smith on taxes upon the wages of labour and the necessaries of life (*Wealth of Nations*, Book V. ch. ii.). McCulloch's remarks on these passages (McCulloch's edition of Adam Smith, Vol. IV. note xxiv.) seem just; his own views (*ibid.* p. 544) human. On this point Professor Seligman, as always where *friction* is the subject, is instructive (*Shifting and Incidence*, p. 174). Among the numbers of other writers who might be referred to, Professor Bastable may be distinguished (*Public Finance*, pp. 358–60, and 436, 2nd edition).

[5] *Shifting and Incidence*, p. 132. Cp. *Wealth of Nations*, Book V. ch. ii: " a tax upon the interest of money could not raise the rate of interest; the quantity of stock or money in the country . . . being supposed to remain the same." But it would not remain the same (*ibid. infra*).

[6] *Cp.* Bastable, *Public Finance*, Book III. ch. v. § 7.

partly account for Mill's statement respecting the " attempt to tax all purchases and sales " that " neither class [buyers or sellers] could throw the burden upon the other." [1] This is true for instantaneous periods, at least of sellers, so far as they are under the necessity of selling what they have brought to market. But can it be affirmed in general of a tax like the Spanish *alcavala*, that " if levied from the sellers " in the long run it would burden sellers more than buyers ? [2]

The difference between the elasticity of supply according as short or long periods are considered is conspicuous in the case of houses.

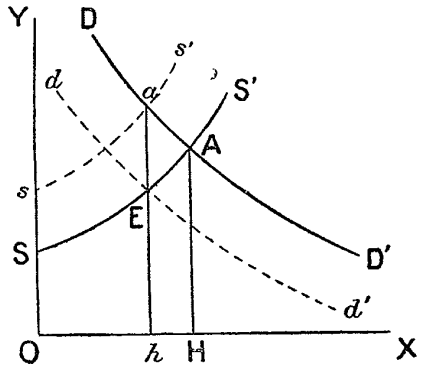

FIG. 11.

For times so short and in places so limited that the number of houses offered may be regarded as a fixed quantity,[3] a tax on house rent, whether imposed on the occupier or owner of the house, is in general borne altogether by the owner. This conclusion of the older economists [4] is verified by the newer methods.[5]

[1] *Political Economy*, Book V. ch. v.

[2] The effect attributed to a " tax on all commodities " by Mill in an earlier passage (Book V. ch. iv. § 1, par. 2) would require a long period.

[3] The case of a commodity of which the quantity cannot be increased may be regarded as a limiting case of one which can only be increased at an increasing cost; and so belongs to our class B.

[4] Mill, *Political Economy*, Book V. ch. iii. § 6, par. 3; Ricardo, *Political Economy*, ch. xiv., first two pars.

[5] In the accompanying figure S S' and D D' are taken as, in Professor Marshall's phrase, " the typical diagram for stable equilibrium for a commodity that obeys the law of diminishing return " (*Principles of Economics*, p. 425, ed. 3; *cp.* p. 524). A is the position of undisturbed equilibrium, O H is then the supply. When equilibrium is disturbed by a tax (of the kind now under con

Some confusion appears to be caused by supposing the law of demand to alter concurrently with the imposition of the tax.[1]

sideration) on the producer, the supply is reduced to O h; h E is the price received by the producer, E a the tax paid per unit of commodity (Marshall, *loc. cit.*). The figure shows that, if the tax is levied from the consumer, the result is the same. For $d\,d'$, the demand-curve as displaced by the tax, strikes the original supply-curve in E. This theorem is given by Professor Carver in his article on "Shifting of Taxes" in the *Yale Review* for November 1896 (compare Auspitz and Lieben, *Theorie der Preise*, Art. 82).

Fig. 12 represents the two limiting cases of this theorem. S S′ is the perfectly elastic curve of constant cost, $s\,s'$ the same displaced by a tax, as in Professor Marshall's Fig. 33; $d\,d'$ has the same import as in the last paragraph.

In the other limiting case, when the supply is perfectly inelastic, let it equal O κ. Then $\kappa\,\alpha$ is the supply-curve. If it is imagined as sloping a little outward, the limiting form not quite reached, the effect of a tax on supply would, as before,

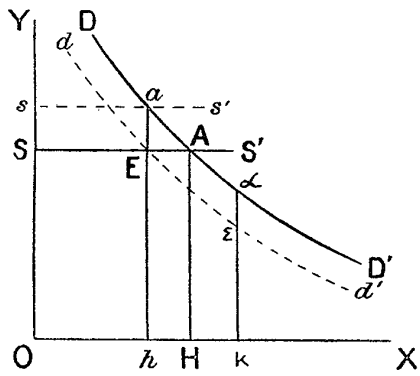

FIG. 12.

be represented by moving (every point of) the curve vertically upwards through a distance corresponding to the extent of the tax. The intersection of this displaced supply-curve—not shown in the figure—would cut the demand-curve in the neighbourhood of a, and accordingly the price paid by the consumer is nearly—in the limit quite—the same as before; the whole tax falls on the other party.

But it is simpler to use the theorem that it comes to the same whether the tax is on supply or on consumption. In the latter case, if $d\,d'$ is as before the demand-curve displaced by the tax, $\kappa\,a$ the price paid by the consumer is unaltered, the whole tax falls on the other party.

Compare Fleeming Jenkin: "If a holder sells unreservedly . . . the whole tax falls on the seller; the supply curve becomes a vertical straight line" ("Incidence of Taxes," p. 114 of *Papers Literary and Scientific*).

Or is it easier to say that, if with Cournot (*Principes Mathématiques*, Art. 51) we represent the equality of *demand* and *supply* before the tax by the equation $F(p) = \Omega(p)$ and after the tax (of u per uni imposed on the supply) by the equation $F(p') = \Omega(p' + u)$; then if Ω is degraded to a constant the equation for p' the disturbed price is the same as the equation for p the original price.

[1] Thus the Report of the London County Council Committee (Lord Farrer,

Is it not competent to the "mechanics of industry" to treat superposed disturbances independently and one at a time? If a person wears high heels, may we not estimate the elevation due to that cause without putting him on a hill? If indeed there is some connection between the artificial elongation and the position of the wearer, it may be proper to note this. Xenophon tells us that the great king alone among the ancient Persians wore his tiara erect. If then the king—as according to Dryden, the conqueror—of the Persians sate "aloft in awful state," the apex of the royal tiara would have been elevated both in itself and on account of the wearer's position. Yet Xenophon's statement is intelligible by itself. So rates on houses when expended in improving the neighbourhood tend to increase the demand for houses.[1] Yet in measuring the burden of the tax to the owner it is allowable in pure theory to abstract its influence on demand.

Another reflex influence of a house-rate on the demand for houses already built—reflected from the quarters where new building is possible—as it presupposes the mobility of capital, must be deferred to a later section. At present we are supposing the offer of built houses to be constant—the *fourth* of the cases so lucidly distinguished and discussed by Mr. Pierson in the second edition of his *Leerboek*.[2]

When it is affirmed that under these circumstances the burden of the tax falls altogether on the owner it is understood that the demand of the occupant is of an ordinary kind—not of that extreme or limiting variety which is perfectly inelastic. The contrary assumption is made by some writers; Mr. Blunden, for instance, who puts houses in the category of those "absolute necessaries

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chairman) concludes that in prosperous communities house-rate falls on tenants; in declining ones on landlords. Similarly the *Dictionnaire d'Écon. Pol.*, art. "Incidence de l'Impôt." So Lord Farrer in his evidence before the Town Holdings Commission (Q. 1,244): "The best authorities seem to think that it depends very much on the state of the market." If it is asserted that the incidence of a tax depends on whether the demand is rising in the sense of the demand-curve being raised as a whole, I altogether dissent; if it is meant that the incidence depends on whether the demand becomes more urgent in the sense of the demand-curve becoming steeper, I give only a qualified assent (see p. 71). It is too true that the "best authorities" express themselves carelessly. Pantaleoni forms a brilliant exception when he explains that a rise of rents does not mean shifting of tax (from the owner to the occupier) if the rents would have risen independently of the tax (*Teoria della Traslazione dei Tributi*, p. 226 *et seq.*).

[1] Mr. Fletcher Moulton, in his evidence before the Town Holdings Commission, has dwelt forcibly on this incident.

[2] Noticed in the Review III. 78.

of life" of which the "prices may rise considerably without appreciably affecting the demand." [1]

No doubt it is so in particular instances, for instance, in the case of the dwelling-houses of the labouring classes in certain localities.[2] But can it be affirmed generally that the demand for dwelling-houses is perfectly inelastic? "If the tax, indeed, was very high," says Adam Smith, "the greater part of people would endeavour to evade it as much as they could by contenting themselves with smaller houses." And even if the tax be not high, is not a consequence similar in kind, if less in degree, to be apprehended by the owner who offers it for hire?

In fine, even granted the premiss that the demand for houses is inelastic, the conclusion that the tax falls wholly or chiefly on the occupier does not follow. The supply of houses (already built) being, as here supposed, also inelastic, the price or rent becomes *indeterminate*.[3]

The extreme cases which have been instanced form rather limits than exceptions to the rule that both sides of the market suffer by a tax. An exception is presented by a species of export-tax analysed in a preceding article; [4] the abstraction of a certain portion of the exports in kind, to be disposed of in a manner not affecting the market under consideration.[5] An instance would be the virtual export-tax which is imposed by the capture of

[1] *Local Taxation and Finance*, p. 49. Compare the author's recapitulations of his views in the *Journal of the Statistical Society* for December 1896.

Similarly Professor Seligman: "The landowner is not compelled to part with his land, but the tenant is compelled to occupy some apartments" (*Shifting and Incidence*, p. 111). Elsewhere, indeed (*ibid.* p. 120), he admits that the tax might be "so high as to cause the tenant to content himself with meaner apartments, or rooms in a less desirable locality."

I regard it as the general case, that the tax on the occupier *tends* to diminish his demand for house accommodation. Thus Mr. Bourne, steward of the London estates of the Duke of Bedford, affirmed, "with the greatest confidence," "from the knowledge that I have of everyday work for many years in London," "that the person taking the house is so free in his choice, that he can afford to throw up the houses when he takes into consideration what the rates and taxes are" (Town Holdings Committee, 1887. Q. 11,288–9).

[2] *Cp.* Cliffe Leslie, *Taxation of the Working Classes.*

[3] The intersection of two coincident perpendiculars!

What the actual effect of a tax under such conditions will be would seem to depend on circumstances which from the point of view of pure theory may be called accidents; among which no doubt the circumstance whether the demand is rising or falling (above, p. 70) may in practice be important.

[4] Above, p. 38.

[5] The condition is stated with much precision by Cournot with respect to the taxation of *monopolies*: "Il peut se faire que le produit de l'impôt en nature soit appliqué à une consommation qui n'aurait pas eu lieu sans l'impôt, et qui n'influe en rien sur la demande que les autres consommateurs font au producteurs" (*Principes Mathématiques*, Art. 42).

smuggled goods; the intercepted goods being destroyed, or so disposed of as to produce the same effect on the demand and supply in the two countries as if they were destroyed. It is not contended that the exception is of any practical importance.[1]

Another class of exceptions comprises what Mill has called "peculiar" or "anomalous" cases of value.[2] Such is the case of "joint production," as defined by Mill, when "the same outlay would have to be incurred for either of the two [commodities] if the other were not wanted or used at all." Akin to this case is that in which the increase of the production of either commodity, though it does not necessitate, yet facilitates, the increased production of the other.[3] I propose to call products connected by this relation, which I have elsewhere defined more precisely,[4] *complementary*.

If we suppose the degrees of complementariness to be gradually diminished, we shall pass through the zero point of absolute independence to a relation which may be distinguished as *rival* production; when the increased production of one commodity renders the increase of the other more difficult. For instance, where a limited amount of time, strength, or other resources may be spent in either of two sorts of otherwise unconnected production.

The following propositions respecting the taxation of products correlated in either of the two ways just defined may easily be proved; it being supposed that the demand for one commodity is independent of the demand for the other. A tax upon one of two rival products will raise the price of both. A tax on one of two complementary products will raise the price of the taxed one, and lower the price of the untaxed one. In the latter case, it is conceivable that the consumers as a whole might be advantaged by the tax, if we may set the gain of one class against the loss of another.

The gain and loss to be balanced would appertain to the same persons in the corresponding case of correlated demand. The demand for two products may be called *complementary* when a rise in the price of one is attended by a fall in the price

[1] As this kind of tax is in practice rare, I have to acknowledge that I have, in a preceding article (ECONOMIC JOURNAL, Vol. IV.) exaggerated the asymmetry between export and import taxes; and to retract my criticism of Professor Bastable on that point (*ibid.* p. 624). *Cp.* above, R, pp. 3, 12, 38.

[2] *Political Economy* Book III. ch. xvi. *passim*, and last par.

[3] *Cp.* Marshall, *Principles*, Book V. ch. vi.

[4] *Giornale degli Economisti*, 1897. E, Vol. I.

of the other, *rival* when a rise in the price of one is attended with a rise in the price of the other.[1]

The following propositions respecting the taxation of commodities for which the demand is correlated may be proved. A tax on one of two rival commodities will raise the price of both. A tax on one of two complementary commodities will raise the price of the taxed one, and lower the price of the one which is not taxed. It is conceivable that the latter effect should so exceed the former that, on balance, a gain results to the consumers.

The possibility of a positive gain resulting to one side of the market—one of the two "nations"—from the imposition of a tax [2] is more evident in the case of commodities which are complementary, both as regards production and consumption.* In this compound case it may be shown—but not, I think, very easily, perhaps not without the use of mathematics—that a tax on one commodity may lower the price of either, but not of both.[3]

[1] *Ceteris paribus*, and in particular the marginal utility of money being supposed constant. I have used a more essential attribute for the definition of *rival* and *complementary* demand in my paper on "Monopoly" already referred to.

[2] Exclusive of the gain accruing from the tax to the importing country, a gain which must in general be included in order that an import-tax may result in a net gain to the importing country; as maintained by Messrs. Auspitz and Lieben (*Theorie der Preise*, Art. 81), and by the present writer (ECONOMIC JOURNAL, Vol. IV.).

* For a fuller statement see the added note at the end of this Paper.

[3] x and y being the quantities purchased, consider the collective total utility (the *Gesammtnutzlichkeit* of Messrs. Auspitz and Lieben), and also the collective total cost (the *Gesammtkoste* of the same authors), each as a function of x and y. Before the tax, the price of the first commodity = its marginal utility (*i. e.* the differential of the total utility with respect to x) = its marginal cost (*i. e.* the differential of the total cost with respect to x). The price of the second quantity is similarly determined. After the tax—which may be at first supposed small and specific, say u per unit of x, and levied from the producer—if x' and y' be the new quantities then (1) marginal utility of x' = the marginal cost thereof + u; (2) the marginal utility of y' = its marginal cost. Substituting $x + \Delta x$, $y + \Delta y$ for x' and y', expanding and neglecting higher powers, we obtain two simultaneous linear equations for Δx and Δy. Solving these, we can find the increments of the prices and the decrement of Consumers' Rent, in terms of three kinds of data: (1) the extent of the tax, (2) the rate of decrease of utility and the rate of increase of cost, and (3) the measures of the correlation between the two commodities in demand, and also in supply (the second differential with regard to x and y of the utility-function, and also that of the cost function). These magnitudes must comply with certain conditions; but those conditions are not inconsistent with the statements in the text. But, if only one of these correlations exists, though the price of the taxed commodity cannot fall, yet the Consumers' Rent may rise.

By parity of reasoning it may be shown that though in the case of a single commodity, "if the commodity obey the law of diminishing return . . . the result [of a tax] will be to raise the supply price by something less than the full amount of the tax" (Marshall, *Principles*, Book V. ch. xii. § 4), yet in the case of

Our estimate of the importance of these exceptions to the rule that neither party gains by a restriction of trade depends partly on the question whether the " peculiar cases " are frequent. According to Jevons the cases of joint production, " far from being ' some peculiar cases,' form the general rule, to which it is difficult to point out any clear or important exceptions." [1]

However that may be, the exceptions which have been adduced do not militate against the general rule considered as expressing the most frequent, the typical case. In all the varieties of correlated demand and supply it is still true that most frequently the price of the taxed commodity will be raised, while the price of the correlated commodity will as often be raised as it will be lowered in consequence of the tax. Whence it follows that the cases in which a balance of gain results to one party are a minority.

In these examples we have insensibly passed the frontier, not very important for the present purpose, which separates the case of two " non-competing groups " from that of several. We may now restore to the various markets involved in " Distribution and Exchange," the interdependence which we at first abstracted. We may now suppose a whole system of countries connected by international trade.

The reader may be referred to a former article for a discussion of this general case—the case of several balls in the bowl. It may be well to remark that when in equilibrium one ball presses against another, and that other against a third, it is not in general indifferent between which two balls a wedge shall be inserted. For example, suppose three islands, A, B, C, engaged in this sort of international trade. A imports from B goods, for

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correlated commodities it is possible that the result of a tax on one may be to raise its price by more than the full amount of the tax; that though in general, the producers' surplus is diminished by a tax, yet in the case of correlated commodities it may be increased. The negative case of this paradox is, that a bounty may prejudice the bountied parties (directly and apart from ulterior effects, and from the cost to their Government).

What has been proved of a small specific tax may be extended (by neglecting higher powers of small quantities) to *any* small marginal tax (increasing with the increase of the commodity). What has been proved for an indefinitely small tax may be extended to a finite tax by reasoning which Cournot has made familiar. (For further explanations see my article on " The Pure Theory of Monopoly," in the *Giornale degli Economisti*, E, Vol. I.)

[1] *Theory*, p. 217. *Cp.* Preface, p. liii. Jevons is speaking of " joint " products in the narrow sense above attributed to Mill. If Jevons is right in using such strong language (which I am disposed to doubt), then *a fortiori* with reference to the wider category of goods that are *complementary* either in production or consumption.

the manufacture of which B has to import materials from C.[1] An import tax in A (or an export tax in B) on the goods exported from B to A will not come to the same as an import tax in B (or an export tax in C) on the materials imported by B from C. As an extreme case, suppose that the materials imported from our island C are supplied there yearly in constant quantities independently of human effort—*e. g.* seaweed deposited on the shores of C. A tax on the price charged by inhabitants of C for permission to inhabitants of B to remove this seaweed would fall altogether on the inhabitants of C; the price of the goods imported from B into A would not be affected. But a tax on these latter imports would be followed by a rise in the price of those imports, and a fall in the price of the materials imported from C; all three parties will be worse off—in general, and except in the limiting case in which the demand in A for the imports from B is perfectly inelastic; in which case the entire burden of the tax will fall on A, B and C will be unaffected.

A B C d.[2] The possibility which has been shown in the preceding section, that a tax upon products may be in part shifted by the producer, even though he has not the power of changing his occupation, no longer exists when the tax is imposed on profits, or generally *surplus*.[3] The case is not now that of a wedge inserted between two balls in a bowl; it is rather as if the position from which one of the two balls was started to run down to equilibrium was lowered. The height at which it would finally settle would not be altered by this abbreviation of its descent to equilibrium [4] (the bowl being supposed spherical). The

[1] Compare Professor Carver's correct decision on the case of a tax that is placed upon an article on its way through the hands of a merchant from the producer to the consumer. (*Yale Review*, Nov. 1896.)

[2] See the explanation of these symbols above, p. 65.

It would have been agreeable to classical tradition to place in this section the theorem, that a tax on rent falls entirely on the landlord. Thus James Mill: "To him [the capitalist cultivator] it is a matter of perfect indifference whether he pays the surplus in the shape of rent, to an individual proprietor, or in that of revenue, to a government collector" (*Elements*, chap. iv. § v. par. 1). So Florez Estrada, Book IV. chap. v.; Professor Seligman, *Shifting and Incidence*, p. 35 and p. 184, and many other authorities.

Yet in spite of the almost universal practice, I venture to think that there is some advantage in the classification here adopted. It may be observed that though under a regime of competition, a tax imposed upon the payment for an article absolutely limited in quantity, such as land, may be viewed as falling either upon margin or surplus, it is otherwise in a regime of monopoly: the tax is there certainly marginal.

[3] Compare Hadley, *Economics*, 512, 3.

[4] Energy representing total utility by a metaphor familiar to the mathematical economist. *Cp.* Irving Fisher's *Mathematical Investigations*, Part II. ch. iii. (*Mechanical Analogies*).

conditions of economic equilibrium are not affected by a tax on surplus.

This is the first approximation. But it must be remembered that in general it is not possible for the tax-collector to hit a surplus which is altogether "intramarginal." A tax on profits—such as Schedule D of our Income Tax, or such as a payment for a licence to carry on a trade—cannot be levied without some little disturbance of economic margins. This proposition might be illustrated by considering the classical theorem that the remission of rent to all farmers would not lower the price of corn. That is the first approximation. But if the farmers' "margin of saving" was displaced by their increased income, they might be willing to invest more capital in agricultural improvement, and so lower the marginal cost of produce.[1] Contrariwise there might be now required a higher rate of remuneration to evoke the same exertion from the cultivator; his new affluence having displaced the margin at which the decrements of the utility of consumption become equal to the increment of the discommodity of labour.[2] If with Jevons, or still more elegantly with Gossen,[3] we represent that margin by the point along a line at which the ordinates to certain two curves become equal, it will be evident that neither tax on profits, nor poll tax, nor licences, nor any other form of impost under category d will be able to reduce the area representing surplus, without disturbing its boundary.

Some little disturbance of this kind is to be attributed to an income tax, in so far as it strikes the shareholders in a joint stock company. But in so far as it strikes those who are entitled to a fixed payment from the proceeds of a going concern, it affects economic margins only in so far as the reduction of income may cause an alteration in the consumers' scale of demand.[4]

To the present section belong also consumers'—as well as producers'—licences. A tax on licence to consume a thing differs in its effect from a tax upon the thing, when more than one unit of the thing are, or would be in the absence of taxation, consumed

[1] *Cp.* Ricardo, *Political Economy*, chap. viii.: "There are no taxes which have not a tendency to lessen the power to accumulate."

[2] Against the probability that taxation will diminish accumulation, there is the possibility that "curtailment of profit may act as a stimulus" (Mill, Book V. chap. iii. § 3). A very bare possibility, according to Bastable (*Public Finance*, Book III. 2nd ed.). For the cognate doctrine that the impoverishment of the labourer will act as a stimulus, see the apt quotations at p. 16 of Professor Seligman's *Shifting and Incidence*.

[3] See Palgrave's *Dictionary*, Art. "Gossen," Fig. 3.

[4] The effect of changes in income upon prices is well analysed by Professor Irving Fisher in his *Mathematical Investigations on Prices*.

during the period within which the licence must be renewed, say a year. If no sportsman wanted more than one gun a year, the effect of a sporting licence in checking demand would be much the same as that of a specific tax on guns. But the licence to drink tea for which, as Adam Smith tells us,[1] people used to pay so much a head in Holland, would act differently from a tax of so much per pound on tea. It would be a tax on surplus. It would knock off all those consumers who do not derive from the consumption of tea a consumers' rent or surplus more than equivalent to the payment of the licence. On the remaining consumers it would act simply as a tax on their income.

A B c.[2] Let us now remove the barriers which have so far been supposed to separate our "non-competing groups." Let us introduce that mobility of the agents of production which is the essential attribute of domestic as distinguished from international trade, which is an important property of long periods as distinguished from short ones. Admitting the classical hypothesis respecting the freedom of capital and labour, we must accept the classical theorems concerning the effects of taxation: that, in the words of Adam Smith—

"A tax . . . upon the profits of stock employed in any particular branch of trade can never fall finally upon the dealers . . . but always upon the consumers, who must be obliged to pay in the price of the goods the tax which the dealer advances" (*Wealth of Nations*, Book V. chap. ii. art. 2).

In the words of Ricardo—

"A tax on the profits of the farmer would raise the price of corn; a tax on the profits of the clothier, the price of cloth" (*Principles*, chap. xv. par. 3).

In the words of J. S. Mill—

"If a tax were laid on the profits of any one branch of productive employment, the tax would be virtually an increase of the cost of production, and the value and price of the article would rise accordingly; by which the tax would be thrown upon the consumers of the commodity, and would not affect profits" (*Principles*, Book V, chap. iii. § 2, par. 1).

I do not know that these expressions can be improved upon. Yet as the attempt to paraphrase our literary classics, which is sometimes made a school exercise, however feeble in itself, brings out more fully the inimitable excellence of the originals, in the

[1] *Wealth of Nations*, Book V. ch. ii.

[2] The category thus designated comprises both A B c A and A B c d.

like humble spirit it may be allowable to expand the above cited authoritative dicta.

As I understand the " industrial competition " with which we are now concerned, the conditions of equilibrium are twofold—(1)[1] one common to the " commercial competition," which was supposed to exist in our first two sections, namely, that in any business the outlay in every direction should be pushed up to the " margin of profitableness," [2] and (2) one which forms the differentia of industrial competition, namely, that the " net advantages " in all businesses between which there is mobility should be equal.[3]

Now let a tax on profits disturb the second condition. If equilibrium is restored by the consumers being " obliged to pay in the price of the goods the tax," it follows from condition (1) that the marginal costs of the business taxed must be raised.

This would, I think, be generally allowed in the most familiar case, that of the " margin of cultivation." Consider the following simplified version of an example which Mill has put among " peculiar cases of value." [4] " For simplicity we will confine our supposition to two kinds of agricultural produce; for instance, wheat and oats." There are supposed (by us, not Mill) to exist only " medium soils which, without being specifically adapted to either, are about equally suited to both " products. The relative value of the two grains will of course be determined by the productivity of the marginal dose of outlay on each species of cultivation. Now let a tax be laid on the profits of oat growers. There will be a rush from the cultivation of oats to that of wheat. There will be established a new equilibrium in which, if the demand is constant, the area of wheat-growing is widened, the marginal cost of cultivation diminished; while the converse is true of oats.[5]

I have been supposing the land to be owned by the cultivators. It comes to the same if the land is rented from competing landlords, and a tax is imposed on the rent of oats-growing land. We have then an example of Professor Marshall's theorem that

[1] See *Address to the British Association, Sec. F*, Report for 1889. I have endeavoured to defend this view in an article in the *Revue d'Économie Politique* for January 1891, and in a passage in the ECONOMIC JOURNAL for 1896, Vol. V. p. 173.

[2] *Cp.* Marshall, *Principles of Economics*, p. 433 *et passim*.

[3] Or rather equally attractive, as explained by Professor Marshall in the *Principles of Economics*. I suppose the condition to hold not only for the typical entrepreneur, but also when enterprise delegates the task of superintendence—*e. g.* shareholders in a joint-stock undertaking. *Ceteris paribus*, the chance of profit tends to be the same in one undertaking as another.

[4] *Principles*, Book III. ch. xvi. § 2.

[5] *Cp.* Marshall, *Principles of Economics*, p. 483, note; 3rd edition.

partial rent does enter into the cost of production, taking as a test of such "entrance" the circumstance that a tax on rent will affect price.

The action of mobility is similar when the tax is not on *surplus*, as we began in this section by supposing, but on *margin* —specific, *ad valorem*, or in kind.[1]

I propose to illustrate these principles by considering a tax affecting an industry which presumably obeys the law of decreasing returns,[2] a tax on the rent of urban dwelling-houses.

Let us take as sufficiently general the case put by the Select Committee on Town Holdings in their Final Report :— [3]

"The typical condition of a town holding under this system [the 'leasehold system'] as regards the parties and their respective interests . . . may be described as follows :—

"(A) The occupier of the house holding at a rack-rent, whether on a yearly tenancy or for a longer or shorter term.

"(B) His immediate landlord, the receiver of the rack-rent, who is ordinarily called 'the owner of the house,' and who holds for a term of years, paying during such term to the freeholder a fixed annual sum, generally called the 'ground rent' . . .

"(C) The freeholder, who receives the ground rent during the term, and on its completion is entitled to the entire property absolutely."

For a first approximation, neglecting the distance in time between the different bargians, we may substitute for the three interests A, B, C, described in the Report the three "nations" A, B, C defined at the end of our first section ; [4] A importing from B, goods for the manufacture of which B has to import materials from C, materials obtained in constant quantities independently of human effort. Only now B is no longer completely insulated, but is connected with a continent of capitalists, whereby the producers in B are kept as it were at a constant level of advantage. With allowance for this difference the solution is as before. A tax on the product houses—whether levied from the occupant or owner—will have the following effects. The occupants will suffer by having to pay a raised price, not in general raised to the full extent of the tax. The capitalist owners will not suffer

[1] Regarding the ascending curve SS′ on Fig. 11 as an ordinary short-period supply-curve, we are to consider that it is first raised up, as explained in that context, by a marginal tax, and then further—in general and except when the demand of the consumer is perfectly inelastic—furled in by the migration of entrepreneurs from the industry. In the case of a tax on surplus the curve is not raised up; it is (theoretically) always, not merely generally, furled in.

[2] See Marshall, *Principles of Economics*, *sub voce* "Margin of building."

[3] 1892, No. 214, p. 6.

[4] Above, p. 75.

though the price which they receive for their product falls; [1] the net advantage of the industry being kept constant by migration into other industries. The ground landlord will suffer by a diminution of the ground rent. A tax on ground rent, whether levied from the ground landlord or the "owner," falls entirely on the ground landlord.

This is, of course, very pure theory, making abstraction of differences in time, that great source of complications in Economics.[2]

For a second approximation let us distinguish three periods, (1) the average duration of the occupant's lease, (2) the average duration of the owner's lease, (3) longer periods.

(1) It is not questioned that a tax imposed while the occupant's lease is running rests where it strikes during that period.

(2) A first approximation has already been obtained for this case,[3] on the assumption that during this period the offer consists entirely of houses already built. On that supposition the tax falls entirely upon the owner.[4] But we have now to take into account that the offer in general consists partly of houses already built in parts of the town already occupied, say the central area, partly of new houses which may be built on land which has hitherto been agricultural, say for brevity the suburbs.[5] Now if we had an exact measure of the advantage of the central area above the suburban periphery we should have an exact measure of the effect of the tax on house-rent. Suppose, for instance, in the vein of von Thünen, that the net advantage offered by houses of equally costly structure in the respective sites differed only on account of the different fares from each site to a central point. Then since the landlord at the suburbs can only stand out for a certain minimum of rent, that which he might have obtained in the way of agriculture,[6] the occupant in the suburbs has in general

[1] The law of decreasing returns being supposed to act.

[2] *Cp.* Marshall, Preface to *Principles of Economics*.

[3] Above, p. 68.

[4] Mr. Cannan clenches the matter thus, "We are not really 'mostly fools.' Who will stand up and confess that he took 76 —— Street at £100 a year, and subject to £20 of rates, when an exactly similar house next door, but in another parish, was to let at £100 a year, and only £12 of rates?" (*History of Local Taxation*, p. 134.)

[5] Our problem is here the same as that which forms Mr. Pierson's *third* case (noticed in the ECONOMIC JOURNAL, Vol. V. p. 436); but our solution is not quite the same as his.

[6] Professor Seligman is alone, as far as I know, in disputing this theorem (*Shifting and Incidence*, p. 106). I cannot agree with him that Mill's reasoning postulates the existence of a no-rent tract. The reasoning is akin to that on p. 78 above, relating to the taxation of rent.

to pay the entire tax, except so far as he reduces his use of house-accommodation; and since the occupants of the central area are better off only in respect of the fares they also have to pay the same price for the same accommodation. The case would be exactly parallel to the familiar case of a tax on agricultural produce. The consumer thereof pays the tax except so far as he reduces his consumption; otherwise the landlords are untouched. This would be the solution, if there were perfect rivalry between central and suburban habitations.

But of course the capacity of houses at the circumference to act as substitutes for houses in the centre is not perfect; there is only a partial rivalry.[1] Since, then, when there is no rivalry, the owners of the central area suffer to the full extent of the tax, and when there is perfect rivalry, they do not suffer at all, it might be inferred that in the intermediate case of partial rivalry the owners would suffer, but not to the full extent of the tax. The inference, however, would not be correct. It is one of the *curiosa* of the theory of *correlated* demand[2] that a tax on house rent might so disturb the balance of demand for urban and suburban accommodation respectively as to cause a positive benefit to the owners.[3] The truth of this proposition is not impaired, because

[1] Compare Fleeming Jenkin: "The rents through the whole town are ruled by those of the new districts. There is a certain selective value between every house in the town, and if the rents of the new houses are dearer, the rents of the old houses are increased in due proportion" ("Incidence of Taxes," p. 117, *Papers, Literary and Scientific.*)

[2] *Cp.* above, p. 73.

[3] Suppose, for simplicity of enunciation, that all the houses in the suburbs are of one kind; and also all the houses in the central area of another kind. Before the tax, let p_1 be the rent of a house, and x the number of houses taken, in the suburbs; and let the corresponding amounts for the central area be p_2 and y. By hypothesis, y is constant. Also, for a first approximation, we may make the classical assumption that p_1, the rent received by the capitalist-builder in the suburbs, does not vary with the tax. Under these conditions, if a tax proportional to the rent, say the ith part thereof (where i is small), is levied from the occupiers in both quarters; for the disturbed equilibrium we have the following equations:—

$$\begin{cases} (1+i)p_1 = p_1 + dx\left(\dfrac{dp_1}{dx}\right) \\ (1+i)(p_2+dp_2) = p_2 + dx\left(\dfrac{dp_2}{dx}\right) \end{cases}$$

Whence

$$dp_2 = ip_2\left(-1 + \frac{1}{p_2}\left(\frac{dp_2}{dx}\right) \div \frac{1}{p_1}\left(\frac{dp_1}{dx}\right)\right)$$

Whence it appears that the rent received by the urban owner falls to the full extent of the tax when the demands for residence in the respective quarters are quite independent, and does not fall at all when the two articles are perfect substitutes. In the intermediate case the owner's rent falls, or rises, according as $\frac{1}{p_1}\left(\frac{dp_1}{dx}\right) >$, or $< \frac{1}{p_2}\left(\frac{dp_2}{dx}\right)$. The former case is, I think, the more probable; but the

there may be in fact from other causes a centrifugal movement of residents from central quarters. *Pro tanto* the tax may have the effect of diminishing the loss which from other causes is accruing to the owners of residential houses in these quarters.[1]

(3) For long periods the solution above given [2] as a first approximation holds good. We might regard the three interests A, B, C, as three bodies held one above the other by a press or "vice," so that the sum of the depths of three bodies is constant. A wedge being driven in between A and B, the bodies—each obeying its peculiar law of compressibility and resilience—will behave as follows. At first A will be compressed to the full extent of the thickness of the wedge; B and C retaining their full dimensions. After a time A will re-expand, in part at least; B will be compressed to some extent, perhaps nearly to the whole extent of the thickness of the wedge; C will remain firm. But leave the bodies alone for a longer time and B will regain its original amplitude, and the compression due to the insertion of the wedge will be divided in uncertain proportions between A and C. It is not to be denied that during the long time required for the working out of these forces, other forces may have come into play. The bodies may have expanded from other causes, the press may have been warped so as to allow room for their expansion. But because the given forces are compounded with others known imperfectly, we are not precluded from calculating the resultant of the given ones.

The proof of the general theory relating to long periods may be verified by an examination of some limiting cases in which the statement of the theory requires modification; the frequent occurrence of which cases may account for the prevalence of opposed theories.

(*a*) In the limiting case when the demand of the consumer,

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latter is by no means impossible; for all that we know about the relative magnitude of these partial differentials is that $\left(\frac{dp_1}{dx}\right) \times \left(\frac{dp_2}{dy}\right) > \left(\frac{dp_2}{dx}\right)^2$. (*Cp.* E, p. 117.) Probably $\left(\frac{dp_2}{dx}\right)$ is less than either of the two factors of which the product is greater than its square; but not necessarily. *Ceteris paribus*, the event is more likely to occur when the demand for urban houses is very inelastic; for suburban houses very elastic. As to the conditions which the demand-functions must fulfil, see article on "Monopoly" in the *Giornale degli Economisti*, 1897, (E).

The proposition is less likely to be true when p_1 is supposed to be lowered (in virtue of the law of diminishing returns). It is strictly proved only for infinitesimal values of i, but may with probability be extended further. (See Index, s.v. *Differential*.

[1] *Cp.* above, p. 70.

[2] Above, p. 80.

the occupier, is perfectly inelastic he will bear the whole tax. This assumption is more readily made, as it is usual, perhaps proper, to make it in problems about agricultural rent. The possibility of this incident has already been admitted with respect to the comparatively short period (2), and may also, though I think less easily, be admitted with regard to period (3).

(β) Suppose that ground-rents are in general very small in comparison with the tax, then of course they can only bear a small part of the tax. May we not explain by this supposition Ricardo's *dictum ?*

" In ordinary cases it may be presumed that the whole tax would be paid both immediately and finally by the occupier." [1]

So J. S. Mill :— [2]

" In the vast majority of houses, the ground rent forms but a small proportion of the annual payment of the house." [3]

This is of course true of houses in the country ; [4] not so true now as fifty years ago of urban rates.

(γ) Again, suppose conditions such that only one " dose," so to speak, of building capital can be applied to one parcel of land—say in China or Peru—through the fixity of custom and the mobility of the earth, only a single-storied dwelling of uniform pattern can be placed on each unit of the area available for building. On such a supposition a tax on house-rent would fall in general entirely on ground-rent.[5] For the accommodation of the occupants could not be reduced without some of the sites being left unoccupied. Each landlord threatened with the loss of his entire ground-rent will lower his terms until ground-rent all round has been, if it can be, reduced to the full extent of the tax.

Upon this or some adjacent less extreme hypothesis, we may account for the opinion of some distinguished writers that the tax on house rent in the long run tends to be mostly borne by the ground landlord. Thus, too, we may perhaps explain what otherwise may seem inexplicable—why the successors of Ricardo should attempt to allocate a certain portion of the house rent to the ground rent.

Thus McCulloch :—

" Were the supply of houses easily diminished and increased, a tax on their rents would fall wholly on the occupiers and ground

[1] *Political Economy*, chap. xiv. par. 3.

[2] A different view of the Ricardian dictum appears to be taken by Esquiros de Parieu, *Traité de l'Impôt*, p. 74, and some other eminent writers.

Political Economy, Book V. chap. iii. § 6, par. 5.

[4] Cp. *Wealth of Nations*, Book V ch. ii.

[5] Of course supposing the tax not to exceed the rent.

landlords, and be divided between them in the proportion which the profit of the capital required to build them bears to the rent of the ground on which they stand." [1]

So J. S. Mill :—[2]

"A tax of so much per cent. on the gross rent falls on both these portions [ground rent and building rent]. . . . The incidence, however, of these two portions of the tax must be considered separately.

"As much of it as is a tax on building rent must ultimately fall on the consumer, in other words the occupier." [3]

So Professor Sidgwick distinguishes "the portion of the tax which is paid for the value of the house itself" and the "portion that falls on the ground rent." [4]

Now on our present hypothesis (γ) these statements would be true in a particular case, namely, when the tax was equal to the original ground rent plus the constant building rent.[5] In that case the effects of the tax would be exactly as Mill and McCulloch lay it down. And it was, perhaps, natural to regard this case as typical; at any rate, when the consumer's demand is supposed perfectly inelastic, when our (α) as well as (γ) is present. In that sub-case the true solution, I submit, is that the division of the burden between the occupier and the ground landlord is indeterminate. But the divisions suggested by McCulloch and Mill are plausible.

The consonance of this incident (γ) with authoritative *dicta* moves me to suppose the prevalence of the incident. A house is naturally thought of by Jevons as an instance of an "indivisible" commodity which forms an exception to the general theory of value.[6] And yet, though a house is indivisible, residential accommodation is not. There may be many "mansions" not only in the archaic sense, but in that which is applicable to the modern "flats." "Increments of villa accommodation," in Professor Marshall's phrase, may be added up to the point [7] where

[1] *Taxation and Funding*, Part I. chap. i. § 2.

[2] As against Mill's precise apportionment, Mr. Sidney Webb's contention that "the freeholder . . . has no fixed point of resistance" (*Town Holdings Commission*, 1890, Q. 42–44) is just. His "large jump in value" from agricultural to building land, is not necessary for this conclusion.

[3] *Political Economy*, Book V. chap. iii. § 6.

[4] *Principles of Political Economy*, Book III. chap. viii. § 8.

[5] In the spirit of the classical writers we may here suppose the cost of building constant, even though the supply of buildings should be reduced.

[6] *Theory*, chap. iv.

[7] "The cases in which a man has to live in a house of a size widely different from that which he prefers, because there is none other available," are exceptional (*Principles of Economics*, Book V., p. 592, note, 3rd ed.).

the price just measures both the marginal cost and the marginal utility of an increment. In short, the law of value for house accommodation is (for long periods) essentially the same for house accommodation as for corn. It might be all very well for Adam Smith, who held that "in the price of corn . . . one part pays the rent of the landlord," [1] to say that "the rent of a house may be distinguished into two parts,[2] . . . the building rent and the ground rent." But what have we to do with such apportionment of price, or tax, we who have received the doctrine of Ricardo that "rent does not and cannot enter in the least degree as a component of price"; the doctrine of Professor Marshall that "ground rent does not enter into the expenses of manufacture," on an understanding "exactly parallel to that which has to be supplied in order to make Ricardo's doctrine true, when applied to agriculture?" [3]

Dwelling-houses, then, belonging to the general category of consumable products, as the highest authorities are agreed,[4] the taxation of such houses obeys the general laws of the taxation of products which have been enunciated above as pertaining to long periods (3);[5] abstracting the peculiarities of the "leasehold system" which have been allowed for with reference to short periods (2).

A practical corollary is that a tax on ground rent hurts the ground landlord more, and the occupier less, than *ceteris paribus* a tax on the occupation rent, in the long run;[6] theoretically even,

[1] *Wealth of Nations*, Book I. [2] *Ibid.* Book V. chap. i.

[3] See the whole passage relating to the margin of building, in *Principles of Economics*, Book V.

[4] Thus Mr. Goschen in his *Draft Report on Local Taxation*: "The inhabitant of the house . . . is in reality the consumer of the commodity produced by the builder" (*Local Taxation*, p. 164). So Professor Bastable regards "houses as a particular manufactured commodity" (*Public Finance*, p. 371, 2nd ed.). *Cp.* Mr. Pierson, *Leerboek*, p. 146, 2nd ed.

[5] Above, p. 82. I am confirmed in this view by finding myself able to agree with all that Professor Bastable has said on this subject (*Public Finance*, Book IV. chap. ii. § 5). I concur with his criticism of Professor Seligman that "he seems to give too little weight to the forces that shift taxation on the ground owner."

[6] The reluctance on the part of common-sense and even of trained intelligence to accept the theory here maintained, that there is an essential difference between the effects of a tax on ground rent and a tax on occupation rent, may be accounted for by the tacit assumption that the amount of building is given and constant, irrespectively of the tax. Consider, for instance, the remarkably clear statements of Mr. Clements in his evidence before the Town Holdings Commission (Q. 1,969).

The argument which he illustrated by the example of a particular actual house (Q. 1,970) tacitly assumes that the amount of house accommodation demanded by the occupier is constant, whether or not the occupier pays an *ad valorem* tax (see notes).

For other direct contradictions of the theory here advanced see *Town Holdings Committee*, 1887, Q. 3,360; 1888, Q. 2,736, 2,837, 4,446, 9,357 *et passim*; or put the question to any practical man.

and apart from friction (*a fortiori*, of course, when we restore the concrete circumstance that taxes are very apt to rest where they strike).[1]

In this argument no use has been made of the circumstance that all the leases do not fall in simultaneously. But it will be found that this concrete circumstance does not invalidate the broad conclusion of pure theory, that there is an essential distinction between the effect of a tax on ground rent, and that of a tax on occupation rent. To fix the ideas, we might suppose occupation leases to be on an average for seven years, ground leases for seventy years. Thus every year on an average the leases of a seventh part of the houses in any given urban area would fall in; and in a tenth of these cases the ground leases would also fall in, and the sites would be offered to capitalist-builders; supposing as a first approximation that the duration of the ground lease coincides with the duration of the house. Upon this supposition a tax on ground rent would as before fall entirely on the landlord; a tax on occupation rent would not in general have that effect. The effect of the latter kind of tax would not indeed be exactly the same in the concrete case of rotation and the imaginary case of simultaneous bargains.

But the differences between the abstract and concrete cases will not, I think, repay examination. I am already sensible indeed that the investigation of economic forces which require some seventy years to work themselves out may seem to have been prolonged beyond the limits of applied theory. I submit, however, that the argument is not so abstract, the reasoning is not deduced through so artificial a chain of remote consequences as at least one of the classical theorems of taxation which are still accepted by economists;[2] I mean Senior's doctrine of tithes. In this argument, as interpreted by Mill, the links seem to be as follows: rise in the price of agricultural wages, check to profits, check to accumulation, check to the production of food, check to the growth of population, check to the rise of rent; comparatively to what would have occurred in the absence of the tax. But it is not my design to determine the limits of applied theory, or to uniformly cover with examples the field so demarcated. I aim only, at least in this first article, at a restatement, with slight modifications, of the classical laws of incidence, and a partial exemplification of the restated theory.

[1] Professor Thorold Rogers advocated this view very strongly in his evidence before the Town Holdings Commission.

[2] *E. g.* Bastable, *Public Finance*, and Seligman, *Shifting and Incidence*.

It will be understood that the application of the theory in this section has been adapted to the typical case propounded; modifications of statement would be required by the circumstances that there may be not only one, but several lessees between the ground landlord A, and the occupier C; that the duration of a house may exceed the period of the building lease; that the transfer of accommodation may be effected by sale or "feu," instead of lease; that houses may be used for business, as well as for habitation; that the ground landlord may act as a capitalist; that rates may differ in different parts of the same town; that house accommodation in different towns forms "rival" commodities; and many other incidents more or less important in practice.

From the point of view of pure theory the following modifications are more interesting. Perhaps the investments which are open as an alternative to an intending builder are not indefinitely extensive in comparison with the house-building industry —at any rate for periods not indefinitely long. The effect of mobility would then be to have connected our island B, not with a vast continent, but only with another island.[1] The joint island might then form a "nation" of capitalists, virtually appertaining to our first section, rather than the present one, obliged to submit in consequence of the house-tax to some permanent reduction in profits.[2]

Again, the building industry may be affected by the law of increasing returns, the operation of which we have next to consider.

A b. The old distinction between increasing and constant (or decreasing) cost presents difficulties to the newer analysis. For if any producer can continually increase his supply at a constant or diminished cost, there appears no general reason why he should not, cutting out his competitors, supply the entire market.[3] The classical conception of constant cost presupposes a limit to the production of each individual. The newer idea of expenditure pushed up to the margin of profitableness, in a regime of competition, implies the law of increased cost.[4] The law of diminishing

[1] Above, p. 79.

[2] This sort of intermediate case between perfect mobility and immobility is treated by Professor Pantaleoni in his highly original *Traslazione dei Tributi*.

[3] *Cp.* Marshall's *Principles of Economics*, 2nd and 3rd editions, Book V. ch. xi. *et passim*.

[4] Thus in the luminous illustration which Professor Marshall has given in note xiv. of the Appendix to his *Principles*, 3rd edition, the total outlay of a master-builder, considered as a function of different classes of labour x_1, x_2, and different kinds of raw materials y_1, y_2, etc., and other kinds of variables, must

costs, as Cournot argues, is only intelligible on the supposition of monopoly.[1]

How then can the law of diminishing cost co-exist with competition? How can a larger offer go with a smaller price? How can the supply-curves of the kind which Professor Marshall has made familiar be ever conceived as *descending?*

The better opinion appears to be that such a downward trending *locus* is not to be regarded as a supply-curve in the primary [2] and obvious sense, not as representing the offer which in a given state of industry would be forthcoming at different prices; but as compounded of, or derived from, a series of such primary curves, which Mr. Cunynghame in his path-breaking essay on the subject [3] has called "successive cost curves."

It has happened to some of us to ascend a mountain slope just up to the point where the desire was just compensated by the difficulty, of further progress. Such is the position of the economic man on a primary short-period supply-curve sloping upwards.

Suppose that, as a party of mountaineers press up a steep slope, the opposing crest gives way, and they are carried down by a sort of avalanche, and landed on a new inclined plane. Again they urge their toilsome march upwards; and again, before the crest is reached, they are precipitated on to another ledge below; and so on till they are brought to a stop on some steep and comparatively firm slope. Their path in space, though in reality saw-shaped, might appear to one taking a general view to be a curve-line. Such, perhaps, is the nature of a competitive industry obeying the law of increasing returns: confined for short periods on an ascending supply-curve, extended during long periods down a descending supply-curve.[4]

Suppose that our party, after coming to a stop on a short slope, were to be incited by some fresh stimulus; they might break through another crest and descend through a distance out of all

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be such that the second term of its expansion fulfils all the conditions of a *maximum* (above, p. 65, note 1); otherwise the statements made, *e. g. op. cit.*, p. 802, par. 2, would not hold good.

The theoretical difficulties connected with the law of increasing returns are frequently referred to by Professor Marshall in his later editions.

[1] *Principes Mathématiques*, Art. 50, p. 102.

[2] See the reference to the subject in a former article, ECONOMIC JOURNAL, Vol. IV. p. 436.

[3] ECONOMIC JOURNAL, Vol. II. p. 41.

[4] The idea of a curve of many branches was propounded by the present writer in his Address to Section F of the British Association (note J), 1889. The date explains one serious omission, that of "external economics," pointed out by Professor Marshall in the *Principles of Economics*. (See *a*, below, p. 305.)

proportion to the exciting cause. Conversely, the imposition of a new burden might have prevented such progress from occurring. It is thus that, in an industrial regime of the kind considered, a bounty is apt to lower price,[1] a tax to raise it,[2] to a disproportionate extent.[3]

But, if the law of increasing cost is fulfilled in its natural and obvious sense, if the primary or short-period curves are descending, presumably the case belongs to *Monopoly*, the subject to which we next proceed.

Monopoly,[4] the branch of the subject we have now reached, presents a bifurcation peculiar to itself. There is an essential difference between (a_1) the cases in which we are concerned with only one monopolist, and (a_2) those in which two or more monopolists enter. The first subdivision presents ramifications parallel to those which have been traced in the case of competition. The first of these varieties is obtained by combining the just now defined attribute a_1 with the attributes which form the first members of cross divisions enunciated in the first article: namely, B prevalence of the law of decreasing returns, C immobility of capital and labour, D taxation of margin (*e. g.* specific or *ad valorem*, or in kind).

[1] Marshall, *Principles of Economics*, Book V. chap. xii. § 4.

[2] *Ibid.* *Cp.* Mill, *Political Economy*, Book V. chap. iv. § 2, end.

[3] A tax on a rival might of course act as a bounty; *e. g.* duty on foreign imports as a bounty to native producers.

Professor Carver's argument (*Yale Review*, November 1896) that, when an import tax is levied on a commodity which is produced at home under the law of increasing returns, the consumers may possibly bear no part of the tax, is not, I think, as he seems to apprehend, "opposed to the best orthodox teaching"; unless orthodoxy be defined very straitly. The argument is used by some of the highest modern authorities, to whom I have referred in a former article [ECONOMIC JOURNAL, Vol. IV. p. 48, E]. I don't know that they would accept his reply to the objection that the price—after being lowered in consequence of the tax—"might be further lowered by removing the tax." "This might be temporarily," says Professor Carver, "while it is probable that the same forces which kept the price up before the duty was first levied would ultimately bring about the same conditions after it was removed."

[4] The taxation of monopolists and monopolised goods, which is the subject of this article, is not to be confounded with the taxation by monopoly which is practised by several modern governments. The pure theory of the latter form of taxation is simply the pure theory of monopoly in general—a subject which I have attempted to handle in the *Giornale degli Economisti* for 1897 (E). The taxation of monopoly and the taxation by monopoly are connected by a certain analogy, which, as indicated by Messrs. Auspitz and Lieben (*Theorie*, p. 427 and context), exists between monopoly and taxation in general. In ordinary taxation government alters, in taxation by monopoly it makes, prices in its own interest—or rather at its own discretion, as Government need not be perfectly self-interested (*see* Marshall on "Compromise Benefit," *Principles of Economics*, Book V. ch. xiii.).

a_1 B C D. The simplest case under this head is obtained from the case which was first discussed under the heading A B C D, namely a simple market, by supposing one side of the market to be as it were solidified into a monopoly. We have thus the typical case discussed by Cournot: a single monopolist dealing with a body of customers competing with each other, at the same price for all the customers. As demonstrated by Cournot,[1] the effect of a specific tax on the product will be to raise its price to an extent which may be either greater than, equal to, or less than the amount of the tax "selon les circonstances."[2] To which it may be added, I think, that it will probably be less, at least for the case of decreasing returns.[3] In general the addition to the price will be a substantial proportion of the tax.

What is said by many popular writers, and even by some distinguished economists, that a tax (of the kind now under consideration) will not affect the consumer, for that the monopolist has already done his worst, is true only in two special cases. (1) Where it is not in the power of the monopolist to increase or limit his output at will, he will very generally have to bear the whole tax. Cournot has noticed the case of the monopolist who is unable to

[1] *Principes Mathématiques*, chs. v. and vi.

Professor Seligman appears to be under the impression that the only reason advanced by Cournot for the phenomenon that a tax on a monopolised article may raise the price to an extent greater than the amount of the tax was that the price paid by the consumer must include not only the tax but also interest on the sum advanced in order to pay the tax and the profits of middlemen (*Shifting and Incidence*, p. 156). "This theory," says Professor Seligman, "which Cournot invested with elaborate apparatus of mathematical diagrams, is, however, nothing but the accepted doctrine of Adam Smith, Ricardo, and Mill" (*ibid.* p. 157; *cp.* p. 159, par. 1). But it will be evident to any one who studies Cournot's theory of monopoly (*Principes Mathématiques*, p. 78, referred to by Mr. Seligman) that Cournot rests the phenomenon in question, in the case of monopoly, upon a principle other than the accepted doctrine of Adam Smith. Cournot in the passage referred to, has "invested with mathematical apparatus" the law of taxation stated in our text; which is so far from being the accepted doctrine of Adam Smith that it has escaped even Mr. Seligman.

I do not deny that Cournot has also employed the "accepted doctrine"; whether mistakenly, as Professor Seligman holds, will depend on the validity of "the old doctrine of normal or natural profits" (Seligman, p. 158 and *cp.* p. 145), a subject on which I am not called upon in this connection to express an opinion (*cp.* ante, p. 77).

[2] *Principes Mathématiques*, *p.* 77.

[3] As proved by Cournot (*Principes Mathématiques*, Art. 31) the increase of price due to a small tax u per unit of commodity is of the form $u \times A \div (2A+B+C)$ where A is always negative; B is negative when the law of decreasing returns prevails (and positive in the converse case); of C nothing is known in general, either as to its sign or magnitude, except that the expression $(2A+B+C)$ must be always negative. In such a case, I submit, we are justified in regarding it as probable that A will be (in absolute quantity) less than $2A+B+C$, and therefore the addition to the price less than u—at any rate when B is of the same sign as A, as in the present section.

increase his output.[1] The converse exception may be illustrated by an owner of urban land, if prevented by public opinion from keeping it out of the market. (2) The second exception is where the monopolist is a sole *buyer*, and the supply of the article bought is perfectly inelastic: for instance, a combination of tenants dealing with landlords incapable of combining. But in general the addition to the price will not be zero. I am unable to follow Professor Seligman when he asserts the contrary.

These theorems may be extended from a specific to an *ad valorem* tax. The demonstration is given by Cournot in a passage already referred to (*Principes Mathématiques*, p. 78 *et seq.*). I have endeavoured to give a simplified version of Cournot's reasoning in a note.[2]

[1] *Principes Mathématiques*, last paragraph of ch. v.; first paragraph of Art. 39.

[2] Before the tax, the monopolist will have fixed the price which renders his net profits a *maximum*. He may be supposed to begin tentatively with a very low price, and to go on raising the price as long as the increment of net profit which corresponds to a rise of price continues positive. He will stop just at the point at which that increment ceases to be positive and begins to be negative. That is the required position of *maximum*. A rise of price above that point is attended with a fall in *gross* receipts. For, by hypothesis, the price of rise is attended with a fall in net receipts; and net receipts equal gross receipts *minus* total cost; and total cost decreases with the rise of price, since total cost diminishes with the diminution of the quantity supplied, and that quantity diminishes with the rise of price. If, then, gross receipts minus total cost diminishes while the subtrahend total cost diminishes, much more must gross receipts diminish.

After the tax, the quantity which the monopolist seeks to maximise is the net profits in the same sense as before (that is, gross receipts minus total cost) *minus* the amount of the tax, a certain percentage of gross receipts. This quantity will not be a maximum at the point before determined. For if the price be raised above that point, the increment of the quantity to be maximised (net profits *minus* tax) will be 0 minus increment of the amount abstracted by the tax. But the increment of the amount abstracted is negative, since when the price is raised the gross receipts are diminished (as shown in the last paragraph), and therefore the tax which is a fixed percentage of the gross returns is diminished. It will be the interest then of the monopolist to raise the price beyond the old maximum point up to a new limit; at which the loss in respect of net profits (gross receipts *minus* total cost, which diminishes as the price is raised above the old maximum point) is just compensated by the gain in respect of the diminution of the tax. The new price therefore will be higher than the old.

A scruple may be felt whether, as the price is raised above the old maximum point the loss will overtake the gain, as the reasoning requires. The answer is that the loss (attending the increase of price above the old maximum point) is proportional to the *square* of the increment of the price, while the gain is simply proportional, it being supposed that the tax is small (*cp.* above, p. 74). Whence in general there is a determinate value for the increase of the new price above the old, such that the gain of the monopolist—the net receipts *minus* the tax—should be a maximum.

The mathematical reader will not expect the accuracy of a purist in this popular version; the general reader will perhaps be disappointed in his expectation of simplicity. I don't know that much has been effected by this cumbrous simplification, except to show the great superiority of the genuine mathematical method.

Cournot's reasoning may be extended to any (small) marginal tax, provided that the aggregate tax increases with the amount supplied.[1]

Throughout all the cases which have so far been considered, there prevails what may be called the general rule, both for competition and monopoly, that both parties suffer more or less from a tax. It remains to point out that in monopoly, as in competition, there are exceptions to this rule. The monopolist, indeed, always suffers, but his customers may be benefited in certain cases.

There is first a peculiar tax in kind noticed by Cournot with respect to monopoly, and described in our former article with respect to competition: "the abstraction of a certain portion of the exports in kind, to be disposed of in a manner not affecting the market under consideration."[2]

The case of several commodities presents a second class of exceptions, even more paradoxical in a regime of monopoly than in that of competition. If a monopolist supply two commodities for which the demand is *correlated*, that is either "rival" or "complementary," then a tax on one commodity may benefit the consumers of both.[3] To fix the ideas, let there be a railway, like our Midland, with two classes of passengers, first class and third class. Let there be imposed a small tax of say 2 or 3 per cent. *ad valorem* on the gross receipts of the first class passenger traffic; or there being, as in fact, a tax already, let there be

[1] As pointed out by Professor Marshall in the passage to which Professor Seligman refers when he says that "most writers, including Marshall (*Principles*, 460) [see p. 462, 1st ed., note, par. 1, p. 538, 3rd ed.] and Pantaleoni (*Traslazione*, 76), overlook this, and confuse a tax on gross receipts with a tax on sales or amount produced."

If the views, here stated summarily, and more explicitly in my article on "The Pure Theory of Monopoly" (*Giornale degli Economisti*, 1897), are correct, there is not the slightest confusion or mistake to be attributed to either of the authors thus disparagingly referred to by Professor Seligman. It is to be regretted that his intellectual sympathy is not always proportioned to his learning.

[2] Above, p. 72. This kind of tax is presupposed in Colonel Barone's theorem respecting a contribution proportional to the quantity; forming the fourth theorem of his "Teoria Matematica dell' Imposta" (*Giornale degli Economisti*, March 1894), p. 207.

[3] In the case of *rival* demand, the theorem makes no postulate about the cost of production. Either the law of increasing or decreasing (marginal) cost may prevail. There may even be no cost, two "mineral springs" controlled by a monopolist (*cp.* Cournot and Marshall) supplying without human effort two waters which may, to some extent, act as substitutes for each other. Or the (total) cost may be constant, not changing with that change in the amounts produced which is consequent on the tax; as it is allowable, though not necessary, to suppose in our example of first and third class accommodation. In the case of *complementary* demand, it must be postulated that production is *correlated* in a certain manner.

superimposed a small additional tax; I say the consequence of the new tax on first-class tickets may be to benefit passengers by lowering the fares of both kinds, both the third class and also the first class.

A mathematical demonstration of this theorem has been offered elsewhere.[1] The reader who has seen how difficult it is to state in ordinary language the proof of the simple proposition that a tax on a monopolised article tends to raise its price (above p. 91, note), will not expect here a full statement of the more complicated argument relating to two commodities. He may be put on the track of the investigation by the following hints.

Let x and y be respectively the first and third class fares (per mile or other unit) before the tax. After the tax let us suppose the directorate of the railway to alter the fares one at a time; and first the first-class fares.[2] By the theory above stated, the first-class fares will be raised in consequence of the tax (the third class being for the time fixed), say to x_1. There will result a diversion from first-class traffic, a rise in the demand for third-class accommodation. There might be expected then a rise of third-class fares in sympathy with the rise in the price of the rival accommodation; for when the demand for an article is raised it is probable[3] that its price will rise, *ceteris paribus*. But other things are not the same here. For, while the purchasers are now disposed to give more for third-class accommodation, the sellers are more ready to offer third-class accommodation now that it has an advantage over the first-class in not being taxed. Thus the change in the third-class fare will be the resultant of two tendencies, one making for a rise, the other for a fall. There is nothing to show that the latter tendency may not preponderate. Accordingly, the new y, say y_1, may be less than the old one. This low third-class fare will tend—by the sort of sympathy between the prices of substitutes which has just been noticed—to drag down the first-class fare, when we come to the third step, which consists in determining the first-class fare, x_2, which is most profitable to the monopolist, the third-class fare being fixed at y_1.

[1] See *Giornale degli Economisti*, 1887, Vol. I. p. 131 (E). In that article I give an explicit example of a possible curve, or rather numerical law, of demand for first and third class accommodation, such that if a tax of 2 or 3 per cent. is put on first-class tickets, it will be the interest of the management to lower *both* fares.

[2] Of course I do not suppose so delicate an adjustment—such a frictionless movement towards the position of maximum profit—to be realised in the concrete management of an English railway. But I think that it may be of scientific interest to establish the theoretic possibility of the paradox enounced in the text.

[3] Though in a regime of monopoly, not necessary. See Index, s.v. *Paradox*.

It is probable then that x_2 will be less than x_1. And there is nothing to show that it may not possibly be less even than x. There is nothing to show that the series of subsequent steps will not converge to a system of two fares each of which is lower than the original one.[1]

The preceding theorem illustrates a general characteristic of monopoly, that the laws of incidence relating to that regime resemble, but are less exact than, those relating to a regime of competition.[2] To take another example from the case of

[1] An acute friend has objected: If it is advantageous to the monopolist to lower prices after the imposition of the tax, why is it not before? I reply: Because the conditions of maximum profit are altered by the tax. Let x be the price of first-class accommodation, and y that of third-class accommodation, before the tax; and let the corresponding quantities demanded be D_1 and D_2. After the tax, let x' and y' be the prices, D'_1 and D'_2 the corresponding quantities. Then the profit of the monopolist is D_1x+D_2y before the tax, after the tax $\frac{1}{20}D'_1x'+D'_2y'$; if, to fix the ideas, we suppose the tax to be 5 per cent. *ad valorem* (on the proceeds of first-class tickets), and for simplicity we abstract expenses of production (supposed constant). There is no inconsistency in supposing that it is neither the interest of the monopolist to change the fares from x and y to x' and y' before the tax, nor from x' and y' to x and y after the tax; x being greater than x', and y than y'. It is necessary only that D_1x+D_2y should be greater than $D'_1x'+D'_2y'$, and also $\frac{1}{20}D'_1x'+D'_2y'$ greater than $\frac{1}{20}D_1x+D_2y$; or that we should have at the same time $(D'_2y'-D_2y)$ less than $(D_1x-D'_1x')$, and $(D'_2y'-D_2y)$ greater than $\frac{1}{19}(D_1x-D'_1x')$. It is not surprising that of two quantities which are known not to be widely different (the tax being small), one should be greater than nineteen-twentieths, and less than the whole, of the other, provided that the quantities are *positive*. Now it is probable that $D_1x-D'_1x'$ is positive, if x is greater than x_1', as D_1 is probably greater than D_1' (the monopolist reducing his offer of the taxed commodity). Nor is it improbable that $D'_2y'-D_2y$ should be positive; nor inconsistent with the supposition that y' is less than y.

The objector may still insist: If Dx is greater than $D'x'$, why not raise x' to x, leaving y' what it is? To which I reply that the proceeds of the first-class traffic will no longer be Dx in that case, but $x \times$ first-class accommodation demanded at the prices x (first-class) and y' (third class), say $x \times \Delta$. Now Δ may fall off so rapidly, as the first-class fare is raised from x' to x, owing to the counter attractions of the third class (at the price y') that the monopolist will lose more by the decrease in the demand for first class than he gains by that increased demand for the third class which he at the same time causes.

[2] This greater latitude is explained by the circumstance that in monopoly, unlike competition, the producer [or, *mutatis mutandis*, the monopolist buyer] must take account of the change in demand price caused by variation in the amount of product which he may offer (*cp.* Marshall, *Principle of Economics*, Part II. p. 802, 3rd ed.). If β be that amount and p the price (*ibid.*), the marginal cost is equated in competition to $p\Delta\beta$, but in monopoly $p\Delta\beta + \beta\Delta p$. Accordingly, in determining the variation in the position of equilibrium due to a (small) tax (*cp.* Cournot, *Principes Mathématiques*, ch. vi.), whereas in competition the highest order of differential (of p with respect to β, taking β as the independent variable) which we need take account of, is of the form $\frac{dp}{d\beta}$, in monopoly we have also to take account of differentials of the form $\frac{d_2p}{d\beta^2}$. The sign of the former is given by the law of diminishing utility. But the sign of the latter is not usually a *datum*.

correlation. We have seen that when production only is correlated, "it being supposed that the demand for one commodity is independent of the demand for the other," in a regime of competition (1) a tax upon two rival products will raise the price of the taxed one, and lower the price of the untaxed one; (2) a tax on one of two complementary products will raise the price of both.[1]

Now let the correlated production be in the hands of a monopolist, the demands as before being uncorrelated. Then (1), as before, a tax upon one of two rival products will raise the price of the taxed one; but it will not, as before, necessarily—it will only probably—lower the price of the untaxed one. Also (2) as before, a tax on one of two complementary products will certainly raise the price of the taxed one, but will only probably raise the price of the untaxed one.

For example in a regime of monopoly it is probable, though not so certain as in a regime of competition, that a tax on malt would tend to lower the price of wheat in a country dependent on a limited area for its supply of both products; assuming that the production was not (otherwise) correlated. Contrariwise if the products are *complementary* in respect of the rotation of crops, as the Malt Tax Committee of 1862 suppose. "The effect of the malt tax," they say, "is to interfere with the due rotation of crops,"[2] and therefore presumably to cause wheat to be grown in more unfavourable conditions, at a higher price.

Some peculiar cases of rival production are constituted by the property that in monopoly identical objects may be sold at different prices: for instance seats for men or women at a theatre. If instead of the theatre tax which is now levied in Italy there were imposed a tax on men's tickets only, the ladies would be likely to gain not only in exemption from the tax, but also in having less to pay for their tickets. That is supposing the demand for one kind of ticket to be independent of the demand for the other kind. *A fortiori* if, as it is natural to suppose, the demands are *complementary*.[3] For it is probable in monopoly, as it is normally true in competition, that "a tax on one of two complementary commodities [*i. e.* "for which the demand is correlated"] will

[1] Above, p. 73, par. 2, where, by a misprint, or *lapsus plumæ*, the predicates of two propositions were transposed in the original version.

[2] Parly. Papers, 1868, 420. Cp. *Evidence*, 1867: Q. 2616, 2952, 3023, 3305.

[3] The young man who treats his lady friends to tickets, and the paterfamilias who provides for mixed parties, would be disposed *ceteris paribus* to give more for men's tickets if ladies' tickets were lowered, or more for ladies' tickets if men's tickets were lowered.

raise the price of the taxed one, and lower the price of the one which is not taxed." [1]

Following an order similar to that of the corresponding section on competition, we shall now leave correlation out of sight, and go on to consider the general case in which a monopolist deals with two or more competitive groups. For example, we might imagine one of our islands A, B, C,[2] to be now held by a monopolist. As before, we shall find that it is not indifferent whether a certain tax is levied on transactions between A and B, or on transactions between B and C. If, as before, we suppose the materials supplied by C to be constant in quantity and independent of human effort, we have now three cases, according as A (the consumers), B (the manufacturers), C (the landlords) form a monopoly. In the first case a tax on the product will prejudice both parties, not only in general as in competition, but always—correlation of supply or demand not being now supposed. For the circumstance which forms an exception in competition, the inelasticity of the consumer's [3] demand, cannot occur in monopoly.[4] In the second case a tax on the goods produced by the monopolist B will not prejudice C; for the monopolist will have already done his worst.[5] In the third case a tax on the product will, on the general supposition that the demand of the consumers is not perfectly inelastic, prejudice the landlord in general in monopoly as in competition; but not in monopoly always. For, though in consequence of the tax the demand on the part of the (competitive) producers in B for the (monopolised) article supplied by C falls, yet it does not follow that the price of C will fall. For it is one of the irregularities of monopoly as compared with competition that a rise [or fall] in the demand for a monopolised article is not necessarily, but only probably, attended with a rise [or fall] in its price.[6]

[1] Above, p. 73. [2] Above, p. 75. [3] Above, *loc. cit.*

[4] This may be elegantly exhibited by the curves which Messrs. Auspitz and Lieben have employed to illustrate the case in which the monopolist is a sole *buyer*. [5] See E, Vol. I., p. 113.

[6] Suppose that the demand for the monopolised commodity increases in the sense that the average consumer "will buy more of it than he would before at the same price, and that he will buy as much of it as before at a higher price" (*Principles of Economics*, Book III. chap. iii. § 4, 3rd ed.). Then the demand-curve will be shifted outwards, as in Professor Marshall's figures 26, 27, 28. The *Monopoly revenue curve* (*ibid.*) will accordingly be modified. The new q_3 will be on a *constant revenue curve* (*ibid.*), which is further "out," or away from the axis than the corresponding old curve. But there is nothing to prove that the new Lq_1 is greater than the old one; the displacement of the demand curve may be such that—whatever the shape of the "supply-curve"—the new Lq_1 is less than the old one.

This survey of marginal taxes of monopolised articles may conclude with the reflection that the consequence of such taxes appear to be in general more unpredictable in monopoly than in competition.

a_1 B C d. That taxation upon the profits of a monopolist cannot be shifted is universally acknowledged. It may be observed that this is true not only as stated in the books of a capitation tax consisting of a lump sum, and an *ad valorem* tax directly proportionate to profits, but also of a *progressive* tax on profits (the proportional contribution increasing with the amount).[1] Of course the alteration of the monopolist's revenue may produce, as in the case of competition,[2] an indirect result on the margins of work and saving.

a_1 B c. To attribute mobility to monopoly may appear a contradiction in terms. The designation may, however, be appropriate to a certain mixed case intermediate between monopoly and competition when there is present mobility of labour and capital, but not the other characteristic of perfect competition, unique price determined by higgling of the market.

Crossing Mont Blanc from Italy into France, at a little mountain inn which is perched high above Chamonix I had some refreshment in the charge for which one item was a franc for a slice of bread. I don't suppose that this franc had any exact correspondence with the marginal efforts and sacrifices requisite to produce bread and carry it up to that height. It might have been so if several of the Chamonix bakers had erected each a shop or booth on the rock and competed with each other. Supposing this to be impracticable under the circumstances, it might still be open to any citizens of Chamonix to start a rival *auberge*, if it appeared that the proprietor of the existing one was making more than ordinary profits (account being taken of all the hardships incident to the business). In such a case we have " industrial " without " commercial " competition. The case is perhaps

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The rationale of this uncertainty, as well as others which have been noticed, is to be found in the same " dominant fact in the theory of Monopoly " (above, p. 94, note). In competition we are concerned only with the rise in the amount demanded at each price, the *variation* of Cournot's function $F(p)$, say $\delta F(p)$. If this is positive, the price must rise, the law of decreasing returns prevailing. In monopoly we have also to look to the sign of $\frac{d}{dp}\delta F(p)$, which is not usually given.

[1] See article in *Giornale degli Economisti* (E.)

[2] Above, p. 76. The effect of such a tax on the margin of production is exhibited by Colonel Barone in the second theorem of " Teoria Matematica dell' Imposta " (*Giornale degli Economisti*), 1894.

common where there is a large establishment and a comparatively small number of customers.

As Professor Walras has well observed, these conditions are realised much more often than is supposed—

" tel fabricant de chocolat, qui vend son chocolat 3 francs la livre quand il le débite sous le nom modeste de ' chocolat superfin' enveloppé simplement de papier glacé le vendra 4 francs la livre à la condition de le parfumer à la vanille et de le débiter sous le nom de ' chocolat des princes,' enveloppé de papier doré. Il en est de même des différentes places d'un théâtre, dont les différents prix ne sont nullement proportionels au frais de production de ces places." [1]

In such cases the incidence of taxation will partly obey the law of monopoly and partly that of competition. For marginal taxes not large enough to overcome the friction which resists mobility, it should seem that the law is rather that of monopoly than of commercial competition, our a B C D rather than our A B C D. One difference would be that, where the demand of the consumer is very urgent, the tax would fall on the producer to a greater extent than is to be expected in a regime of perfect competition.[2] In general the consequences of the tax become more unpredictable in virtue of the property under consideration; a difference which is perhaps aggravated by the circumstance that the case is apt to be one of joint production.

a_1 b. The law of " increasing returns " does not constitute such an important subdivision of monopoly as of competition. There is no difficulty in the case of monopoly in understanding how production should stop short in the full career of the law of increasing returns, the diminution of the cost being more than counterbalanced by the falling off of demand.[3] The principal difference between the case of increasing and decreasing cost, is, I

[1] *Économie Politique Pure*, p. 416; *cp.* Schönberg's *Handbuch ;* article by F. Neumann, especially ii. 3; Acworth, *Railways and Traders*, ch. iv. So Hermann (*Untersuchungen*, ch. vi. p. 419, ed. 1870) of a shopkeeper who uses smuggled wares along with wares which have paid duty : " Stellen sich die preise höher als die schmuggelpreise tiefer als die preise der verzollten waare." It is not easy to see how this can be, unless the shopkeeper enjoys some of the properties of a monopolist. Professor Marshall points out that the " dominant fact in the theory of monopolies " (*cp.* above, p. 94) " is dominant also in the case of any producer who has a limited trade connection which he cannot quickly enlarge."

[2] Above, p. 90, note 3.

[3] Regarding the action of increasing returns as essentially different in the case of monopoly and competition, I cannot quite accept Professor Seligman's statement with respect to a (specific) tax on a monopolised article. " Of course the same qualifications are to be introduced as before [in a regime of competition] according as the monopoly industry obeys the law of the constant diminishing or increasing returns " (*Shifting and Incidence*, p. 161). Not " of course," surely.

think, that the price is apt to be raised—will be raised *ceteris paribus*—to a greater extent in the case of decreasing than in that of increasing cost.[1]

a_2. The remaining branch is the shortest, yet not perhaps the least fruitful with regard to general economic theory, which rather than finance is here cultivated. Where two or more monopolists take part in a system of bargains there are no laws of incidence, for there are none of value. It has long been recognised that the bargain between two monopolists [2] is indeterminate; it is now submitted that a system of bargains in which two or more monopolists deal, not directly with each other, but with one or more competitive groups, is also indeterminate. Suppose, for instance, a miller dealing with two monopolists, one the owner of the ground, the other of the water-power—as in an example put by Professor Marshall,[3] the only writer, so far as I know, who has given a hint of this theory. Let there be a number of such millers in competition with each other; and let the ground rent and the water rent be subject to revision from time to time, *e. g.* every year. The monopolists might go on for ever shifting the rents, making moves against each other like two chess-players when on each side there survives only the king with one or two inferior pieces.

The theorem may have some bearing on a system which is regarded by some as the ideal of the economic future, that each industry should be consolidated into a "Trust" or combination. Such a system would be characterised by instability, by fluctuations of prices such as now occur in railway wars, but more prolonged; for in so far as the combatants, like the two landlords in the example given, are not direct competitors, the combat seems less likely to be terminated by either the ruin of one party or the amalgamation of the two. That consummation may be more apt to occur when the two monopolists supply, not complementary articles, like land and water-power, but rival commodities. But even in this case the proposition that value is between

[1] I am compelled to differ from Professor Graziani (*Istituzioni delle Finanze*, p. 338) on this point. As already pointed out (above, p. 90) the imposition of a tax u per unit of commodity is in general of the form $u \times A \div (2A+B+C)$; where A is always negative, B is negative or positive according as the law of decreasing or increasing returns prevails; nothing is known of the sign of C—it represents the element of chance in the theory of monopoly (Index, *A priori probabilities*). Accordingly, *ceteris paribus*, A and C being constant, the increase of price is *greater* when the law of increasing returns prevails.

[2] Sidgwick, *Political Economy*, Book II., ch. x., § 3; Jevons, *State in Relation to Labour*, p. 154; Böhm-Bawerk, *Positive Theory of Capital*, Book IV. ch. ii.; cp. *Mathematical Psychics*, pp. 21 *et seq.*, by the present writer.

[3] *Principles of Economics*, Book V. ch. x.

certain limits—over a certain range of price—indeterminate may well be of considerable theoretical importance.[1]

For further discussion of this and other subjects touched upon here, I once more refer to the contemporary article on the Pure Theory of Monopoly in the *Giornale degli Economisti.*

The character of pure theory, deduction from received first principles, attaches not only to the incidence of taxes, which has been considered in the preceding paragraphs, but also—in a minor degree, doubtless—to the distribution of the fiscal burden among the taxpayers, which is to be considered in what follows. There is at least one aspect of this subject which may present sufficient length of reasoning and strength of premises to deserve the title " pure." The view thus distinguished is that according to which the sacrifice felt by the taxpayer is a dominant factor in the apportionment of the fiscal burden, the hedonistic, or in a special sense utilitarian, principle of taxation, as it may be called. Some other principle may be held—for instance, that of " ability," or " faculty," in a more objective sense,[2] but can hardly be held to belong to the domain of pure theory.

[1] The case of wayleaves on mines to which Professor Marshall directs attention (*loc. cit.*) may illustrate the general principle, or rather absence of principle. It appears from evidence given before the Commission on Mining Royalties that it is not always possible to make the negotiation for royalty and wayleave concurrent (Q. 590). The lessee who, having sunk much capital on a mine, wants to take up an adjoining mine " deals with a halter round his neck " (Q. 561); he has been " got into a cleft stick " (Q. 673; *cp.* Q. 5690). There is also much evidence that " the royalty is reduced by the rate of wayleave paid " (Q. 13,151). " A lower royalty is taken than would be if not subject to wayleave " (Q. 1933; *cp.* Q. 13,218, 11,306, 13,151, 13,749, etc.). Let us imagine these transactions to be effected successively, as thus : The first step is to take a lease of a coal-field for a certain period. The second step is to take a wayleave or some other subsidiary privilege. The terms exacted for this privilege being onerous, the third step would be on a revision of the first lease to lower the rent for the coal-field. The fourth step, it may be supposed, would be on a revision of the other lease to still further raise the wayleave. But this result is not certain, or not certain to continue as the series of steps is prolonged. It may become the interest of the owner of the subsidiary privilege to lower his terms in order to encourage the industry, wayleave being a charge of so much per ton. The principal landlord may retort by raising his terms; and the see-saw may go on *ad infinitum.* The lessee would not necessarily be worse off than if he had to deal (for both coal-field and subsidiary privileges) with a single monopolist—except so far as the instability of value is harassing to industry.

Of course I do not suppose such a prolonged series of steps as I have described to occur in the concrete. But I think it is a legitimate fiction in order to bring out the contrast that, whereas in a regime of competition a series thus continued theoretically tends to a definite position of equilibrium (as illustrated *e. g.* by Professor Walras in his *Économie Politique Pure*), in a regime of monopoly there does not exist, even theoretically, a determinate position of equilibrium.

[2] These terms are often applied to the principle of subjective sacrifice. *Cp.* Cohen-Stuart, *Bijdrage tot de Theorie der Progressieve inkomstenbelastung,* ch. i

The purest, as being the most deductive form of utilitarianism is that from which Bentham reasoned down to equality.[1] There are those who regard this form as also purest, in that its first principle is the most apt to be universally accepted. That principle proposes as the end of action, or criterion of conduct, the greatest sum-total of happiness; the intensification of pleasure, its prolongation and distribution among increased numbers being approved only when they conduce to that end. The conception has been formulated mathematically by the present writer.[2] In extending the summation of pleasure, according to the formula, over all time and all sentience, it is to be considered that, just as egoism is never so perfect but that distance in time renders pleasure less attractive, so utilitarianism is never so perfect but that persons whose interests are widely separate will not each "count for one" to the other.[3]

This remark may seem particularly appropriate to the adoption of utilitarianism as the rule of political action. The average citizen cannot be expected to care much for the interests of the foreigner, perhaps not very much for the interests of fellow-citizens outside his own class, nor at all for a remote posterity.

The proof of utilitarianism as the principle of political action has been variously conceived. The same speculative height is reached by different paths.[4] There is an approach on the economic side which it may be allowable to point out here. Let it be granted that there is a certain analogy between political and industrial co-operation or concerted action—an analogy admitted by many high authorities with respect to the fiscal action of the State.[5] We must not regard as an essential feature of the

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The connection between the two meanings is very happily explained by Professor Seligman, *Progressive Taxation*, p. 191. Bentham inveighs with characteristic vehemence against the ambiguity of the term "faculties" in the French Revolutionary Declaration of Rights (*Works*, Vol. II. p. 518).

[1] "Propositions of Pathology upon which the Advantage of Equality is Founded," *Principles of the Civil Code*, Part I. ch. vi. (*Works*, Vol. I. p. 304); *Constitutional Code*, Book I. ch. iii. sec. v. (*Works*, Vol. IX. pp. 14–18) *et passim*. J. S. Mill's doctrine of equality is not so clear (*Utilitarianism*, p. 93). Fitzjames Stephen complains of its obscurity with some reason (*Liberty, Equality, and Fraternity*, ch. v.).

[2] *Mathematical Psychics*, p. 57 *et seq.* On the pleonastic words "of the greatest number" commonly suffixed to "greatest-happiness," see *ibid.* p. 117, and *cp.* some good remarks by Professor Montague at p. 34, of his preface to Bentham's *Fragment of Government*.

[3] *Ibid.*, Appendix IV., "On Imperfect Egoism."

[4] There are some valuable reflections on Bentham's proof, or want of proof, in Professor Sidgwick's article on Bentham in the *Fortnightly Review* for 1877, Vol. XXI. p. 647.

[5] *E. g.* the well-conceived analogy between the State in its fiscal capacity and a

analogy the circumstance that in economic bargains there generally prevails a rate of exchange corresponding to final utility. That circumstance is brought about by competition, which does not exist in the case of the political contract. It is therefore improper, with Lord Auckland, to call income-tax " a fair price for protection," [1] to ask with Thiers, " What is society if not a stock company in which every one has more or fewer shares ? " [2] This is the fundamental fallacy of the " *quid pro quo* principle " or " benefit theory," which is justly rejected by J. S. Mill,[3] Professor Seligman, and other high authorities. The " economic " theory of taxation propounded by Professor Sax, his " conception of tax as a value-phenomenon," [4] appears open to a similar objection. The truer analogy is with those economic bargains which are not governed by competition; for instance, an agreement between an employer or an association of employers and a trade union, or, as in the case of a " boundary " dispute, two trade unions. Is there any general principle governing such agreements ?

The present writer has suggested, as the principle apt to be adopted by two [or, *mutatis mutandis*, a few] self-interested parties contracting in the absence of competition, the greatest-happiness principle, slightly modified : that arrangement to be made which conduces to the greatest sum-total welfare of both parties, subject to the condition that neither should lose by the contract.[5] Of course each party would rather have his own way completely. But the action of self-interest being suspended by mutual opposition, the more delicate force of amity which even in economic men is not entirely wanting,[6] may become felt. Moreover, each party may reflect that, in the long run of various cases, the maximum sum-total utility corresponds to the maximum individual utility. He cannot expect in the long run to obtain the larger share of the total welfare. But of all principles of distribution which would afford him now a greater, now a smaller

co-operative institution in Professor De Viti's *Carattere Teoricetico dell' Economia Finanzieria*, p. 103, *et seq.* *Cp.* Professor Graziani, *Istituzioni*, lib. 3, cap. 4, and authorities there cited.

[1] Quoted by Professor Seligman, *Progressive Taxation*, p. 96, in which context many similar references will be found. [2] *Ibid.*

[3] *Political Economy*, Book V. ch. ii. § 2, par. 2.

[4] " Die Progressivsteuer," p. 87 *et seq.*, in the Austrian *Zeitschrift für Volkswirtschaft*, Vol. I. Part I. Cp. *Grundlegung, passim.*

[5] *Mathematical Psychics*, p. 53.

[6] Much evidence was given before the Labour Commission as to the beneficial effects of " closer acquaintanceship between the parties." (Group A, Q. 607. *Cp.* Q. 2,019, 15,072-3, etc.).

proportion of the sum-total utility obtainable on each occasion, the principle that the collective utility should be on each occasion a maximum is most likely to afford the greatest utility in the long run to him individually.[1] Thus the recommendation of utilitarianism to self-interested parties would not be—as Bentham's teaching has been said to be—like making ropes out of sand.[2] *A fortiori*, the higher the degree of public spirit which is ascribed to the parties.

On these or other grounds assuming the greatest-happiness principle to be the test of governmental action, at least with respect to taxation, let us proceed to apply the principle. The primary problem is to determine the distribution of those taxes which are applied to common purposes, the benefits whereof cannot be allocated to particular classes of citizens. The condition that the total net utility procured by taxation should be a maximum then reduces to the condition that the total disutility should be a minimum.[3] From the condition that the total disutility should be a minimum, it follows in general that the marginal disutility incurred by each taxpayer should be the same. But if the inequality of fortunes is considerable with respect to the specified amount of taxation, there may not be taxation enough to go round, so to speak. The solution of the problem is that the higher incomes should be cut down to a certain level. At the same time the fact that the general marginal condition is not perfectly satisfied, suggests the solution of a wider, a *secondary* problem, namely, to determine the distribution of taxation, not being limited to that amount of which the benefit is indiscriminate. The solution of this problem in the abstract is that the richer should be taxed for the benefit of the poorer up to the point at which complete equality of fortunes is attained.

[1] Thus it would appear reasonable that a foreman who is insolent to the workpeople, and not particularly serviceable to the employer, should be dismissed, if thereby the employer's profits are not sensibly diminished, while the workpeople gain considerably in freedom from annoyance; or, again, that workmen should consent, on terms not extravagant, to do a little extra work on an emergency, if thereby the employer is saved from considerable loss. But to illustrate fully the applicability of the principle would be out of place in this article.

[2] *Cp.* Professor Sidgwick in *Fortnightly Review*, *loc. cit.*

[3] The authority of Bentham may be cited in favour of this theory of taxation :—

"It is therefore necessary that those who create wealth by their labour should give up a portion of it to supply the wants of the guardians of the State. . . .

"All government is only a tissue of sacrifices. The best government is that in which the value of those sacrifices is reduced to the smallest amount (*Principles of the Civil Code*, Part I. ch. xiii., *Works*, Vol. I. p. 13, ed. 1859).

"To take care that this pain of constraint and privation be reduced to the lowest term" (*View of a Complete Code of Laws*, ch. xxix., *Works*, Vol. III. p. 204).

The *acme* of socialism is thus for a moment sighted; but it is immediately clouded over by doubts and reservations.

In this misty and precipitous region let us take Professor Sidgwick as our chief guide. He best has contemplated the crowning height of the utilitarian first principle, from which the steps of a sublime deduction lead to the high tableland of equality; [1] but he also discerns the enormous interposing chasms which deter practical wisdom from moving directly towards that ideal.

"In the first place it is conceivable that a greater equality in the distribution of produce would lead ultimately to a reduction in the total amount to be distributed in consequence of a general preference of leisure to the results of labour on the part of the classes whose shares of produce had increased." [2]

There is also the danger—

"That the increase through equalisation of the incomes of the poorer classes will cause the population to increase at a more rapid rate than at present; so that ultimately the increment of an average worker's share will be partly spent in supporting a larger number of children, and partly reduced through the decrease in the efficiency of the more crowded labour." [3]

It is remarkable that Mill should have apprehended the dangers of deficient production and excessive population less than the danger to liberty. The weighty sentence into which he condenses the substance of his teaching on liberty deserves to be repeated.

"It is yet to be ascertained whether the Communistic scheme would be consistent with that multiform development of human nature, those manifold unlikenesses, that diversity of tastes and talents, and variety of intellectual points of view which not only form a great part of the interest of human life, but in bringing intellects into stimulating collision and by presenting to each innumerable notions that he would not have conceived of himself, are the mainspring of mental and moral progression." [4]

Liberty is not the only one of the higher goods which is threatened by a dull equality: there is also the "function of maintaining and developing knowledge and culture," the performance of which function, as pointed out by Professor Sidgwick, has hitherto been largely due to "rich and leisured persons." [5]

[1] *Principles of Political Economy*, Book III. ch. vii. § 1.
[2] *Loc. cit.* § 2.
[3] *Ibid.*
[4] *Political Economy*, Book II. § 3.
[5] *Ibid.*

The transition is easy to another reservation, which is in some sense more intrinsic than the preceding. The Benthamic argument that equality of means tends to maximum of happiness, presupposes a certain equality of natures: but if the capacity for happiness [1] of different classes is different, the argument leads not to equal, but to unequal, distribution. The testimony of Professor Sidgwick that Bentham would probably have recognised this reservation [2] carries a double weight of authority. The possibility corroborated by so high evidence is calculated to temper the more drastic applications of utilitarianism.

The preceding reservations relate to the pursuit of socialistic equality by any methods; the following relate more particularly to the pursuit of that end by means of taxation. A progressive tax rising to such a rate that it would not be the interest of the taxpayer to increase his fortune by saving or enterprise above a certain amount,[3] while improving the distribution, would check the augmentation of the community's wealth. There is, however, to be set off the probable increase of saving among the poorer classes.[4] Especially the investment of capital in persons by way of education might be increased.[5] There would be an increase of production also so far as the proceeds of socialistic taxation are applied to render the poorer classes more efficient.[6] But against this increase in the efficiency of the poor might

[1] This terminology has been employed by the present writer (*op. cit.*, p. 57; *cp.* pp. 64, 125) to designate differences both in the amount of means which different individuals may require in order to attain the threshold or zero-point of happiness, and in the amounts of utility which they may derive from the same additions of means above that point. Compare Professor Carver's weighty observations (*American Acad. of Polit. Sci.* 1895, p. 82) upon difference in *wants*—a term which may also refer to differences in the amount of means needed for efficiency.

[2] Professor Sidgwick says: "I do not however think that Bentham intended to deny (1) that one person may be more capable of happiness than another, or (2) that, if so, the former's happiness is more important than the latter's, as an element of general happiness." (*Elements of Politics*, p. 583, note 2. *Cp.* note 3 for a fuller statement of Professor Sidgwick's own view.)

[3] The condition which a progressive tax must fulfil in order not to have this effect, is clearly expressed by Professor Ulisse Gobbi in a recent tract to which the chaste use of mathematics lends a peculiar charm. (*Sul carattere razionale dell' imposta progressiva*, § 3.)

[4] *Cp.* Bastable, *Public Finance*, 2nd edition, p. 295, and, with special reference to the problem of distribution with which Trade Unionism is concerned, Professor Marshall's *Economics of Industry* (1879), p. 202.

[5] *Cp.* Marshall, *Principles of Economics*, *sub voce*, Discount of future pleasure; also Sidgwick's *Principles of Political Economy*, Book III. ch. ii.

[6] The exemption of a minimum is to be defended not only after Bentham (*Constitutional Code*, ch. xv., *Works*, Vol. I. p. 319) on the ground of least sacrifice, but also on the ground of greatest efficiency.

have to be set some decrease in the efficiency of the not very rich.[1]

Again, there is the general presumption against governmental action, the special danger that taxation extended beyond its proper objects will be abused. The warning comes with less weight from those who are ready to employ taxation for a collateral purpose of which they themselves approve—the correction of intemperance.

In fine, the increase of taxation is limited by evasion.

These extensive, though briefly indicated, reservations reduce the *prima facie* revolutionary dictates of pure utilitarianism to the limits of common sense. The position thus defined is much the same as Mill's.[2] "That the State should use the instrument of taxation as a means of mitigating the inequality of wealth" is not to be demanded when a "tax on industry and economy," a check to the growth of wealth, is thereby imposed.[3] But the utilitarian will be as "desirous as any one that means should be taken to diminish those inequalities": such means as the limitation of inheritances and the taxation of unearned increments, so far as these means are free from the dangers above enumerated. A similar reconcilement between equality and security [4] is taught in an article replete with utilitarian wisdom on *The Ethical Basis of Distribution*, by Professor T. N. Carver.

"The minimum amount of repression [or check to the growth of wealth] is secured by imposing an equal sacrifice on all members of the community, but the minimum amount of sacrifice is secured by collecting the whole tax from those few incomes which have the lowest final utility. No rational writer advocates the latter plan exclusively, but many rational writers do advocate the former plan. Yet it is not beyond dispute that the former plan ought to be followed exclusively." [5]

This passage, read with the context, almost exactly expresses the thesis here maintained; except that the last sentence is asserted rather too diffidently, and the first clause much too confidently. *Minimum sacrifice*, the direct emanation of pure

[1] For an instance see below, p. 120.

[2] For some differences from Mill see below, p. 115.

[3] Mill, *Political Economy*, Book V. ch. ii. § 3.

[4] The reconcilement is taught by Bentham with respect to legislation in general. See the forcible chapter on "Security and equality—their opposition," with the context (*Principles of Civil Code*, Part I. ch. xi. *et passim*); *e.g.* "It is not equality itself but only a tendency towards equality, after all the others [security, subsistence, and abundance] are provided for, that is the proper object of endeavour" (*Works*, Book III. p. 294).

[5] *Annals of the American Academy*, 1895, p. 97.

utilitarianism, is the sovereign principle of taxation; it requires no doubt to be limited in practice; but query whether the requisite limitation is to be obtained from *equal sacrifice*, or any of the cognate subsidiary forms of the hedonic principle which are presently to be considered?

Before leaving the principle of minimum sacrifice, let it be observed that, under the limitations which have been described, this principle may also be applied to justify differential taxation on the ground of differences in other respects besides size of income: for instance, difference in the permanence of the income,[1] differences in civil state,[2] number of children,[3] age,[4] and other attributes.[5]

Besides the principle of minimum sacrifice, which has been considered, there are other species of the hedonic theory of taxation. The most familiar are the principles of equal and of proportional sacrifice: that each taxpayer should sacrifice an equal amount of utility, or an equal proportion of the total utility which he derives from material resources. The former species is the commonest in England; the latter flourishes in Holland.[6] The two species might be included in a genus termed "like sacrifice."

It will be convenient to consider first the practical consequences, next the theoretical proof of these two principles.

In order to deduce conclusions from either premiss, there is required another premiss relating to the law of diminishing utility. There are some reasons for assuming—it is at least the simplest hypothesis—that utility diminishes in inverse ratio to

[1] It is hardly necessary to refer to Mill's magisterial discussion of this matter.

[2] An instance of taxation varying according as the taxpayer is married or not is referred to below on p. 133, note 2.

[3] All would not agree with Mill that having a large family, so far as concerns the public interest, is a thing rather to be discouraged than promoted.

[4] See the curious remarks of McCulloch on this subject (*Edinburgh Review*, 1833, vol. 57, p. 156).

[5] Dr. Robert Meyer has given a list of attributes (*Principien der gerechten Besteuerung*, p. 53). Of course it must be remembered that, as Mr. Cohen-Stuart points out, by attempting to make corrections without sufficient data we run the risk of making our result worse.

[6] The honour of clearly distinguishing these principles appears to belong to Mr. Cohen-Stuart (*Bijdrage tot de Theorie der Progressieve Inkomstenbelastung*, ch. i. § 4). He does not seem to deserve Professor Seligman's disrespectful criticisms: "Much ado about nothing" (*Political Science Quarterly*, Vol. VII. p. 337); "Cohen-Stuart takes a long time explaining this, but as we know it is nothing new, being precisely what Mill expressed in other words" (*Progressive Taxation*, p. 184; *cp.* as to Mill's formula, p. 136). It may well be doubted whether Mill entertained the notion of proportional sacrifice, or distinguished it from that of equal sacrifice. It is certain that the ideas have been confounded by other writers.

means, after the law of Bernoulli.[1] Upon this assumption the principle of equal sacrifice gives proportional taxation;[2] the principle of proportional sacrifice gives progressive taxation.[3]

But there seem to be better reasons for assuming that the utility diminishes with the increase of income at a faster rate. There is the testimony of high authorities, Montesquieu,[4] Paley,[5] J. B. Say[6] and many others cited in the learned pages of Professor Seligman. True, Mill regards the doctrine as "too disputable altogether" with regard to the higher incomes. But neither Mill nor any other considerable authority has held that the diminution is *less* than in the inverse ratio of the income. Mill's estimate being the lowest, we may take as the most probable estimate one intermediate between his and others, and assume that the utility diminishes at a rate exceeding the increase of income, if not for the highest incomes, at any rate for incomes considerably above the usually exempted minimum.

This presumption is confirmed by the observation that the property in question, the diminution of utility out of proportion to the inverse income, almost certainly holds for large differences as distinguished from differential variations; as Dr. Robert Meyer has well argued.[7] But, if such is the character of the utility-curve as to finite differences, it is probably also its character as to differential variations.[8] The observed circumstances would not be consistent with the prevalence of Bernoulli's law throughout. It must be assumed that for a considerable tract of the curve—supposed not violently discontinuous—the property in question prevails.

[1] The grounds of this provisional assumption are well stated by Professor Sax, *Die Progressivsteuer* (p. 78).

[2] As reasoned by Fauveau (*Considérations mathématiques sur l'Impôt*). It is important to observe that the proposition applies not only to taxes, which may be treated as infinitesimal, but also to integral imposts (*cp.* Cohen-Stuart, *Bijdrage*, Appendix I. p. 190).

[3] As reasoned by Cohen-Stuart, *op. cit.*

[4] *Esprit des Lois*, liv., xiii., ch. vii.

[5] *Elements of Political Knowledge*, forming Book VI. of *Moral and Political Philosophy*.

[6] *Cours*, Part VIII. ch. iv.; *Traité*, liv. iii., ch. ix.: "Si l'on voulait asseoir l'impôt de chaque famille de manière qu'il fût d'autant plus léger qu'il portât sur un revenu plus nécessaire, il faudrait qu'il diminuât non pas simplement proportionellement, mais progressivement."

[7] *Die principien der gerechten Besteuerung*, p. 333. Professor Sax's criticisms of this passage seem unnecessarily severe (*Progressivsteuer*, pp. 52–3). Professor Sax's own reflections (*ibid.*) confirm the assumption here made as to the character of the utility-curve, up to a certain point at least of the curve.

[8] The converse of this statement is proved by Cohen-Stuart in his first Appendix.

Some doubt may remain as to the extremity of the curve which corresponds to very high incomes. It has been supposed by several high and independent authorities, that ultimately the law of Bernoulli holds good. Some of the reasons assigned are to be found in the passages cited below from eminent authors.[1] It is here submitted that the character ascribed to the extremity of the utility-curve is not sufficiently evidenced. First, as to capitalisation, regarding it as an application of income to future gratifications (whether personal or vicarious), one does not see why it should not approach satiety with a rapidity greater than that which is assigned by the Bernoullian law. It may be suspected, too, that an improper inference is drawn from the circumstance that as the income is increased by equal increments the differences between the successive increments of utility become less. But it is not with these differences that we are concerned, but with the *ratio* between successive increments of utility. And there is nothing to show that this ratio does not increase more rapidly than according to the Bernoullian law. The pleasure derived from a certain income may well increase with the income somewhat as, according to the theory of errors of observation, the probability that an error will occur within a certain distance increases with the distance.[2] Ultimately the

[1] Professor Sax, *Die Progressivsteuer*, *loc. cit.*, p. 101 : " Durch diese Einbeziehung künftiger Bedürfnisse erfährt der Bedürfnissstand eine Erweiterung welche die grössten Dimensionen annehmen kann, sich dann aber eben auch auf Bedürfnisse von minimalen Stärkegraden erstrekt. Dieser Umstand in Verbindung mit der einleuchtenden Folgerung dass, da der Bedürfnisgrad nicht auf Null sinken kann, von einem gewissen Punkte niedriger Bedürfnissintensität an die Differenzen der durchschnittlichen Intensität auf einander folgender Bedürfnisgruppen rasch abnehmen müssen und somit das Verhältnis der umgekehrten Proportionalität der Einkommen annehmen, bedingt schliesslich die Aufhebung der Progression des Wertstandes, was folgerichtig die Progression der Steuer zum Stillstande bringt."

Professor F. J. Neumann, *Progressive Einkommen Steuer*, p. 146 : " Da in den höchsten Beträgen ein *sehr grosses* Einkommen . . . regelmässig etwas gleich entbehrlichen Genüssen oder aber der Capitalisirung dient."

Professor Traub, in a passage quoted at length by Mr. Cohen-Stuart (*op. cit.* p. 148), speaks of " het punt waarop de nuttigheidsgrad van het inkomen constant bligft."

Professor Graziani, *Giornale degli Economisti*, 1891, p. 164 : " E escluso il concetto d' un saggio [d' imposta] continuamente crescente, poichè s' è dimostrata l'impossibilita d' un continuo accrescimento nella differenza di valutazione fra ciascuna frazione successiva di ricchezza." Cp. *ibid.* p. 167. Professor Graziani here takes proportional sacrifice as the desideratum (*ibid.* p. 160).

[2] The marginal utility of money—the measure of the increment of welfare which corresponds to an increment of income—might quite well have some such form as the probability-curve, viz.

$$y\left(=\frac{du}{dx}\right) = Ae^{-(x-a)^2}$$

additions become imperceptible, but not the less do they obey the law that a disproportionately large increment of the independent variable is required to produce the same increment of the dependent one. In fine the view here combated has no doubt derived some adventitious aid from the supposed practical necessity of adopting a proportional income-tax for very high incomes; which could only be justified by the principle of equal sacrifice upon the assumption of the Bernoullian law.

It is to be admitted, however, that the property in question has been accepted by Mr. Cohen-Stuart, who cannot be suspected of mathematical confusion, and who has expressly distinguished the theoretical and practical points of view.[1]

Here are his reasons :—

"For the millionaire—or rather . . . the *milliardaire*—the possession of his income signifies no more than a cipher, the increase of which has no longer any influence on his consumption. To see the cipher increased by 4 per cent. for instance, if it is a pleasure to a man with 10 millions [francs per annum] or one with 100 or 500 millions, would be, I should say, about the same pleasure to each. . . . As soon as all personal wants are pretty well satisfied, and, *a fortiori*, after the income has passed this limit, its increase proportionately, that is by an equal percentage, must, as it seems to me, tend to afford an equal pleasure. That the addition of the same *amount* should be as strongly desired, should produce equal pleasure, however great the income, seems to me absurd; that the same *proportion* of the income should have this effect strikes me as rational." [2]

The mathematical reader who is not convinced by Mr. Cohen-Stuart on this point will hardly defer to others.

Upon the assumption that the diminution of marginal utility with income is (throughout) in excess of Bernoulli's law, the principle of equal sacrifice and that of proportional sacrifice both give progressive taxation, the latter in a higher degree than the former.[3] Either principle, but more probably the

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where x, the independent variable, is the amount of income, y, the dependent variable, is the marginal utility of income (the differential of u, the total utility of income); a is the minimum of existence, and A another constant. In order that the sacrifices (first supposed small) made by two individuals having incomes x_1 and x_2 should be equal, the respective contributions should be, not as $x_1 : x_2$, but as $e^{+(x_1-a)^2} : e^{+(x_2-a)^2}$. And this disproportion of contribution to income would not only be maintained, but increased, as the income is indefinitely increased. *A fortiori*, if proportional, not equal, sacrifice is aimed at. *A fortiori*, too, if the sacrifices are not small.

[1] *Op. cit.* p. 134. [2] *Op. cit.* p. 155.

[3] *I.e.* higher for any assigned form of the utility-curve, and amount of taxation.

latter, may (upon the assumption above made) lead to a subtraction of income so great as to leave the possessor little interest in increasing his income beyond a certain limit. The two varieties of like sacrifice may in this respect resemble the principle of minimum sacrifice in requiring to be limited by a regard for other disutilities beside the constraint and privation occasioned to the taxpayer.[1]

[1] The relation between the different modes of the sacrifice theory might thus be exhibited diagrammatically. Let Y measured along the axis OY represent size of income; and construct the curve II' such that the co-ordinate to any assigned Y represents the number of incomes smaller than Y. Thus the strip of area $Y\Delta X$

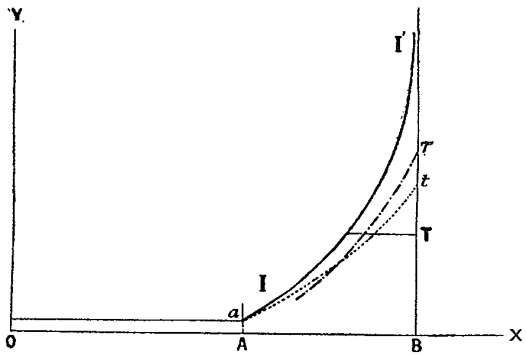

FIG. 13.

represents the portion of the national income which consists of incomes of the size Y. The curve II' is probably shaped as in the figure; in conformity to Professor Pareto's beautiful theory of income-curves ("Cours d'Économie politique pure: Courbe des révenus." *Cp.* ECONOMIC JOURNAL, Vol. VI. p. 666) II' may be regarded as asymptotic to—or at least terminating on—a perpendicular through B, where OB is the total number of incomes; Aa may be taken as a minimum exempted from taxation.

Then to apply the principle of minimum sacrifice, find a point, T, on the ordinate through B, such that the area intercepted by that ordinate, the horizontal through T and the curve II' should represent a portion of the national income equal to the required amount of taxation. To apply the principle of proportional taxation, find t on the same ordinate such that the required amount of taxation may be equal to the amount represented by the area intercepted between that ordinate, the curve II', and a curve at which the ordinate η at every point fulfils the condition

$$(y - \eta) \times \frac{du}{dy} \div u = \text{constant};$$

where u is the total amount of utility derived (on an average) from the income y. This curve is represented by the continuously dotted curve line in the figure (upon a certain supposition as to the minimum of exemption, *cp.* Cohen-Stuart, *op. cit.*, on the "Bestaans-minimum"). To apply the principle of equal sacrifice, find τ, on the same ordinate, such that the required amount of taxation may be

The method of applying the limitation might well be, for all the forms of the sacrifice theory, the use of such a scale of progression as would be given by the principle of proportional taxation *upon the supposition* that the extreme tract of the utility-curve was such as it has been conceived by Mr. Cohen-Stuart and others. Practical reasons, not deductions from any form of the first principle, would thus lead to a " degressive progression " culminating in a simply proportionate tax of the higher incomes, such as in fact seems to be coming into vogue.[1] Then those who hold the principle of proportional sacrifice might avail themselves of the curious theorem given by Mr. Cohen-Stuart, that an approximately proportional tax being imposed on the higher incomes, the law of progression for the tax on the lower incomes, as deduced from the principle of proportional sacrifice, would be much the same, however the law of utility might vary, between wide limits.[2]

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represented by the area contained between the ordinate, the curve II', and the curve of which the ordinate η fulfils the condition

$$(y - \eta) \times \frac{du}{dy} = \text{const.}$$

A part of this curve is represented by the discontinuously dotted (broken) line in the figure. If completed this curve may be expected to meet II' asymptotically below A. But, doubtless, regard to efficiency and other practical considerations may lead to the deformation of the curve, so as to join on at an earlier point to II'.

It will be apparent from this illustration that equal sacrifice is less socialistic than proportional sacrifice, and proportional sacrifice less so than minimum. But in what degree either mode of like sacrifice is less socialistic than minimum sacrifice, we have no data, it is submitted, for determining. It is quite possible that the curve through τ, and, *a fortiori*, the curve through t, should prescribe a taxation of the higher incomes, which in the interests of production it would be fatal to carry out. Like sacrifice can no more, or not much more, than minimum sacrifice be trusted to act without checks. What, then, is the ground for preferring like sacrifice ?

[1] Thus Dr. Westerdijk, in his very able article on the progressive income-tax in the Dutch communes in *De Economist* for March 1897, holds that " all reasonable supporters of progression " are at one in approving of Mr. Troub's principle which in practice is the same as Mr. Cohen-Stuart's, though not grounded on the same reasons. The formula which is given by Professor Ulisse Gobbi in his *Carattere razionale dell Imposta Progressiva* (1897) deserves consideration not only on account of its elegance, but also because it is based on an assumption which is frankly arbitrary, not identified with the sacrifice theory. The assumption is that there ought to be an inverse relation between the portion of tax incident on each successive increment of income and the importance of the wants provided for by that increment. Whence for extremely large incomes, as the difference in the importance of the wants provided by successive increments of income becomes indefinitely small, the difference between the portion of tax charged to each increment of income should become indefinitely small; or the tax should become more and more nearly proportional.

[2] Professor Sax's criticism of this arrangement as illogical is not justified by the logic of mathematical science. " From unproved premisses can never follow a

The distinction between like sacrifice and minimum sacrifice is not more serious when the principles are applied to differences in other respects besides amount of income (*cp.* above, p. 556). For example, whether is it easier to say that incomes which are not equally permanent should not be taxed equally, because the sacrifices would not be equal to each other, as Mill has it,[1] or because their total would not be a minimum, as here proposed? So the doctrine of minimum sacrifice, as well as that of proportional sacrifice, may use the proposition affirmed by Mr. Cohen-Stuart, that for the purposes of taxation the married differ from the unmarried chiefly in having a higher minimum of exemption.[2]

To have deduced the precepts will aid us in estimating the authority of *like sacrifice*. This may best be effected, from the utilitarian point of view adopted in this article, by determining the relation of the principle under consideration to the supreme principle. That relation is one of complete autonomy, if like sacrifice is prescribed by intuitive justice. But the utilitarian will not accept this *imperium in imperio*. He will object to like *sacrifice* thus supported, what several acute dialecticians [3] have objected to proportional sacrifice, that its propriety is not self-evident.

A position more neutral with regard to utilitarianism is taken by Mr. Cohen-Stuart when he maintains that proportional sacrifice, leaving the relation between all the parties in respect of welfare unchanged, is *the* principle for the Manchester man.[4]

conclusion that is to be regarded as proved; from mere assumptions, never a real fact." And much to the same effect in the context (*Progressivsteuer, loc. cit.* p. 82). Now the character of certainty in the conclusion with uncertainty in the premisses is one which frequently is presented in that branch of mathematical science which, as being applied to human affairs, is nearest akin to pure economics, namely, the calculus of probabilities. In ascertaining the probability that a given effect has resulted from a certain cause, it is generally necessary to deal with certain quantities termed *a priori* or antecedent probabilities, about which nothing is known, except that they are not very small [or very great, or very unequal]. Thus Mill, of such an argument: "it would be impossible to estimate that probability with anything like mathematical precision," yet "a practical decision can generally be come to without much hesitation." *A priori* probabilities of this character are involved in the received treatment of physical observations.

[1] *Political Economy*, Book V. ch. ii. § 4, note.

[2] *Bijdrage*, pp. 140–145. The judicious author here repeats the warning that we may overshoot the mark by attempting too great accuracy. Compare the exemptions in favour of the married in the scheme proposed by the Grand Council of Berne (but rejected by the referendum), *Report on Graduated Taxation in Switzerland*. C. 6856.—15 (1892).

[3] *E. g.* Graziani, *Istituzioni*, p. 301; Sax, *Progressivsteuer*, p. 62.

[4] *Bijdrage*, ch. v. This proposition can hardly be said to have the character of an axiom. What Walker and Professor Seligman call the "leave-them-as-you-

Somewhat similar appears to be the position of Professor Sidgwick, except that he has explicitly recognised the supremacy of the greatest-happiness principle, and admits the possibility of its being employed to promote equality by taxation. But he regards this direct interposition of the supreme principle as liable to a dangerous excess. The principle of equal sacrifice is therefore deputed to act, a deputy not liable, like the principal, to be betrayed into imprudent concessions. This paraphrase is based on the following passages and their context :—

"The obviously equitable principle—assuming that the existing distribution of wealth is accepted as just, or not unjust—is that equal sacrifices should be imposed on all; and this [is] also obviously the most economic [1] adjustment of the burden except so far as it is thought desirable to make taxation a means of redressing the inequalities of income that would exist apart from governmental interference."

"The introduction, however, of this latter principle to any marked extent involves the danger" [described in the preceding chapter]. "And the danger is much greater here . . . because if the principle is applied at all, any limit to its application seems quite arbitrary."

The position of Mr. Cohen-Stuart with respect to proportional sacrifice and that of Professor Sidgwick with respect to equal sacrifice are tenable, so long as we hold with Mr. Cohen-Stuart that the utility-curve is ultimately of the Bernoullian form. But if, as above contended, this premiss is not tenable, then a rapidly progressive taxation following from the principle of proportional sacrifice, the Manchester man could hardly be expected to acquiesce in that principle. Nor could the principle of equal sacrifice be safely deputed to act on behalf of the supreme principle. Of the deputy as well as of the sovereign, we might then say: "if the principle is applied at all, any limit to its application seems quite arbitrary." At any rate, the only certain limit to the application of equal sacrifice—viz. that more than the necessary minimum of taxation should not be

find-them theory" is, in the opinion of many, best carried out by a simply proportional income-tax. Thus, McCulloch, in the *Edinburgh Review* (Vol. LVII. p. 162), protests against a graduated income-tax, on the ground that "no tax is a just tax unless it leaves individuals in the same relative condition in which it found them." So Courcelle-Seneuil speaks of the proportional income tax as "qui altererait le moins l'état de distribution résultant de la liberté" (*Traité théorique et pratique*, ii., p. 218).

[1] "Economic" is, of course, used here in the same sense as in the preceding chapter; that is, nearly equivalent to utilitarian in our first sense. (See Book III. ch. vii. § 1, par. 1)

raised and more should not be required from the higher incomes than would bring down all above a certain level to that level [1]— is greatly in excess of practical limitations. Is it not simpler to dismiss the deputed principle of equal sacrifice, and to adopt as the true norm of taxation minimum sacrifice tempered by a regard for the growth of wealth and other advantages above enumerated?

The capacity of like sacrifice to act independently is even less upon another view of its authority. What if, as compared with the utilitarian code, it is not a sort of by-law, as just now in effect supposed, but simply a clause, a badly-transcribed clause, of the code itself? What if *equal sacrifice* is but a corrupt reading for *equi-marginal sacrifice*, the condition of minimum disutility? [2] Thus Mill, in that classical passage which has influenced the most influential of his successors,[3] in the same breath proclaims the principles of equal and of least sacrifice :—

" Whatever sacrifices it [a government] requires from them [persons or classes] should be made to bear as nearly as possible with the same pressure upon all, which, it must be observed, is the mode by which least sacrifice is occasioned on the whole." [4]

It is remarkable that in support of one of the principal reforms with respect to taxation which he advocated, Mill should have employed the genuine utilitarian reasoning in favour of equality rather than the questionable principle of equal sacrifice.[5] Thus when he first introduces the proposal to limit inheritances :—

" It must be apparent to every one that the difference to the happiness of the possessor between a moderate independence and five times as much is insignificant when weighed against the enjoyment that might be given, and the permanent benefit diffused by some other disposal of the four-fifths " (*Political Economy*, Book II. ch. ii. § 4).

And in a later chapter on inheritance, he refers to " the

[1] As appears from the position of the curve through t in note to p. 111.

[2] Above, p. 103.

[3] Both Mr. N. G. Pierson (*Staathuishoudkunde* 1886, p. 310) and Sir Robert Giffen (Evidence before the Financial Relations Commission Q. 7777) profess to follow Mill.

[4] Mill, Book V. ch. ii. § 2, par 1. The divergence between the principle of equal sacrifice presented by Mill and that of minimum sacrifice is indicated by the present writer (*op. cit.* p. 118). Professor Carver calls attention to the fact that Mill affirmed the two divergent principles in the same passage (*Amer. Acad. for Pol. Sci.*, 1895, p. 95).

[5] Bentham is always clearer than Mill in the deduction of equality from greatest-happiness, because he virtually employs the differential calculus : adding and subtracting " particles of wealth," as in *Pannomial Fragments* (*Works*, Vol. III. p. 231).

deeper consideration that the diffusion of wealth, not its concentration, is desirable " (Book V. ch. ix. § 2).

So McCulloch can see no halting-place, such as the principle of equal sacrifice is supposed to supply, between a proportional income-tax and that levelling of the higher incomes which, as above shown, is the inference from the principle of minimum sacrifice.[1]

So Dr. Robert Meyer describes as " the commonest argument " in favour of progressive taxation one which rests upon an interpretation of equal sacrifice which makes it virtually identical with equi-marginal sacrifice.[2]

So some of the high authorities who have advocated progressive taxation on the ground of equal sacrifice may be credited with an " unconsciously implicit " [3] utilitarianism of the pure type. Mr. Cohen-Stuart indeed has argued that several of these high authorities hold the principle of proportional sacrifice. For whereas they have deduced progressive taxation from the principle of equal sacrifice, *simpliciter* and without any *datum* as to the law according to which utility diminishes, this fallacious reasoning is explicable, he thinks, on the former supposition, but on the latter inexplicable. But what if there was in the confused minds of these distinguished publicists not *equal* sacrifice nor yet *proportional* sacrifice, but *equi-marginal* sacrifice (leading to *minimum* sacrifice)? It is true that this premiss is less consonant to their statements than the other. But then their conclusion really does follow from this premiss. Obliged as we are to make a compromise between obscure premisses and fallacious reasoning, may not the line of least confusion, so to speak, be—not the assumption that the premiss was somewhat obscure and the reasoning somewhat erroneous, but—that the premiss was quite confused and the reasoning quite exact?

Altogether, whatever view we take of the relation of the principle of like sacrifice to pure utilitarianism, the sphere of its action independently of that supreme principle appears to be insignificant.

The proposal here made to substitute *minimum* for *equal* in

[1] *Edinburgh Review*, Vol. LVII. p. 164 (1833).

[2] " Das gewönhlichste Argument dass die proportionale Steuer bei grösseren Einkomen deswegen ein geringeres Opfer verursache, als bei kleineren, weil sie nur entbehrlicheren Bedürfnissen die Befriedigungsmittel entzieht beweist zu viel " [*i. e.* it leads to the conclusion above deduced from the principle of least sacrifice, that the higher incomes should be cut down to a lower level.] *Op. cit.* p. 331.

[3] The happy term applied by Professor Sidgwick to the utilitarianism which is latent in current ethical opinion.

the " sacrifice " theory of taxation will not modify considerably the practical directions afforded by that theory. Rather the obligation to follow those directions is made clearer. As of old, before the invention of the compass, the star-steered sailor would not sensibly have altered his course if he could have discovered, by the use of a telescope, that what he had regarded as a luminous point was really a double star, composed of bodies separated from each other many million miles; so in the present state of financial science, affairs being at such an enormous distance from principles, the discovery that the sacrifice theory comprises several distinct ends is not calculated to result in a serious alteration of the line of conduct indicated. Rather the use of our speculative instruments in separating the species of this theory of taxation conduces to keeping in view the generic principle, in spite of distance and obscurity. The use of *minimum*, instead of *equal*,[1] sacrifice enables us to pierce the sort of metaphysical mist which has been raised by the question: *Why* should the principle be adopted?[2] The question is not embarrassing to those who regard minimum sacrifice as a deduction from the greatest-happiness principle—" the only possible, the only conceivable principle which can guide legislation on a great scale."[3] Again, there is a want of clearness in the reasoning from the principle of equal sacrifice, because in order to obtain any conclusion some assumption must be made as to the rate at which the increase of utility tends to diminish with the increase of means;[4] while " to ascertain the exact relations between something psychical and something material is impossible."[5] But the reasoning from the principle of minimum sacrifice assumes no exact relation between utility and means; it assumes only what is universally admitted, that utility does not increase proportionately to means, the Jevonian " law of diminishing utility." Again, some confusion is caused by the conflict between the two forms of equal sacrifice; equal in a proper sense and proportional. But the pure utilitarian has no difficulty in accepting both principles as equally inexact but equally useful approximations to the true principle; or rather that of proportional sacrifice as more exact, being more in accordance with minimum sacrifice,

[1] In the following paragraphs dealing with practice it has seemed best as most agreeable to usage, to occasionally employ the term " equal " generally, covering *proportional* as well as *equal* in the proper sense (see above, p. 107).

[2] Asked, *e.g.*, by Professor Graziani with reference to the Dutch form of the doctrine, *Istituzioni*, p. 301.

[3] Sir Henry Maine, *Political Institutions*, last page.

[4] Above, p. 107. [5] Seligman, *Progressive Taxation*, p. 136.

equal sacrifice as more useful, in this country at least, as being more familiar.

To take as a concrete example the problem presented by the financial relations between England and Ireland, the general result of the preceding considerations would be to strengthen that argument in favour of the Irish claim which is grounded by some high authorities on the principle of equal sacrifice, and to weaken those objections to the argument which are grounded by other high authorities on the indefiniteness of the principle. This correction would be the less important in so far as the consideration of sacrifice has to be combined with other considerations not admitting of precision.

There are several considerations special to the concrete problem which can only be adverted to here—put in the form of questions to which the answers are not given. (1) What disbursements from the Exchequer are applied to common purposes, the benefits of which cannot be allocated to particular regions, any more than to particular individuals? Does "the expenditure . . . . for the general objects for which a government is carried on, namely, for the administration of justice, the maintenance of order, the maintenance of the poor" [1] and so forth, belong to this class? (2) Whether, if much of the taxation complained of is raised by duties on stimulants, the privation thereby occasioned to the consumers should be left entirely out of account, and no part of it should be reckoned in that total of sacrifice which it is sought to minimise, because some part of the taxed commodities is consumed intemperately? (3) Whether it is any mitigation of such sacrifice that the consumer of the taxed articles was free to abstain therefrom; or whether it is not only demonstrated by the classical political economy [2] that taxes on commodities are not less burdensome, but also demonstrable by the modern mathematical method [3] that they are

[1] Sir Robert Giffen, in evidence before the Royal Commission on the Financial Relations between England and Ireland (Q. 11,024).

[2] See Mill, Book V. ch. vi. § 1. *Cp.* Bastable, ECONOMIC JOURNAL, Vol. VI.; note to p. 202.

[3] If a given amount—which may at first be supposed finite and small—is raised by taxing a few commodities (not specially selected in the interest of the consumer as suggested by Professor Marshall, *Principles*, Book V. ch. xii. § 7), there will in general occur under the head of those commodities a loss of "consumers' rent" which does not occur when the amount is directly subtracted from income, the consumer being free to reduce his expenditure on all commodities without disturbance of prices. The proposition may easily be extended to larger amounts of taxation.

Abstraction is here made of certain secondary advantages attending indirect taxation: that it is apt to escape attention, and that it is taken at a time and in a

more burdensome, than direct taxation? (4) Whether, if a special rate of taxation is claimed by the inhabitants of a peculiarly circumstanced part of a united kingdom, such as Ireland, on the ground of a special predilection for certain now heavily taxed commodities, or on any other *prima facie* reasonable ground, such a claim is not to be entertained, because it would not be practicable to allow a special rate to the inhabitants of some other part of the United Kingdom, such as Wiltshire; or whether, as Mill says, with reference to his proposal to allow a specially low rate of taxation for a class whose sacrifice is specially great, "the difficulty of doing perfect justice is no reason against doing as much as we can?" [1] (5) Whether, if Ireland is overtaxed only because she is poor, it may not be a problem of practical interest to determine by how much her taxation is in excess of what it would be if a just distribution of taxation as between rich and poor prevailed throughout the United Kingdom? (6) Whether and how much the Irish claim is strengthened by the treaty of the union providing for "particular exemptions or abatements"? or (7) on account of the alleged "economic drain from Ireland"? or (8) on old scores, in particular the generally admitted overtaxation in the middle part of the century, and the much-disputed transactions in the first part of the century?

To attempt to evaluate these items in the account between the two countries would be out of place in a discussion of pure theory. It will merely be assumed here, for the purpose of illustration, that the items referred to are not so preponderant as to make the consideration of "sacrifice" of no account. Upon this assumption the first approximation to the solution of the problem is obtained by minimising the total sacrifice, subject to the condition that production is not much diminished. The condition, as above explained,[2] operates at two points. There is danger of diminishing by differential taxation the accumulations of the very rich, the efficiency of the not very rich.

The latter consideration has been urged with great force by Professor Sidgwick and Professor Bastable in the Memoranda submitted by them to the Financial Relations Commission.[3] It

manner very convenient. The former advantage, as Mill observes (*loc. cit.*), is dwindling; the latter may find a set-off in the practice of stopping income-tax at the source.

[1] *Political Economy*, Book V. ch. ii. § 4. *Cp.* "It is no objection to this principle [equal sacrifice] that we cannot apply it consistently to all cases."

[2] Above, p. 105.

[3] Vol. II., 182 col. *b.*, and 184 col. *b.*

must be remembered, however, that the consideration figures on both sides of the account. If the efficiency of the richer country may be somewhat diminished by increase of taxation, the efficiency of the poorer country may be considerably increased by relief from taxation. The gain on balance is especially evident in the case where the aggravation of taxes is experienced by those whose income exceeds the necessaries for efficiency, and the relief from taxation is experienced by those whose income falls short of the necessaries for efficiency; necessaries being here defined, as proposed by Professor Marshall, in such wise that the income of any class is said to be below the necessary level when an increase in their income conduces to an increase by a greater amount in their efficiency.[1]

It is not quite clear how far this consideration of efficiency forms part of the ground on which the exemption of a minimum income from taxation is ordinarily claimed. It is, at any rate, a consideration only subordinate to a consideration for the "sufferings of privation" which, as Bentham says, are caused by taxes levied upon persons who may not have wherewith to pay.

The balance of minimum sacrifice against maximum production being necessarily rough, it is no great objection to any part of the calculation that it is devoid of numerical precision. Yet that is the sort of objection which has been brought against Sir Robert Giffen's estimate of the "taxable surplus" for Ireland and Great Britain respectively. On this subject Professor Bastable writes :—

"The fairest rough test—and no test can be looked on as more than an approximation—of taxable capacity is to be found in a comparison of total income rather than in any refined and doubtful calculation as to what is left after necessary expenses have been defrayed."[2]

"The difficulties inherent in any attempt to refine on or manipulate the total income in order to get a measure of ability, seems to indicate the convenience of keeping to the plain rule of taxation according to income."[3]

"The decisive objection to such theories is the difficulty of their practical application. To estimate income is a task, perhaps, too difficult for the statistician; but to discover the amount of 'free' income is quite hopeless, and the employment of conjec-

[1] *Principles of Economics*, Book II. ch. iii. § 3. The level of *necessaries* in the technical sense, is in general different from the "necessaries of life," "the requisites of life and health" (Mill) usually referred to in this connection.

[2] Appendix to *Evidence of Financial Relations Commission*, Vol. II. p. 185.

[3] Economic Journal, Vol. VI. p. 200.

tural amounts as guides in so definite a matter as taxation is a dangerous course which might lead to the most paradoxical results. A slight alteration of figures would supply the premisses for an altogether different conclusion." [1]

"In 1886 Sir R. Giffen estimated the taxable surpluses of Great Britain and Ireland respectively at 800 millions and 15 millions; in 1895 he estimated them at 900 millions and 22 millions. From which it at once follows that Ireland's taxable capacity had risen from being less than 1 : 53 to more than 1 : 41. . . . Mr. Lough gets 1,092 millions to 15 millions or $72\frac{4}{5}$: 1 as the ratio." [2]

This argument seems to exaggerate the determinateness of the problem. Taxation is "not so definite a matter." The claim to a special rate of taxation, as Mill says of the claim on behalf of temporary incomes, "does not rest on grounds of arithmetic, but of human wants and feelings." As involving an estimate of immaterial quantities the distribution of burdens in the way of taxation might be compared to the distribution of prizes by way of examination. It is not so definite a matter measuring intellectual ability by numerical marks. "The difficulties inherent in any refined attempt to get a measure of ability" might seem to indicate the convenience of keeping to some "plain rule" of examination, *e.g.* that the marks of a candidate should be proportioned to the *length* of his answers. But this plain rule being plainly unfair is not preferable to more refined estimates which though affected with "personal equations," and all kinds of fortuitous aberrations, still aim at least at ideal fairness. It is quite possible that marks so different as 53 and 41 assigned to the same candidate by different assistant examiners might assist the head examiner in placing the candidate. So the very divergent estimates of damages made by separate jurymen are compounded into a result by which substantial justice is secured. The estimates of taxable surplus must be similarly treated as liable to a considerable "error" or uncertainty. It is in this spirit that the distinguished propounder of the estimates in question has understood his figures.

"When we come to deal with the matter equitably, that is a thing which ought to be allowed for." [3]

"But then of course these are very rough computations indeed,

[1] ECONOMIC JOURNAL, pp. 199, 200. [2] *Ibid.* p. 200, notes.

[3] Evidence of Sir Robert Giffen before the Commission on Financial Relations between England and Ireland (Q. 7777).

and are only meant to be illustrative of what the effect of applying this principle would be." [1]

"Some rough mode of calculation of that kind is all you can do." [2]

"I do not suggest and never suggested such a rule [that the taxable incomes of Ireland and Great Britain are in the proportion 22 : 900] could be applied in a strict arithmetical way." [3]

In short, the general principle for adjusting taxation between two regions—or any classes, whether defined geographically or not—between which it is thought fit to establish distinctions of taxation, is not a simple rule of three applied to incomes, but rather a method such as that which was adopted in Switzerland in 1851, for regulating the contributions of the cantons to the "Federal Expenditure."

"To make oneself familiar with all the different circumstances which have to be taken into consideration in order to form a judgment on the economic situation of a canton; then, using a free judgment, and without taking too rigorous a basis, to tax each canton according to a certain tact." [4]

The alternative plan of "proportioning the normal contributions of different income classes to the *total* incomes of the respective classes, while carrying out Mill's principle of not trenching on the subsistence minimum, by excluding taxes on the necessaries of life as well as direct taxes of the poor," [5] derives, it is here submitted, an undue support from Mill's use of *equal*, instead of *minimum*, sacrifice. Even on the principle of equal sacrifice proportional taxation of income (above the exempted minimum) is an extreme measure.[6] The golden mean is not to be attained by creeping cautiously close to the limiting extreme—*nimium premendo litus iniquum*. The guiding star of Utilitarianism shines in a direction away from that hard coastline, and gilds the bolder course with the light of pure theory.

[Let $F(x, y)$ be the money measure of the satisfaction attending the acquisition of the quantities of the commodity x and y. Let

[1] Evidence of Sir Robert Giffen before the Commission on Financial Relations between England and Ireland (Q. 7778).

[2] *Ibid.* Q. 7780.

[3] *Ibid.* Q. 7787.

[4] Quoted by the Royal Commission on the Financial Relations between England and Ireland, Vol. II., Appendix XVI.

[5] Professor Sidgwick in Appendix I. to Vol. II. of Evidence before the Commission on Financial Relations (C. 7720), p. 183. Compare the very similar language of Professor Bastable on p. 185.

[6] As argued above, p. 114, par. 4.

$f(x, y)$ be the cost of producing those quantities. What the consumers seek to maximise is then

$$(1)\ F(x, y) - p_1x - p_2y;$$

where p_1, p_2, are the prices of the respective commodities. What the producers seek to maximise is

$$(2)\ xp_1 + yp_2 - f(x, y).$$

Whence

$$(3)\ \left(\frac{dF}{dx}\right) = p_1 = \left(\frac{df}{dx}\right);$$

$$(4)\ \left(\frac{dF}{dy}\right) = p_2 = \left(\frac{df}{dy}\right);$$

together with certain conditions which must be fulfilled by the second differential coefficients of the functions in order that (1) and (2) may be each a maximum. We must have $\left(\frac{d^2F}{dx^2}\right)$ and $\left(\frac{d^2F}{dy^2}\right)$ each negative; say $-A$ and $-B$, where A and B are always positive. Also $\left(\frac{d^2f}{dx^2}\right)$, $\left(\frac{d^2f}{dy^2}\right)$ must each be positive, say $+a$ and $+b$, where a and b are always positive. Let $\left(\frac{d^2F}{dxdy}\right) = \mp C$, where C is always positive; the upper sign being used when the demand is rival, the lower when it is complementary. Likewise let $\frac{d^2f}{dxdy} = \pm c$, c always positive; the upper sign being used where the supply is rival, the lower when it is complementary. Then the two remaining conditions which must be satisfied by (1) and (2) may be written,

$$(5)\ AB > C^2;\quad (6)\ ab > c^2.$$

Now let a tax of τ per unit be imposed on one of the commodities, say x. Thereby there is added to the cost of producing x the amount of the tax, viz. τx; and accordingly to the value of $\left(\frac{df}{dx}\right)$ for any x there is added τ. Equation (3) with this addition and equation (4) will now be true of the new values of x and y consequent on the impost, say $x' + \Delta x$, $y' + \Delta y$, where x', y' are the original quantities. Substituting these values and expanding (3) and (4) thus modified in ascending powers of Δx and Δy, recollecting that those equations are satisfied by x' and y', we obtain

$$(7)\ \Delta x \times -A + \Delta y \times \mp C = \Delta p_1 = \Delta x \times a + \Delta y \times \pm c + \tau;$$

$$(8)\ \Delta x \times \mp C + \Delta y \times -B = \Delta p_2 = \Delta x \times \pm c + \Delta y \times b;$$

whence

$$(9)\ \Delta x \times -(A + a) + \Delta y \times \mp (C + c) = \tau;$$

$$(10)\ \Delta x \times \mp (C + c) + \Delta y \times -(B + b) = 0.$$

Solving these simultaneous equations for Δx and Δy we obtain

$$\Delta x = \tau \times -(B + b)/D^2;\quad \Delta y = \pm (C + c)\tau/D^2;$$

where D^2 is the determinant $(A + a)(B + b) - (C + c)^2$; which is known to be positive, whether *a priori* as the condition that the total advantage of both producers and consumers *in globo* should be a maximum (*cp.* Marshall, *Principles of Economics*, mathematical note xiv); or as a deduction from the aforesaid conditions of maximum pertaining to each party separately, in equations (5) and (6). Substituting these values in the expressions for Δp_1 and Δp_2 given by (7) and (8) we obtain

$$(11)\quad \frac{D^2}{\tau}\Delta p_1 = -A \times -(B + b) \mp C \times \pm (C + c);$$

$$(12)\quad \frac{D^2}{\tau}\Delta p_2 = \mp C \times -(B + b) - B \times \pm (C + c).$$

Let us begin with the case in which it is proper to take the upper signs before C and c; that is the case of rival demand and rival production. In this case it appears from (11) and (12) that both prices will fall if

$$(13)\quad A(B + b) < C(C + c);$$
$$(14)\quad C(B + b) < B(C + c);\ Cb < Bc.$$

These conditions are far from exacting. They are satisfied by values of the constants of which two are quite arbitrary, except for the conditions that they must be positive, and the remainder are very slightly restricted. Take *any* (positive) values of a and b and any value of $c < \sqrt{ab}$ (> 0); and *any* (positive) proper fractions r, s, t. Let $C = r\sqrt{AB}$; and for the inequations (13) and (14) write the equations

$$(15)\quad A(B + b) = s^2AB + sr\sqrt{AB} \times c;$$
$$(16)\quad r\sqrt{AB}b = s^2t \times Bc.$$

From (15) we have

$$AB(1 - sr^2) + Ab = src\sqrt{AB}$$
$$B(1 - sr^2) + b = src\sqrt{B/A}$$
$$(17)\quad \sqrt{B/A} = (B(1 - sr^2) + b)/src.$$

From (16) we have

$$(18)\quad rb = s^2tc\sqrt{B/A};\ \sqrt{B/A} = b/src t.$$

Substituting this value of $\sqrt{B/A}$ in (17) and reducing we obtain for B $b(1/t - 1)/(1 - sr^2)$. Whence from (17) or (18) $A = \frac{s^2r^2c^2t^2}{b}\frac{(1/t - 1)}{1 - sr^2}$. $C = r\sqrt{AB} = r^2sct\left(\frac{1}{t} - 1\right)/(1 - sr^2)$. The reader may verify the fulfilment of inequations (13) and (14) by assigning values almost at random to the symbols in (15) and (16). Thus let $a = 1$, $b = 1$, $c(< \sqrt{ab}) = \frac{1}{2}$; $r = s = t = \frac{1}{2}$. Then $A = \frac{1}{2^2 2^1}$, $B = \frac{8}{7}$, $C = \frac{1}{2^3}$, $A(B + b) = \frac{1}{2^2 2^4}(\frac{15}{7}) < \frac{1}{2^3} \times \frac{15}{2^3} = C(C + c)$. $Cb = \frac{1}{2^3} < \frac{4}{7} = Bc$.

Whence ($D^2 = 1{\cdot}865$ nearly) $\Delta p_1 = -{\cdot}005\tau$, $\Delta p_2 = -{\cdot}287\tau$; both prices fall in consequence of the tax.

So far of rival production and rival consumption. When both correlations belong to the other species, "complementary," the tax of one article may, as stated in the text, cause the rise of *either* of the articles but not of both. This case differs from the preceding in that the lower signs in (7) and (8) and the sequel are to be used (C and c still treated as positive). Then, if it be possible, let there coexist the inequations

$$(19) \quad -A \times -(B+b) < -C \times -(C+c),$$

i. e.

$$A(B+b) < C(C+c);$$

$$(20) \quad +C \times -(B+b) - B \times -(C+c) < O,$$

i. e.

$$B(C+c) < C(B+b).$$

$$Bc < bC; \quad c < bC/B.$$

Since $AB > C^2$ we infer from the inequation (19) $Ab < cC$. Substituting in this inequation the superior limit for c derived from the inequation (20) we obtain

$$Ab < bC^2/B; \quad AB < C^2;$$

which is absurd, being contrary to (5).

By putting now c, now $C = 0$ we may verify other statements in the text.]