or less reduced in such wise that the final utility procured by the last penny expended should be the same for both articles. In the case of a stud, indeed, considered as an object of consumption which varies by very large differences, a certain slight significance may be attached to the statement that the value of the (last) horse kept is measured by a "foreign utility." The fractions of the price—the shillings not the pounds; or the units, but not the tens of pounds—are so determined.

Mr. Smart seizes the essence of the new doctrines when he says in conclusion: "What is contended is that the law of cost is a good working secondary law" (p. 82); not "fundamental" (p. 81), like the principle of utility. He quotes Jevons: "Repeated reflection and inquiry have led me to the somewhat novel opinion that *value depends entirely on utility*." It is true that Jevons does say so. But the economists who appeal to his authority in favour of a one-sided theory of value remind us of the logicians who, following Aristotle, attended only to his doctrine of the syllogism, forgetful that the great master had also taught the complementary principle of induction. It is to be remembered that Jevons, in his chapter on labour, in effect makes value depend not "entirely on utility," but also on the *disutility* of labour. After giving (p. 200, 2nd ed.) equations for the distribution of a given fund of labour among different employments—the problem beyond which, as we understand, the Austrians have not thought it necessary to advance—Jevons proceeds:—"We require another equation in addition to the above. . . . Labour will be carried on until the increment of utility from any of the employments just balances the increment of pain." In other words, and as the symbols of the context suggest, utility and disutility are independent variables in that expression, the maximum of which determines economic equilibrium. Among the important lessons which the world owes to the Austrian school, this is not one.

Das Geschlechtverhältniss der Geburten in Preussen. Von Dr. C. Düsing. (Jena: Gustav Fischer), 1892.

The familiar observation that the areas of art and science do not coincide is nowhere more strikingly exemplified than in the field of inquiry which Dr. Düsing has chosen. The proportion between the sexes at birth is not related, like the death-rate at different ages, to the business of insurance or the art of sanitation. The fact that some five per cent. more boys are born

than girls is probably a mere *curiosum* in the eyes of the practitioner; yet it has a theoretical value for the biologist, especially when compared with similar observations for the inferior animals, and even plants, as has been done by Dr. Düsing in an earlier work. Moreover, even if the fact were entirely isolated and remote from physiological inquiries, its investigation would still possess a scientific interest, as affording a particularly perfect study in statistical method. For the proportion between the numbers of male and female births is one of the concrete phenomena most amenable to the delicate rules for appreciating evidence which are furnished by the Theory of Probabilities. Not only is the average rate particularly constant under unaltered circumstances, but also the deviations from the average, the so-called "errors," display a typical regularity not often found elsewhere outside the sphere in which the Calculus of Probabilities took its rise, viz. games of chance. This character of the sex-ratio was first pointed out by Professor Lexis, whom Dr. Düsing has wisely chosen as his guide. Instructed by the author of *Massenerscheinungen*, Dr. Düsing constructs a formula whereby to test the worth of an average in view of the number of observations which it represents. This touchstone being applied reduces many specious and hitherto esteemed generalisations to worthless conjectures. On the other hand, those observations which are tried by this test and not found wanting acquire in the eyes of the connoisseur a value—as it were of refined gold—concentrated into small bulk. Take, for example, the observation that the sex-ratio (as we may call, for brevity, the percentage of male compared with female births) for more than a million plural births (twins and triplets) in Prussia was 104·447 (males : 100 females); whereas the corresponding ratio for all births (single as well as plural) was 106·305, the observations on which this average is based numbering several millions. The difference between the two averages, viz. 1·858, being more than twelve times as great as the *probable error*, 0·14, cannot possibly be accidental. We obtain at a stroke a degree of evidence in favour of law which could only be obtained, according to the common method, by separately considering several batches of statistics for plural births and births in general, and observing whether the respective sex-ratios repeatedly differed from each other in the same sense.[1]

[1] We venture to suggest that the author's argument would have been improved in form, if he had contrasted the sex-ratio for plural births during one period, say, 1824–1887, with the sex-ratio for births in general during the *same* period. As it

The observation that the sex-ratio is less in the case of plural births is one instance of the author's theory that this ratio tends to decrease as the birth-rate increases, and *vice versâ*. The theory is more directly supported by the variations of the two phenomena from month to month. The seasons of maximum birth-rate correspond with the seasons of minimum sex-ratio, and conversely. The variations from year to year during the period 1816–1887 bear similar witness. Each year of maximum birth-rate coincides with, or at least is adjacent to, a year of minimum sex-ratio, and conversely. Adding the births for the ten years in which the birth-rate is particularly high, Dr. Düsing finds the sums 3,650,678 boys and 3,451,820 girls; with the sex-ratio (or proportion of male to female births) 105·761 (: 100). Likewise adding the births for the twenty-four years in which the birth-rate is particularly low, he finds 9,021,366 boys and 8,495,880 girls; with the sex-ratio 106·1852. The difference between the two percentages, viz. 0·424, being some seven times greater than the probable error to which it is liable, is certainly significant. The present writer has obtained a similar result by considering the shorter and more homogeneous period, 1816–1870, omitting the years 1870–1887 which form part of Dr. Düsing's data. The same conclusion is suggested by observing the variations of the sex-ratio from place to place. Some caution, however—not to say ingenuity—is required in interpreting this evidence. If we put in one category the places which on an average of several years have a very high birth-rate, over 42·5 per 1,000, in another category the places which have a birth-rate between 42·5 and 40, and so on in descending degrees of birth-rate, the categories so formed will not present, as they should according to the theory, an increasing scale of sex-ratios. But Dr. Düsing appears justified in accounting for this anomaly by the fact that in a long tract of years there are in each district ups and downs of prosperity, with attendant variations in the birth-rate so violent as to mask the correspondence between high birth-rate and low proportion of males. It appears legitimate therefore with him to put in one category, not simply the place which on an average of years has a high sex-ratio, but the

is, he has taken for the second term of comparison a different period, namely, 1874–1887; thus leaving himself open to the suspicion that the difference on which his argument rests is due to a secular increase of the sex-ratio, which is higher for Prussia during the period 1874–1887 than during the preceding half-century. In fact, the difference between the sex-ratios for plural births and births in general is not so large as he represents it, but quite large enough for the argument. (See *Geschlectverhältniss*, p. 19, referring to p. 1.)

place *with the time; e.g.*, in the category of birth-rate above 45 per 1,000, Dantzig, 1875–77; Marienwerder, 1873–80, etc.; in the category of birth-rate under 45 and over 40, Dantzig, 1873–74..., Marienwerder, 1881–83...; and so on. Thus rearranged the descending scale of birth-rates corresponds exactly with an ascending scale of sex-ratios.

The proposition which connects a small excess of male births with a large birth-rate, connects it also with prosperity and abundance. But an independent principle, as we understand, is involved in the theory that the depletion of the adult male population caused by war is compensated by an increase in the excess of male births. Another independent principle connects a large proportion of females with "crossing" of breeds. Dr. Düsing thus explains the well-known fact that the excess of male births is particularly small in towns. The less familiar variations in the sex-ratio according to the religion of the parents are similarly explained. With the Jews, owing to the purity of their breed, the excess of males is particularly great. Conversely mixed marriages between Christians of different persuasions—corresponding it is presumed to mixture of breed—are favourable to female births. In the case of the mixed marriages between Jews with Christians the large proportion of girls is quite overwhelming. But Dr. Düsing with characteristic judgment and candour does not clutch at this piece of evidence; which, tested by the theory of errors, is found not to be very valuable.

The hypothesis that "crossing" favours female births is used by Dr. Düsing to account for another frequently observed circumstance: namely, that the excess of males is remarkably small in the case of illegitimate births. In a chapter devoted to the latter subject, Dr. Düsing brings out the curious relation that, according as the proportion of illegitimate (to legitimate) births increases, the peculiar characteristics of illegitimate births diminish. May it be, he suggests, that, where illegitimacy is more of an institution, the condition of illicit unions approaches nearer to that of regular marriages? We might thus account for the decrease, with the increase of illegitimacy, of at least one incident of that condition, namely, the high proportion of the still-born which prevails among illegitimate births.

As it has long been known, the number of males who do not cross the threshold of life is much greater than that of females, in the ratio of about 130 : 100. Dr. Düsing has moreover proved, by a rigorous use of the calculus, that this ratio diminishes as the proportion of still-births to births in general increases. He

accounts for this observation by an ingenious and probable theory.

The distinction between fact and theory is present to us while expressing our admiration for Dr. Düsing's researches. It is one thing to determine the mathematical probability that two averages differ significantly from each other; another thing to appreciate what Cournot calls the "philosophical probability" that a particular cause has operated. As that high authority on the doctrine of chance remarks, no attention would be due to a theory that the sex-ratio varied according as the day of birth was odd or even. Some such reflection may perhaps be suggested by our author's hypothesis that the ravages of war upon male life are repaired by the compensating action of generative nature. It must be remembered however that "philosophical" probability, though in a sense *a priori*, yet ultimately rests upon experience. Now Dr. Düsing, in virtue of his earlier studies on the sex-ratio both for man and the inferior creation, may fairly lay claim to just that wide conversance with the phenomena which constitutes a good authority on the question what hypotheses are worth submitting to statistical verification. We do not presume therefore to criticise his physiological speculations. We content ourselves with expressing the wish that the present work might be translated into English as affording a particularly perfect example of the technique of statistics.

Mathematical Investigations in the Theory of Value and Prices. By Dr. Irving Fisher. From *Transactions of the Connecticut Academy* (Vol. IX., July 1892.)

Dr. Fisher is distinguished above most writers on Economics in that he does not attempt to carry the reader over the whole ground, however familiar, but confines himself to those parts where he is himself a path-breaker. Or, if it is necessary to start by beaten ways, yet even these he makes straighter, and improves them by depositing new materials.

The last remark applies especially to the first part of the *Investigations*, in which the author restates many of the conclusions of his predecessors. He imparts new clearness to the idea of marginal utility by introducing a "unit of utility." "The utility of the hundredth loaf per year may be regarded as the unit of utility," it being assumed that the utility of bread (or any other commodity which may be selected as the standard) depends on the quantity of that commodity, "but is *independent*