A Geometrical Political Economy. Being an Elementary Treatise on the method of Explaining Some of the Theories of Pure Economic Science by Means of Diagrams. By H. CUNYNGHAME. (Oxford: Clarendon Press. Pp. 128.) 1905.

As ancient teachers used to incite their pupils to the study of the elements by the offer of cakes, so Mr. Cunynghame makes his elementary treatise attractive by the elegance and sprightliness of his exposition. Wit is made the handmaid of science. The happy use of metaphor contributes to the effect, as in the following passage :—

"Two manufacturing rivals are like men pulling against one another on a rope, one on each side of the summit of a hill. When one of them is once pulled over the summit the other can run away with him. But if the men were pulling on each side of a hollow, as in rival production of the agricultural order, then when one got pulled down a little his opponent's task would become harder; so they would come at last like a marble in a bowl to a position of equilibrium" (p. 89).

A feature that is likely to be very attractive to beginners is the author's practice of using examples taken from real life; for instance, the cost per ton of coal diminishing as the total output increases, in a "recently opened mine in Yorkshire," computed by the manager himself (p. 71), or the similar schedule for the cost of producing a crown 8vo book, obtained from a "leading publisher" (p. 55). While thus showing a grasp of concrete fact, our author does not ignore the abstract character of geometrical political economy. He warns the student that "diagrams cannot decide the question of Free Trade against Protection." We cannot forbear from quoting what he says *obiter* on that question with his usual force and brevity :—

"Perhaps the real truth is that Protection is a medicine, and that before giving it you ought first to find out whether the patient is ill; next whether the proposed drug will make him better; and thirdly how much you are going to administer" (p. 102).

It is given to few to unite like Mr. Cunynghame the powers of popular exposition and scientific investigation. Apparently it is not given even to him to apply both powers at the same point. We had hoped that the path struck out by him in his original article on "Exchange Value" in an early number of the ECONOMIC JOURNAL (1892) would have been now converted by him into a high-road accessible to the wayfarers of science, even though not specialists. But this hope has not been fully gratified. There

still, as it appears to us, remains some difficulty, which we shall endeavour to smooth over by a free restatement.

Let us begin with the theory of Demand. And first let the article demanded be of a species pointed out by Mr. Cunynghame, such that the utility to each purchaser derived from a certain quantity of the article would be less the greater the amount purchased by others. "Orchids" is the happy instance given in the work before us. If orchids became common, we must suppose the demand-curve which pertains to any individual to be altered in such wise that at some or all prices the individual demands fewer orchids. It might seem sufficient, considering two epochs at which orchids were respectively rare and common, to regard the dispositions of the parties as having suffered a change in the interval. There would be a new "collocation" of causes, in the language of Mill's *Logic*. But Mr. Cunynghame is not satisfied with this merely historical account of the change. He demands, as we understand, that the new collocation should itself be explicable by conditions which are pre-existent and co-existent in much the same sense as the dispositions represented by the ordinary demand curve. It is thus that we interpret his doctrine: "a group of successive curves is the expression of a state of facts existing at one time, and is not a group of successive time phenomena" (ECONOMIC JOURNAL, Vol. II. p. 39). We understand that there is here predicted the same sort of permanence as that which belongs to the state of facts designated by an individual's demand for a commodity, say tea: the law of demand does not change when the price changes. As our author well puts it:—

"Demand in its true meaning of the general group of amounts he was prepared to give for tea, each amount being dependent on getting it at a certain price has not changed" (p. 51).

But while we are quite prepared to find the sort of co-existence which we understand Mr. Cunynghame to postulate between the two states of an individual's demand for orchids, pertaining respectively to a period of rarity or abundance—to the scale, small or large, on which those exotics are used by society—still, as orchids cannot be at once rare and abundant, we see no objection to designating the state of demand by the attribute "short periods" (or perhaps "different periods"). Mr. Cunynghame, however, objects strenuously to this nomenclature (Preface and p. 73); with what justice we can better judge after considering his system as a whole.

We should piece the parts together as follows. Let us begin

with an individual's demand for anything, as generally conceived, as described, for instance, by Professor Marshall (*Principles of Economics*, Book III. ch. iii. § 4). The amount of the commodity which he demands (efficiently) depends upon the price. To this received conception we are now to add that the amount of a commodity like orchids demanded by the individual depends, not only on the price, but also on the amount purchased by others. There may be some little difficulty about the last phrase. Perhaps the amount purchased habitually on an average by the group of purchasers who are in competition with the individual would be an appropriate description. Passing over minor difficulties—for instance, as to the extent of purchases which the average is to cover—let us designate this new quantity, on which, as well as on price, the individual demand depends, as the scale of total consumption. Now let us put together these "individual demand curves," as we shall call them. We thus obtain "the sum of the demands of all the individuals" (Marshall, *loc. cit.*, § 5); dependent, not only on the price, but on the scale of total consumption. There may be some difficulty about this summation on the ground that the scale which effects the demand of each individual is not the same for all, each being affected by the others. But this difficulty disappears if we suppose, as we must suppose in a regime of competition, that the amount purchasable by each is negligible in comparison with the amount purchased by all.

The result of summing the particular demand curves is represented by Mr. Cunynghame—according to our interpretation—by a "successive utility curve" (ECONOMIC JOURNAL, *loc. cit.*). The transition to what he calls "the demand curve" is most easily expressed by mathematical language. Let x be the sum of the individual demands. It is dependent on p, the price, and on the scale of general consumption, which we will call x'. Now drop the *dash*, treating the x on both sides of the equation as one and the same quantity, and you will have a relation between x and p which constitutes "the demand curve proper," as for the sake of distinction we shall designate what we understand Mr. Cunynghame to mean by "the demand curve." It is characterised by this property. Take any amount, x, of the commodity, and form the individual demand curves corresponding to the scale of total consumption, x. The price at which the sum of those particular demands will be x is represented by the ordinate of the demand curve proper, corresponding to the abscissa x.

The construction may be illustrated by supposing some simple

system of "successive utility" curve, *e.g.*, straight lines "negatively inclined" (*cp.* Marshall, *Principles*, p. 174, 4th ed.) (sloping downwards from left to right); as each individual demand curve, and therefore the compound called a "successive utility" curve, must be in general (*ibid.*). The line designating such a curve lies nearer the origin the greater the amount of the article of the "orchid" species, habitually on an average purchased by the whole group of competing purchasers at the period under consideration. The demand curve proper will then be a parabola negatively inclined to the axis of x (or rather to the part of it with which we are concerned, the positive part, on the right of the origin).

Next let the article belong to the same class as certain "hymn books" instanced by Mr. Cunynghame (ECONOMIC JOURNAL, *loc. cit.*, p. 39): such that the more extended use thereof by the society is accompanied with a more urgent demand on the part of each individual. If, as before, the "successive utility" curves are negatively inclined straight lines, they are now to be conceived as *further from* the origin the larger the scale of social consumption. The demand curve proper will be a parabola as before, but one that is not throughout negatively inclined to the (positive part of) axis x. A part of the demand curve will be positively inclined to (the positive part of) the axis x.[1]

[1] Let ξ_r be the amount demanded by a certain individual; and let the equation of his "individual demand curve" be

$$\xi_r = A_r - B_r p - C_r x'^2;$$

where p is the price, x' is the total purchased in some such sense as above indicated; A_r, B_r, C_r are positive coefficients (depending on the nature of the individual). Adding together the equations pertaining to each individual, we have the "successive utility" curve $x = A - Bp - Cx'^2$; if $x = \Sigma\xi_r$, $A = \Sigma a_r$, and so on, the summation extending over all the individuals with which we are concerned. Now, treating x', no longer as a constant, but as a variable identical with x, we obtain for the demand curve (proper)

$$Bp = A - x - Cx^2,$$

representing a parabola of which the vertex is at the point $x = -\frac{1}{2}/C$; $p = (A + \frac{1}{4}/C)/B$. The right arm passes through the space enclosed by the axes $+x$ and $+p$, $\frac{dp}{dx}$ being negative throughout that space.

Next let the C_rs, and accordingly C, be negative. As before, we obtain for the demand curve proper a parabola. But the vertex is now on the right of the axis p; the arms stretch upwards; and there will always be a part at least of one of them (the one on the right) for which $\frac{dp}{dx}$ is $+$, while p and x are $+$.

These conclusions may be generalised by putting for the equation to a "successive cost curve"

$$x = \Sigma f_r(p,x') = F(p,x').$$

If the function f_r is such that not only $\frac{d}{dp}f_r$ is throughout negative, but also

We must confess that this elaboration of the theory of demand appears to us chiefly important as an introduction to the more difficult theory of supply. Corresponding to the three kinds of curve above described, we have now (1) "individual supply" curves, each connecting the amount which any individual producer is willing to supply with the price and the scale of total production; (2) "successive cost curves" (Cunynghame, ECONOMIC JOURNAL, *loc cit.*), formed by putting together curves of the first kind; (3) the supply curve proper. In a regime of competition it must be supposed that the particular supply curves are positively inclined to the axis x. For there must be some impediment preventing an individual producer from cutting out all his competitors by producing more and more at an ever cheaper rate. "The necessity of carriage from one place to another is an obstacle. The impossibility of suddenly creating the necessary skill is another" (Cunynghame, *Geom. Pol. Econ.*, p. 90, and *cp.* Marshall, *Principles*, 4th ed., Book IV., ch. xi. § 5, p. 365; p. 522, par. 4). If in a certain industrial regime we ascertained how much any individual would produce at a certain price up to the point at which it ceased to be his interest owing to some such impediment, and plotted a curve representing the amount of product corresponding to each price, that curve would be the individual supply curve (analogous to the individual demand curve). It cannot be a negatively inclined curve like that which represents the cost per ton of coal diminishing with the total output in a certain colliery to which Mr. Cunynghame refers (p. 71). For that curve is not, as it ought to be (analogously to the particular demand curve), the *locus* of the points at which the amount produced at any assigned price affords a *maximum* advantage to the producer. It is rather the locus of *zero* advantage; at any given price fixed by the outside market the firm would do well to increase its output. The statistics presented by Mr. Cunynghame in a graphical form may be described as constituting a *cost of production curve* for a particular coal-mine.

In a regime of monopoly, indeed, there need not be supposed impediments resulting in an upward curl of the individual supply curve. In fact, the conception of an individual supply curve,

---

$\frac{d}{dx}f_r$ for all the particular functions, then for the demand curve (proper), viz., $x = F(p,x)$, $\frac{dp}{dx}\left(= \left(1 - \frac{dF}{dx}\right) \Big/ \frac{dF}{dp}\right)$ is negative throughout. But if the conditions specified are not fulfilled, it may happen that $\frac{dp}{dx}$ is positive for a part, or even the whole, of the region with which we are concerned.

which represents the amount which a producer is just willing to offer at an assigned price, becomes insignificant in a regime of monopoly, a characteristic of the monopolist being his power to modify the price (*cp.* Marshall, *Principles*, Appendix, Note XIV.). We are concerned only with the cost of production curve, which may be either positively or negatively inclined.

The liability of an industry to be monopolised when it obeys the law of increasing returns creates peculiar difficulty in the application of the geometrical method to supply. In order that the theory which has been above set forth with reference to demand should be extended to supply, it must be postulated that the output of each producer is small in comparison with the collective output of all his competitors. But this postulate is apt not to be adequately fulfilled in modern manufacturing industry; as Mr. Cunynghame reminds us in many a striking passage (pp. 79, 86–89).

Keeping to the regime of competition we may illustrate the successive cost curves by parallel right lines positively inclined to the axis x. First let the height, or distance from the origin in an upward direction, of a successive cost curve be *greater* the larger the scale of total production (corresponding to the case of agriculture, if as the total produce is increased a higher price is required to evoke any assigned amount of produce from the individual cultivators). Then the supply curve proper will be a parabola positively inclined to (the positive part of) the axis x throughout.

Next let the height of a successive cost curve be *less* the larger the scale of production (corresponding to the case of some manufactures). Then the supply curve proper will be a parabola, with one branch negatively, and one positively inclined to the axis x.[1] Presumably in the latter case "external economies" are overridden by a tendency to diminishing returns. Under other conditions the supply curve proper might be horizontal. (*Cf.* Marshall, *Principles*, 4th ed., pp. 398, 522.)

The interpretation of Mr. Cunynghame's theory which we have offered in the preceding paragraphs fits fairly well. But it is not to be concealed that the original presents some dark sayings and unaccountable reticences which baffle the interpreter. The

[1] Put for the successive cost curves $x = A + Bp + Cx'^2$, where A and B are positive; and for the supply curve proper what this becomes when x' is substituted for x. When C is positive, $\frac{dp}{dx}$ is throughout positive, but when C is negative, this need not be the case. The conclusion may be generalised, as before in the case of Demand.

relation between the " successive utility " and the " individual demand " curves, such as we have conceived it, is not explicitly affirmed by the author. We have inferred the relation from his use of the " successive utility " curves to measure " consumer's surplus." It follows from that property that the successive utility curves must be made up of curves, or discontinuous loci (polygons), relating each to an individual, which have each, at least transiently or potentially, the character of a demand curve. " Short-period demand curves," we should have thought, would be an adequate description both of the " successive utility " curves and the individual loci of which they are made up. They are " demand " curves for the reason just indicated. They are " short-period " curves because when there occurs a change in the conditions of supply, and accordingly a new point of intersection between the new supply curve and the old " successive utility " curve, then—the correspondence between our x and x' being disturbed—the successive utility curve must be conceived as changing its form until x and x' once more coincide. No such change of form is suffered by the proper, or, as we should like to say, " long-period " demand curve. We should have said so if Mr. Cunynghame had not expressly repudiated this nomenclature.

The treatment of supply presents analogous difficulties to the interpreter, with others that are even more serious. Mr. Cunynghame described as a " supply curve " a diagram representing the cost of production of coal (p. 71) which appears to us, for reasons above stated, to be neither an individual supply curve, nor yet a supply-curve proper. Again, referring to the curve which represents the cost of production for successive editions of a book, Mr. Cunynghame speaks of a " line drawn horizontally through P " [the point corresponding to the first edition produced at a certain cost per book] as " the only supply curve that ever exists in the case of books that do not go to a second edition, and a very long-period supply curve many authors find it." Should not the case of these authors be relegated to the chapter on monopoly?

We trust that Mr. Cunynghame will prove his own interpreter in some future publication. We are sensible that it is a difficult and delicate matter to restate theories originated by another. Mr. Cunynghame himself sets the example of such adaptation when he attempts to translate into his own mathematical language Professor Marshall's celebrated foreign trade curves. The translation of a classic is seldom effected without the loss of some subtle quality which contributed to the excellence of the original. In the case before us it has been impossible to preserve in Mr. Cunynghame's representation the incident that

changes in international transactions are apt to be attended with changes in the general level of prices—the marginal utility of money—within a country. It may be urged, no doubt, that with regard to small changes in the large system of modern commerce (*cf.* Cunynghame, p. 120) the abstraction of this incident may be practised with safety. However this may be, for the purpose of education at least—since the exaggerated importance attached to money is the source of the principal fallacies which beset the subject—is it well to forgo the advantage of expressing the too easily forgotten truth that trade is, in Mill's phrase, " in substance and effect, barter " ?

There is more than meets the eye in Professor Marshall's foreign trade curves. As it has been said by one who used this sort of curve, a movement along a supply-and-demand curve of international trade should be considered as attended with re-arrangements of internal trade; as the movement of the hand of a clock corresponds to considerable unseen movements of the machinery. Mr. Cunynghame has set himself to illustrate these internal movements by a complicated system of his own curves. The beauty and flexibility of the geometrical instrument are well exhibited by this feat. Mr. Cunynghame has shown marvellous skill in surmounting the characteristic limitation of the geometrical method: namely, that, in his own words, " it can at most deal with three variants." " When we get beyond this we want a fourth dimension and our imagery fails us."

Notwithstanding this candid admission, we are not satisfied that Mr. Cunynghame holds the balance evenly between the rival claims of geometry and analysis. As exemplifying the peculiar power of symbols there occurs to us Mr. Pigou's masterly investigation of the incidence of a differential tax on wheat imported from foreign countries into the United Kingdom (*Fortnightly Review*, January 1904). The subject-matter, an interdependent system of markets, foreign and domestic, is similar in kind to that which has afforded to Mr. Cunynghame his most brilliant triumphs of geometrical skill. But could even Mr. Cunynghame marshal in a plane, or even in space, all the variables which enter into this problem of *three* countries ?

Nor are we convinced by the following objection :—" To express an experimental supply curve, as, for instance, Fig. 27, or price of getting coal, Fig. 35, or of producing a book, Fig. 27, or still more the curves of demand for corn or sugar by such an expression as

$$y = f(x)$$

is to invest these curves with an apparently simple law-determined character that they do not really possess." But as all that is knowable—much more than is usually known—is a set of discrete data, so much commodity corresponding to such a price, whether is it more arbitrary to draw a freehand curve through points representing those data, or to use a form which stands for any one of an indefinite number of equations,[1] each representing a curve passing through the given points? Both the methods present the sort of difficulty which the student of Euclid has to jump when, for the purpose of proving some proposition relative to triangles, he draws a figure which unavoidably presents other attributes besides mere triangularity. As Berkely says, "it is true that the diagram I have in view includes all these particulars, but then there is not the least mention made of them in the proof of the proposition." It seems to us quite tenable that the indefinite symbol "f" obtrudes particularity even less than a concrete curve-line. But it is natural that one who has attained such distinguished success as Mr. Cunynghame in geometrical political economy should be partial to that method.

The Theory of Distribution. By Professor T. N. CARVER. (New York: Macmillan Co. London: Macmillan & Co. 1904. Pp. 287.)

PROFESSOR CARVER has not only shed new light upon his subject, but has also collected the rays from all other sources of illumination. Brilliant flashes from the latest literature, along with a dry light of classic origin, are reflected on his pages. The harmony between new and old expressions of truth commands belief. The work reads like a revised version of authorised doctrine. Many an old text which had almost lost its meaning overlaid with comment and controversy, now, as it were, re-translated from the original—not always very lucid—idiom, resumes the character of simplicity and truth. For example, the dictum that "rent does not enter into the cost of production," will nevermore, it may be expected after Professor Carver's explanation, perplex the inexpert. So the truth which Ricardo somewhat harshly expressed when he predicted the same sort of effect whether you "diminish the cost of production of hats," or "diminish the cost of subsistence of men"—the portion of truth

[1] In particular, a rational algebraical function $y=A_0+A_1x+A_2x^2+\ldots+A_nx^n$; whether the constants are considered as numerous as the observations and so fitting them exactly, or less numerous, fitting the observations as well as possible.