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MINIMUM SACRIFICE VERSUS EQUAL SACRIFICE

[THIS Paper, published in the *Quarterly Journal of Economics*, May 1910, under the title "The Subjective Element in the First Principles of Taxation," is a defence of minimum sacrifice as the criterion of taxation against objectors who had not realised its bearing on Progressive Taxation. Whereas in order to deduce progressive taxation from the principle of equal (or that of "proportional") sacrifice, there is required in addition to the law of diminishing utility, some further datum which is not generally given; the principle of least sacrifice does not require us to assume that of which we are ignorant. This difference is illustrated by adducing a not very complicated formula for the relation between means and utility, which satisfies the only condition known to us, the law of diminishing utility. For all we know this might be the true law. If so, according to the principal of equal sacrifice (however interpreted) the taxation would be *regressive*; the rate of taxation increasing as the income decreases. No such paradox is attributable to the principle of minimum sacrifice.]

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The following observations are designed as a supplement to the observations on the first principles of taxation which I have contributed to the *ECONOMIC JOURNAL*.<sup>1</sup> It is unnecessary to restate my theses, as they have been reproduced with a very flattering fulness in a work which commands universal attention, the second edition of Professor Seligman's *Progressive Taxation*. It will be sufficient here to recall that I divide the first principles of taxation—so far as they are subjective, and abstracting from the more objective *productional* conditions—into two classes, say *A* and *B*; characterised by the difference that the criterion of right taxation is for *A* an equation, for *B* a maximum condition. Each class is subdivided into two species. Thus *A*, denoting that *like*—or in a large sense of the term, *equal*—sacrifice should

<sup>1</sup> Vol. VII., "Pure Theory of Taxation"; Vol. X., "The Incidence of Urban Rates"; above, S and U.

be imposed on each taxpayer, is subdivided into equal in a narrow or *proper* sense, and equi-proportional sacrifice in a sense explained and preferred by Professor Seligman; say, respectively,  $A_1$  and  $A_2$ . Likewise  $B$ , the general principle that the total net utility produced by taxation should be a maximum, is subdivided into what I have called the "primary problem," namely, "to determine the distribution of those taxes which are applied to common purposes, the benefits whereof cannot be allocated to any particular classes of citizens"—say  $B_1$ ; and "the secondary problem, namely, to determine the distribution of taxation, not being limited to that amount of which the benefit is indiscriminate,"<sup>1</sup> say  $B_2$ . Some additional reflections on these topics suggested by Professor Seligman's weighty criticism may be arranged under three heads.

I. First as to the main question whether formula  $A$ , in particular  $A_2$ , or formula  $B$ , is to be adopted as the first principle. Proof of this, in the ordinary sense of demonstration or deduction from axioms, on such a subject is not to be expected.<sup>2</sup>

It might be suggested in favour of  $A_2$  that it is less *subjective*, less "sicklied o'er with the pale cast" of speculative thought, than the rival criteria of taxation. For conceivably, in order to apply  $A_2$ , we need not attempt to "compare the amount of feeling in one mind with that in another."<sup>3</sup> But in order to obtain a ratio between two "lots" of satisfaction—one of them, the total amount of satisfaction due to the possession of an income—there is required a precision of hedonic units which few utilitarians would venture to postulate. Practically, I think, in order to apply  $A_2$ —to show, for instance, that the richer class should contribute a larger sum of money (I do not say a larger *proportion* of income)—we must presuppose the sympathetic comparison of wants and feelings experienced by different persons. As thus: if it be possible, let the contribution of the rich man be the same as that of the poor man; then the ratio, which according to  $A_2$  ought to be the same for all the contributions, is a fraction of which—by the law of diminishing utility—both the numerator is smaller, and the denominator is larger than what they are for the poor man. Therefore the ratio in the case of the rich man is too small, and must be increased by augmenting his contribution.

Thus the point which I have adduced in favour of  $A_2$  is not

<sup>1</sup> See above, p. 152.

<sup>2</sup> Compare J. S. Mill, "To be incapable of proof by reasoning is common to all first principles." *Utilitarianism*, p. 52 and context.

<sup>3</sup> Jevons, *Theory of Political Economy*, p. 15 (2nd ed.).

of much practical importance; and I must leave the issue in the obscurity which envelops the first principles of conduct.

II. A subsidiary issue is presented by Professor Seligman's thesis that the authorities who have laid down "equal sacrifice" as the criterion have always meant  $A_2$ , not  $A_1$ . "When economists speak of equal sacrifice they mean relatively proportional sacrifice."<sup>1</sup> . . . 'Equal sacrifice' is then merely a rough way of expressing the idea of 'proportional' sacrifice."<sup>2</sup> "The demand for absolutely equal sacrifice in the formal mathematical sense [our  $A_1$ ] has never, so far as I know, been advanced by any one."<sup>3</sup> Statements so confident, made by one who is so conversant with the history of financial doctrine, naturally carry great weight. They compel me to reconsider my *obiter dictum*: "It may well be doubted whether Mill entertained the notion of proportional sacrifice."<sup>4</sup> On reconsideration, I am disposed to omit the word "well."

It will be admitted, I trust, that I am a fair controversialist when I go on to adduce an additional piece of evidence in favour of my critic's thesis. I have been able to question one of the authorities who have professed the doctrine of equal sacrifice, a singularly clear writer and thinker on economic questions, and one who has carried economic principles<sup>5</sup> into public affairs, Lord—formerly known as Mr. Leonard—Courtney. Having observed that Mr. Courtney, in the answers which he submitted to the Royal Commission on Local Taxation, maintained "that taxation for common purposes should be levied from each member of a community according to the law of equal sacrifice, meaning thereby that each individual should be mulcted of such a sum as would, having relation to his means, involve the same sacrifice to the common want," I lately wrote to Lord Courtney asking which of the two formulæ (here distinguished as  $A_1$  and  $A_2$ ) his words were intended to designate. Lord Courtney replied with his usual lucidity—after explanations and definitions which I have not space here to transcribe—with reference to a certain graphical construction, "the law of equal sacrifice would be represented by the exaction of the same proportions of area, (the area representing the 'total satisfaction of the owner') . . .

<sup>1</sup> *Progressive Taxation* (2nd ed.), p. 213.

<sup>2</sup> *Loc. cit.*, p. 214.

<sup>3</sup> *Loc. cit.*, p. 215.

<sup>4</sup> Above, S, p. 107, note.

<sup>5</sup> The scheme of graduated death duties, introduced by Sir William Harcourt's Budget of 1894, was rested by Mr. Courtney on the first principles of taxation (*Parliamentary Debates*, May 20, 1894).

Seligman and I are practically in agreement as to the measure of equal sacrifice."

I make Professor Seligman a present of this weighty testimony in his favour. I am not like that commentator who, with reference to his own interpretation of a certain vexed passage in a classical work, said that he would not believe the author himself—*Ne ipsi quidem Ciceroni crediderim*—affirming that the passage meant something different.

I ought to confess that the present which I have made to Professor Seligman is not of much importance to myself, as the subsidiary issue (II) is one in which I am not much concerned. In fact the side which Professor Seligman takes in that issue is the one more favourable to my main theory, namely, that formula *B* supersedes, or at least subsumes, the formulæ of Class *A* (*A*<sub>1</sub>, and *A*<sub>2</sub>). For *ceteris paribus*<sup>1</sup> the distribution of fiscal burden which *A*<sub>2</sub> prescribes tends to be more progressive than that of *A*<sub>1</sub>.<sup>2</sup> Accordingly, to show that classical authors have meant by the doctrine of "equal sacrifice" *A*<sub>2</sub> rather than *A*<sub>1</sub> lends additional plausibility to the synthesis which subsumes *A* under *B*. If Mill interpreted equal sacrifice as Professor Seligman contends, it is all the more explicable that Mill should have enounced in the same breath both the principle of equal sacrifice and that of least sacrifice.<sup>3</sup>

III. Professor Seligman raises an issue in which I am more concerned when he concludes that "the minimum sacrifice theory is thus really not a whit more successful than the equal sacrifice theory, and possesses the additional disadvantage of being less applicable to the problems of actual life."<sup>4</sup> "Neither in the version of Professor Edgeworth nor in that of Professor Carver does the doctrine of minimum sacrifice afford us any real help, or constitute any improvement on the doctrine of equal sacrifice."<sup>5</sup>

With reference to these passages and the context I have first to remark that the utilitarian position, as I have conceived it, does not reduce from *B* to *B*<sub>1</sub> so unreservedly as Professor Seligman interprets. The greatest-happiness principle, that the total net utility procured by taxation should be a maximum, reduces to

<sup>1</sup> That is, supposing any form of the law of Diminishing Returns to be assigned.

<sup>2</sup> As noticed in the *Economic Journal*, Vol. VII. p. 561; above, p. 110.

<sup>3</sup> "Whatever sacrifices it (a Government) requires from them (persons or classes) should be made to bear as nearly as possible with the same pressure upon all, which, it must be observed, is the mode by which least sacrifice is occasioned on the whole." J. S. Mill, *Political Economy*, Book V. ch. ii. § 2, p. 1, referred to in the *Economic Journal*, Vol. VII. p. 564; above, p. 115.

<sup>4</sup> *Progressive Taxation* (2nd ed.), p. 280 and context.

<sup>5</sup> *Loc. cit.*, p. 289.

the condition that the total disutility should be a minimum, not *simpliciter*, but *secundum quid*, with reference to the *primary* problem, namely, to determine the distribution of those taxes which are applied to common purposes.<sup>1</sup> The *secondary* problem, namely, to determine the distribution of taxation, not being limited to that amount of which the benefit is indiscriminate, is indeed immediately clouded over by doubts and reservations. It is cut into by productional and other interests; but I am not prepared to say that it is entirely cut away. Rather, I have maintained Mill's advanced, yet guarded, position. "That the State should use the instrument of taxation as a means of mitigating the inequality of wealth," is not to be demanded when by "a tax on industry and economy" a check to the growth of wealth is imposed. But the utilitarian will be as "desirous as anyone that means should be taken to diminish those inequalities": such means as the limitation of inheritances and the taxation of unearned increments, so far as these means are free from the dangers above noted.<sup>2</sup>

But, secondly, even with respect to the narrowed utilitarian formula, I am surprised at Professor Seligman's suggestion that the principle of least sacrifice is not a whit more efficacious than the other forms of subjective canon.<sup>3</sup> I should have thought that one who deduced progressive taxation from premises which involve in part subjective considerations of sacrifice would have welcomed a statement of the subjective premises which leads more directly to the conclusion than the ordinary statement does. But the advantage which formula  $B_1$  possesses over both forms of  $A$  in this respect was no doubt obscured by the comments of a critic whom Professor Seligman has followed, namely, Mr. Weston.<sup>4</sup>

Mr. Weston thinks it strange that a mathematical economist should "find satisfaction in a theory based upon a principle that

<sup>1</sup> See *ECONOMIC JOURNAL*, Vol. VII. p. 553, referred to above. I quote word for word from this page passages which Professor Seligman has done me the honour of quoting. But not to distract the reader's attention, I reserve quotation marks (in this paragraph) for quotations from Mill.

<sup>2</sup> The words within quotation marks are quoted from Mill, *Political Economy*, Book V. ch. ii. § 3. The rest of the passage is mostly quoted from the *ECONOMIC JOURNAL*, *loc. cit.*, p. 555; above, p. 106.

<sup>3</sup> "If we base our doctrine of the equities of taxation on the theory of faculty, both the production and the consumption sides of the theory seem to point to progressive taxation." Seligman, *loc. cit.*, p. 203 and context, from which it appears that the consumption element in faculty is equivalent to the sacrifice theory.

<sup>4</sup> *Justice and Taxation*: Columbia University Studies in History and Economics, Vol. XVII. (1903).

does not admit of an exact mathematical expression."<sup>1</sup> He bases this disparaging remark on my statement that "the reasoning from the principle of minimum sacrifice assumes no exact relations between utility and means."<sup>2</sup> But the meaning of that statement, as the context shows, is not that the principle of minimum sacrifice abandons the character of mathematical reasoning, but that it does not require all the data which are required by the rival principle of equal sacrifice. This is the first time that the parsimony of assumptions has been made a reproach to a mathematical argument. After Clerk Maxwell had shown that the observed laws of pressure and so forth were accounted for by the hypothesis that a gas consisted of an indefinite number of perfectly elastic minute spheres encountering each other in a molecular chaos, was it a sign of satisfaction in the absence of an exact mathematical relation, was it any imperfection, to show that much the same conclusion was deducible even without assuming the sphericity of the molecules? Is it a confutation of the method of least squares, that it does not require us to postulate—what we are commonly ignorant of—the exact relation between the frequency of an error and its extent? The method of least squares is content with some very simple and easily ascertained data as to the character of the observations dealt with;<sup>3</sup> just as the principle of least sacrifice requires *only* the law of diminishing utility, not also some more exact datum as to the rate at which utility (*i.e.*, the increase of utility) diminishes with the increase of means.

This prerogative distinguishes the principle of "equimarginal" sacrifice from the other two principles, which indeed are concerned with sacrifice and involve margins in a certain sense, but do not employ the margin of utility to determine the minimum of sacrifice.<sup>4</sup> For example, suppose that the law of diminishing utility, the relation between total satisfaction,  $Z$ , and amount of income,  $y$ , is represented by the form  $Z = H\sqrt{\frac{y}{b}}$ , where  $H$  is a

<sup>1</sup> *Loc. cit.*, p. 206.

<sup>2</sup> *ECONOMIC JOURNAL*, pp. 566-7; above, p. 117.

<sup>3</sup> In particular the mean square of error (or deviation) pertaining to the class of observations with which we are dealing.

<sup>4</sup> Mr. Weston does not seem adequately to recognize the diversity under the appearance of similarity when he writes: "In fact Professor Edgeworth does not claim to do more than to bring out a little more definitely what was already implied in Mill, Sidgwick, Meyer, and others. At any rate as we have understood these authors the marginal sacrifice occasioned by the tax has meant an equimarginal sacrifice; one in which the tax imposes exactly equal sacrifice upon every tax-payer."

constant, for amounts of income greater than  $b$ , then according to the principle of *equal* sacrifice the rate of taxation ought to be *regressive*, varying in inverse proportion to the square root of  $y$ , for small taxes. But according to the principle of *proportional* sacrifice the rate ought to be constant, neither progressive nor regressive. Again, suppose that the law of diminishing utility is represented by the formula  $Z = He^w$ ,<sup>1</sup> where  $w = \left(\frac{y-a}{b}\right)^{\frac{1}{2}}$ , for amounts of income greater than  $a$ , and less than  $a + 8b$ ; then the rate of taxation ought to be *regressive* both for the principle of *equal* and that of *proportional* sacrifice.<sup>2</sup> But no one is in a position to affirm<sup>3</sup> that the assigned functions do not correspond to the true law of utility. For they both fulfil the only condition which may be taken for granted, namely, that utility should increase with the increase of income at a decreasing rate.<sup>4</sup> The principles of equal and proportional sacrifice give an uncertain sound in cases like the above which may be multiplied indefinitely. But the principle of least sacrifice in trumpet tones proclaims that the rate of taxation ought to be progressive; except so far as this distributional presumption is cut into by the productional and other utilitarian conditions.

Not being certain that Mr. Weston has understood the distinctive characteristics of my formula, I am naturally not much affected by his dissent; the rather that I am unable to understand what is distinctive in the formula which he prefers. "The ideal," he thinks, "would seem to be the minimum of sacrifice to the greatest number." This formula appears to be open to the sort of criticism which I have elsewhere ventured to apply to the more familiar phrase "greatest happiness of the greatest number";

<sup>1</sup> Where  $e$  is the well-known constant (which forms the base of the Napierian logarithms) namely 2.71828 . . .

<sup>2</sup> These conclusions may be deduced from the proposition that, if  $\eta$  (a function of  $y$ ) denotes the rate of taxation for any amount of income,  $y$ , and accordingly  $\eta y$  is the contribution of each tax-payer whose income is  $y$ , then for small taxes, (1) according to the principle of equal sacrifice  $\eta y \frac{dZ}{dy} = c_1$ ; (2) according to the

principle of proportional sacrifice  $\eta y \frac{dZ}{dy} = c_2 Z$ ;  $c_1$  and  $c_2$  denoting appropriate constants. The conclusions obtained for small taxes may be extended to taxes of finite magnitude on the principle employed by Cournot (*Principes Mathématiques de la Théorie de la Richesse*, Section 32). [See note added at p. 242, below.]

<sup>3</sup> Some probability that the law of utility is not on what may be called the regressive side of Bernoulli's law may be discerned; above, §, p. 208.

<sup>4</sup>  $\frac{dZ}{dy}$  positive, and  $\frac{d^2Z}{dy^2}$  negative, for the values of  $y$  with which we are concerned.

which I contend is a loose synonym for "the greatest quantum of happiness." "The more familiar statement has, indeed, some advantages. That it is more familiar is no small advantage; another is that it emphasises an essential condition of greatest happiness, that the means of happiness should not be monopolised by a few. The popular, as compared with the exact, formula has only one disadvantage; that it is nonsense. To find the maximum of one quantity *A* 'of,' or in relation to, the maximum of another quantity *B* is a statement of a problem in the calculus of variations which no amount of authority can render other than inaccurate—not the authority of Mill, not even that of Bentham."<sup>1</sup> Analogously, a phrase like that proposed by Mr. Weston might be used to emphasise a condition of minimum sacrifice: that the burden of taxation should not be *very* unequally distributed. The phrase might also serve to recall the productional considerations which I have thus worded, "The large relief from the burden of taxation which *primæ facie* on distributional grounds should be afforded to the less prosperous, is restricted by the productional principle that those who have a share in calling the tune should have a share in paying the piper."<sup>2</sup>

I do not deny that for popular use other expressions of the fiscal first principle may be more effective than the utilitarian formula. Among such variants the one which Professor Seligman prefers is pre-eminent. It was a master-stroke of practical wisdom to include the distributional, as well as the productional, criterion of taxation under the category of "faculty,"<sup>3</sup> which has the appearance of being more definite than the *summum genus* utility.<sup>4</sup> Let those who with M. Leroy-Beaulieu deny the collateral authority of the subjective principle describe this diction as "en quelque sort un jeu des mots." To one who believes in the double nature of the fiscal *summum bonum* the happy ambiguity of the proposed canon renders it all the more acceptable. It has a Parliamentary sound. It is like the celebrated resolution of the House of Commons declaring the throne vacant after the flight

<sup>1</sup> Above, U, p. 155, referring to S.

<sup>2</sup> Above, U, p. 159; there referring to municipalities, but with the omission of a couple of words equally applicable to individuals.

<sup>3</sup> "The elements of faculty, then, are twofold, those connected with acquisition or production, and those connected with outlay or consumption . . . the elements of faculty which are connected with outlay or consumption bring us right back again to the sacrifice theory. Faculty is the larger, sacrifice the smaller conception." *Progressive Taxation*, pp. 291-2.

<sup>4</sup> The *summum genus* under which I include the two modes of detriment: diminution of the total production and aggravation of unequal distribution. Above, U, p. 162.



of James II; in which, Macaulay says, "there was a phrase for every subdivision of the majority. The one beauty of the resolution was its inconsistency."<sup>1</sup>

It is pleasant to believe that, with respect to practical application, I am in complete accord with the eminent critic of my theory. The differences which remain are perhaps not more than verbal. We could both, I think, subscribe to Professor Nicholson's moderate doctrine, "that the ideal of equality of sacrifice, although vague and ill-defined, is one of the supports of certain kinds of exemption."<sup>2</sup> Professor Nicholson expresses entire agreement, in which I entirely concur, with Professor Seligman's reservations as to the equal sacrifice theory "regarded as the paramount consideration in the construction of any definite rate," rather than only one factor in the problem.<sup>3</sup> With regard to the proposal "to state the ideal as 'minimum' sacrifice instead of equal," Professor Nicholson thinks "this statement seems the more logical on the pure utilitarian theory. On grounds of formal justice the equality of sacrifice may be preferred."<sup>4</sup>

(Note referring to p. 239.)

If  $Z = He^w$ , where  $w = \left(\frac{y-a}{b}\right)^{\frac{1}{3}}$ ,  $\frac{dZ}{dy} = Z^{\frac{1}{3}}(y-a)^{-2/3}/b^{\frac{1}{3}} > 0$  if  $y > a$ ;  $\frac{d^2Z}{dy^2} = Z^{\frac{1}{3}}(y-a)^{-4/3}/b^{\frac{2}{3}} - \frac{2}{3}(y-a)^{-5/3}/b^{\frac{1}{3}} < 0$ , if  $y > a$ , and  $(y-a)^{1/3} < 2b^{1/3}$ , or  $y < a + 8b$ .

To observe whether the rate  $\eta$  decreases (or otherwise) with the increase of the income  $y$ ; consider first the case of *proportional* sacrifice, where  $\eta \frac{dZ}{dy} = C_2 Z$ . The sign of  $\frac{d\eta}{dy}$  is then the opposite to that of  $\frac{d}{dy} \frac{1}{Z} \frac{dZ}{dy} = \frac{d}{dy} y^{\frac{1}{3}}(y-a)^{-2/3}/b^{\frac{1}{3}} = \frac{\frac{1}{3}(y-a)^{-5/3}}{b^{1/3}} \{(y-a) - \frac{2}{3}y\}$ ; which is *positive* if  $y > 3a$ . Accordingly  $\frac{d\eta}{dy}$  is *negative*, the rate is *regressive* for values of  $y$  between  $3a$  and  $a + 8b$ . In the case of *equal* sacrifice regression sets in at an earlier value of  $y$ , before  $y = 3a$ . That equal sacrifice should thus be *more regressive* is consonant with the fact that it is in general *less progressive* than proportional sacrifice. (See above, p. 237.)

<sup>1</sup> *History of England*, ch. x. The whole of the brilliant context may be read with advantage by those who would put a finer point on fiscal apparatus than the nature of the material permits.

<sup>2</sup> *Principles of Political Economy*, Book III. § 5.

<sup>3</sup> *Loc. cit.*

<sup>4</sup> *Loc. cit.*, § 4, note.