

Researches into the Mathematical Principles of the Theory of Wealth. By AUGUSTIN COURNOT. 1838. Translated by NATHANIEL T. BACON; with a Bibliography of Mathematical Economics by IRVING FISHER. (New York: The Macmillan Co. London. 1898.)

A *Brief Introduction to the Infinitesimal Calculus, designed especially to aid in reading Mathematical Economics and Statistics.* By IRVING FISHER. (New York: The Macmillan Co. London: Macmillan & Co.). 1898.

THE character of path-breaking originality attaches in a particularly high degree to Cournot's *Recherches* of 1838. What has been said by some one not belonging to the historical school—J. B. Say, if we recollect rightly—that there was no use in studying the writers who preceded Adam Smith as they were all wrong, might be said with more truth of Cournot's mathematical predecessors. At any rate Cournot was not much indebted to them. Those who care to verify this statement and explore *les origines* will be greatly assisted by the bibliography of mathematical economics which Professor Fisher has appended to the translation of Cournot. In his judicious classification of these works the predecessors of Cournot form one category; three others begin with the works of Cournot, Jevons, and Marshall respectively. The stimulus which the *Principles of Economics* gave to the new study may be measured by the fact that since its first appearance mathematico-economic writings have been produced at the rate of eighteen a year, while for the period between Jevons and Professor Marshall the rate was only six.

We take this fact from Professor Fisher's article on "Cournot and Mathematical Economics" in the current number of the *Quarterly Journal of Economics*—an *apparatus criticus* which ought certainly to be read, and might with advantage be bound up, with the translation of Cournot. We accordingly regard it as within our province to acknowledge the very great assistance which the student of mathematical economics derives from Professor Fisher's commentary and notes on Cournot.

It will be gathered from Professor Fisher's criticisms that all the parts of Cournot's work are not of equal merit. The commentator marks off the last two chapters, some 45 pages of the translation, out of a total of 166, as "an ambitious but erroneous theory . . . vitiated throughout by erroneous conceptions of income." "About 18 other pages may be omitted without loss of continuity and without great loss of substance.

The remaining 103 pages are almost uniformly excellent, and will repay very thorough study by all who care for exact ideas and demonstrations in Political Economy." The eulogium contained in the last sentence appears to us too sweeping. There are degrees within the selected 103 pages; the student should be warned that all are not uniformly rewarding.

The highest place—on a level with whatever is most original and classical in political economy—must be assigned to the chapters (IV.—VI.) in which the now familiar demand curve was introduced, and employed in deducing theorems relating to monopoly, some of them not yet familiar.

Cournot's transition from the case of pure monopoly to that of perfect competition by the introduction of first one and then more competitors is, as Professor Fisher has remarked, "brilliant and suggestive, but not free from serious objections." It is so, in our view, not merely because the solution given by Cournot in the particular case put by him—namely, where two competitors deal in an article which, like the supplies from a mineral spring, can be multiplied without expense (*Recherches*, Art. 43)—is erroneous, but rather because he has missed the general theorem: that the solution is indeterminate where the number of competitors is small.

The last stage in the transition from monopoly to perfect competition is reached when the effect on the price which any individual can produce by varying his supply is so small as to be neglected. The condition that the increment in price corresponding to the increment of the quantity supplied by a particular individual is small admits of being elegantly expressed in symbols. But Cournot has not employed the proper expression. On the contrary, as Professor Fisher points out (in the Appendix to the *Quarterly Journal* article, Note 34), Cournot seems to assume that the quantity supplied to the individual is small in a sense which justifies its omission from the statement of economic equilibrium (the omission of D_k in the first equation of Chapter VIII). This assumption is not in general allowable. However, "in the same chapter, Cournot enunciates two other principles which have become classic, though . . . they are seldom duly credited to him." One is that in the regime of competition a tax on an article, subject to the law of increasing cost, will raise the price by an amount less than the tax; a proposition which is to be found in Mill's *Political Economy* (Book V. ch. iv. § 3, par. 1). The second proposition has, perhaps, hardly yet become classic—namely, that where the law of cost, as conceived by Cournot (his ϕ'), is diminishing [or it may be added constant]—where,

as we might now say, the supply curve in that primary sense in which it is applicable to short periods corresponding to one of the "cost curves," considered by Mr. Henry Cunynghame in the *ECONOMIC JOURNAL*, Vol. II. p. 41, is a continually descending [or horizontal] curve; there economic equilibrium is not possible.

Cournot next treats what we should now call "Joint demand," or rather a special case thereof. We agree with Professor Fisher that "his analysis of motives in the minds of the two monopolists is subject to much the same objection as in the case of two competitors." But we cannot agree that "Cournot's conclusions are in the main consonant with facts." "He shows among other things," says Professor Fisher, "that the control by a single monopolist of both copper and zinc will result in a lower price of brass than the control of copper by one monopoly and zinc by another." This paradox has been disputed by the present writer. Both in the case of "joint" as of "rival" demand, it is submitted that where there are two monopolists in the field, the system of bargains will be *indeterminate*. The assumption underlying both investigations, and expressly applied to the latter (*Translation*, p. 105), that "where all the conditions of an economic system are accounted for, there is no article of which the price is not completely determined"—this assumption, we submit, is fundamentally erroneous.¹

Of the 103 pages distinguished by Professor Fisher as "uniformly excellent," there remains now only the first part of Chapter X; the latter part, Articles 72 and 73, belonging to the category of what may be omitted without much loss. The retained part is vitiated by one of those mathematical slips from which Cournot was not free.

Fortunately, there is only one other instance in which the flaw in the mathematical reasoning causes an error in the economic conclusion. But the numerous inaccuracies—not all of them mere misprints, but some actual mistakes in the algebra—prove very embarrassing to the student of the original. A debt of gratitude is due to Mr. Bacon for having carefully corrected these errors—thirty-five in all. He has also by his creditable and clear translation assisted the English reader to contemplate Cournot's work.

If our view of that work is not distorted, it is the *alpha*, but not the *omega*, of mathematical economics. It is the vestibule, but not the complete edifice.

Professor Fisher has not only "restored" the porch of economic science, but he has reconstructed the steps which lead

¹ Cp. above, E.

up to that entrance and the adjoining structures. We refer to his introduction to the Infinitesimal Calculus. The ascent is short but steep. The feeble-kneed will sometimes desiderate the zigzag windings of a more gradual approach. For instance, beginners will probably find a difficulty in surmounting the passage on the fourth page—where a distinction is drawn between the “*limit of the ratio of vanishing quantities* (e.g., $\lim. \frac{\Delta s}{\Delta t}$),

and the *ratio of their limits* (e.g., $\frac{\lim. \Delta s}{\lim. \Delta t}$).” But doubtless the conceptions of the calculus—like the principles of virtue—are not to be acquired by explanation only; practice also is required. Professor Fisher has provided adequate exercises; and he cheers the labours of the student by indicating from time to time their economic significance. Probably it would not be possible to obtain what may be called a saving knowledge of the subject at the cost of less trouble than that which is required to master these 85 pages. They contain what is necessary and sufficient to fulfil the purpose proposed by the author, namely to “enable a person without special mathematical training or aptitude to understand the works of Jevons, Walras, Marshall, or Pareto, or the mathematical articles constantly appearing in the *ECONOMIC JOURNAL*, the *Journal of the Royal Statistical Society*, the *Giornale degli Economisti*, and elsewhere.”

La Co-opération (Conférences de propagande). By CHARLES GIDE.
(Paris: Larose. 1900. Pp. 311.)

THE appearance in English of Professor Gide's *Introduction* to selections from Fourier reminds us of his collected discourses on co-operation, one of which is the lecture given at Nîmes on the prophecies of Fourier. With characteristic sprightliness, Professor Gide begins by telling his audience that he is about to speak about a perfect madman (*du fou le plus complet qui se puisse imaginer*). This power of fixing attention by lively sallies is effective in a preacher. To take another example, speaking of the advantages of co-operation our lecturer says:—“Perhaps you have sometimes been so imprudent as to enter one of those chambers of dentistry, outside which one reads the enticing inscription, ‘Teeth extracted here without pain.’ Well, outside all co-operative shops there ought to be put up the much truer inscription, ‘Saving effected here without pain.’” The encouragement to saving is by no means the principal advantage attributed by Professor Gide to co-operation. Among the “twelve virtues”