

accounts for this observation by an ingenious and probable theory.

The distinction between fact and theory is present to us while expressing our admiration for Dr. Düsing's researches. It is one thing to determine the mathematical probability that two averages differ significantly from each other; another thing to appreciate what Cournot calls the "philosophical probability" that a particular cause has operated. As that high authority on the doctrine of chance remarks, no attention would be due to a theory that the sex-ratio varied according as the day of birth was odd or even. Some such reflection may perhaps be suggested by our author's hypothesis that the ravages of war upon male life are repaired by the compensating action of generative nature. It must be remembered however that "philosophical" probability, though in a sense *a priori*, yet ultimately rests upon experience. Now Dr. Düsing, in virtue of his earlier studies on the sex-ratio both for man and the inferior creation, may fairly lay claim to just that wide conversance with the phenomena which constitutes a good authority on the question what hypotheses are worth submitting to statistical verification. We do not presume therefore to criticise his physiological speculations. We content ourselves with expressing the wish that the present work might be translated into English as affording a particularly perfect example of the technique of statistics.

Mathematical Investigations in the Theory of Value and Prices.

By DR. IRVING FISHER. From *Transactions of the Connecticut Academy* (Vol. IX., July 1892.)

DR. FISHER is distinguished above most writers on Economics in that he does not attempt to carry the reader over the whole ground, however familiar, but confines himself to those parts where he is himself a path-breaker. Or, if it is necessary to start by beaten ways, yet even these he makes straighter, and improves them by depositing new materials.

The last remark applies especially to the first part of the *Investigations*, in which the author restates many of the conclusions of his predecessors. He imparts new clearness to the idea of marginal utility by introducing a "unit of utility." "The utility of the hundredth loaf per year may be regarded as the unit of utility," it being assumed that the utility of bread (or any other commodity which may be selected as the standard) depends on the quantity of that commodity, "but is *independent*

of the quantities of other commodities and services." This condition, it may be observed, underlies the construction of a Demand-curve as usually conceived, *e.g.*, by Cournot (*Théorie Mathématique*, chap. xi.) and by Messrs. Auspitz and Lieben. A unit thus rigorously defined might be named a "util." But "perhaps utility is an unfortunate word to express the magnitude intended. Desirability would be less misleading, and its opposite undesirability is certainly preferable to dis-utility."

The theory of exchange which is based upon marginal utility has received from Dr. Fisher some very happy illustrations. Observing that most economists employ largely the vocabulary of mechanics—equilibrium, stability, elasticity, level, friction and so forth—and profoundly impressed with the analogy between mechanical and economic equilibrium, Dr. Fisher has employed the principle that water seeks its level to illustrate some of the leading propositions of pure economics.

"A consumer will so arrange his consumption that the marginal utility per dollar's worth of each commodity shall be the same. . . . The marginal utilities of all articles consumed by a given individual are proportional to the marginal utilities of the same series of articles for each other consumer. . . . Price, production, and consumption, are determined by the equality of marginal utility and marginal cost of production."

We cannot attempt to describe the elaborate construction of tubes and vessels by which these truths are bodied forth. Indeed we must warn the reader that, even with the help of the author's diagrams, a considerable strain of attention will be required in order to follow explanations like the following, which we take at random out of several pages of like matter:

"There remains to be described the system of levers. The purpose of these levers is to keep the continuous ratio of marginal utilities the same for all individuals and equal to the ratio of prices. First, there is a system of oblique levers (F 12, etc., Fig. 9) connected by sliding pivots with the tops of the cisterns and having their lower extremities hinged to wooden floats F, the hinges being on the level of the water of the tank. These floats are free only to shift laterally. . . ."

Whether it is worth while taking much trouble in getting into these conceptions depends upon what may be called their final utility—how much cognate training the reader has already received. For those who are not already familiar with mathematical functions and fluxions Dr. Fisher's illustrations of the interdependence of the different parts of the economic system

must be invaluable. Even mathematicians, while confining themselves to symbols, might miss some of the theories to which Dr. Fisher is conducted. For instance, suppose the income of one individual to increase, *ceteris paribus*; prices in general will rise, but not necessarily of all articles; and thus some exceptional individuals may be benefited, not injured, by the increase of the income of their neighbour. There are mathematicians who have not yet perfectly realised what Dr. Fisher, after recent writers, calls the "fundamental symmetry" between the forces of demand and supply. It is not long since an accredited teacher of the mathematical branch of economics confidently stated:

"Value is determined by something independent of cost of production, and itself determines the maximum cost at which any one will be willing to produce."¹

As long as we restrict ourselves to the first approximation proper to Dr. Fisher's Part I.—"the utility of each commodity assumed to be dependent only on the quantities"—it is possible to represent the relation between price and quantity of commodity by the mechanism which Dr. Fisher uses; the price corresponding to the depth (measured downward from a fixed point) of the level of a liquid in a certain vessel or cistern, the configuration of which corresponds to what is known as an individual's Demand-curve. But when we suppose "the utility of one commodity a function of the quantities of other commodities," mechanical illustration is no longer adequate to the complexities which arise. In the case of two "competing"—or as they are sometimes called, rival—commodities (such as tea and coffee) or two "completing" commodities—or commodities which are the subject of a *joint demand*—(such as tea and sugar) we should have to suppose that, as one vessel is filled with liquid, the shape of the other shrinks or bulges. In the most general case "the shape of each cistern is a function of the whole state of equilibrium and differs so soon as that differs." The whole rigid system bursts up in a universal *débâcle*; as we relax the assumption that the utility of one commodity is independent of that of others. To represent the more complicated cases, we must resort to the more plastic medium of symbol; and, beginning with the case of two commodities interdependent in their use, put " $U = \phi(A, B)$, where U is the total utility to [an individual], of the consumption combination A and B ."

"Consider horizontal sections of this surface, that is, sections parallel to the plane of the A and B axes. Each section forms a

¹ *Statist*, December 17, 1892.

curve which may be called an *indifference curve*. It is the locus of points representing all consumption combinations of A and B which have a given total utility." Suppose an individual, possessed of the quantities A and B to receive an increment to his income; in what *directions* will he increase his expenditure on the two commodities? Not of course along the indifference-curve which passes through the point A, B; for thus he would not increase his total utility; but in the direction of "maximum increase of utility," which proves to be perpendicular to the indifference-curve. These conceptions, received by Dr. Fisher from his predecessors, have been greatly improved by him; they have been applied to a variety of concrete cases; *e.g.*, with reference to competing commodities, such as the "grades" of the same article virtually form. "If the rich consumers predominate," a certain line becomes steeper, and—

"The two prices of the two qualities separate widely. This interprets the fact that in a rich market like New York City a slight difference in quality will make an enormous divergence in price, while in some country towns different grades either do not exist, or sell for nearly the same price. In the country districts of 'the West' all cuts of beef sell for the same price (about 10d. per lb.). In the cities of the West two or three qualities are commonly distinguished, while in New York a grocer will enumerate over a dozen prices in the same beef, varying from 10 to 25 cents per lb."

When we suppose the total utility to be a function of three commodities we have no longer a dimension to spare for the representation of that utility. But we may imagine the "indifference surfaces" like so many shells (*e.g.*, shaped like egg-shells) over which utility is distributed as a *density*, like electricity over a conductor.

"The utility distributions may be very complicated. If the three articles are substitutes, like oats, corn, and rye, the indifference-surfaces may be almost plane. . . . If they are competing articles, as cuffs, collars, and ties, the indifference surfaces are arranged like concentric cocoons directed towards the origin."

The "maximum directions," the normal to the indifference-surface, are the same (parallel) for all individuals in a market.

The dimensions of space fail us when we advance to the general case of any number of interdependent commodities. Soaring into the fourth and higher dimensions, beyond the furthest flights of his predecessors, Dr. Fisher employs the "quaternion analysis of Hamilton, the *Ausdehnungslehre* of Grassman," to

indicate the analogues of the theorems which he has proved for simpler cases in more familiar symbols. He attains in this higher region a conclusion which seems to us of unexpected importance. Unless certain mathematical conditions are fulfilled by the "maximum directions" and the prices which act along them, "integration is impossible and there is no such quantity as total utility or gain"!

The importance of this theorem appears in the conclusion that "if we seek only the causation of the *objective facts of prices and commodity distribution*" certain "attributes of utility as a quality" are unessential. We need not assume that "for the same individual the marginal utilities at one consumption-combination can be compared with those at another, or at one time with another." While in the elementary stage, fixing attention on a single commodity, we require to assume some standard unit, in the higher stage, dealing at once with several commodities, "we may dispense with the total utility density¹ and conceive the economic world to be filled merely with lines of force or 'maximum directions.'"

This appears to us a very remarkable result; contrasted with the view of the ordinary mathematical economist, very much as Bailey's contention that values are nothing but ratios, with the opposed position of Malthus that something of the nature of a real standard underlies value. One way of realising the significance of the new idea would be to observe that, so far as it is accepted, the principle of "Consumer's Rent," as explained by Professor Marshall, becomes inapplicable. The case would arise which has been carefully excepted by Professor Marshall. (See *Principles of Economics*, Book III. ch. iii. § 6, and p. 753.)

We do not understand that Dr. Fisher carries scepticism so far as to consider this case as general. On the contrary, he regards the assumption that the utility of one commodity is independent of that of another as a first approximation which holds fairly well and widely. "In general the interdependence in the shapes of the cisterns (the Demand-curves for different articles) is very slight." It may be added that the more realistic view is at least favoured by the analogies accepted by Dr. Fisher between the principles of maximum energy in Physics and maximum utility in Economics.

There is another "attribute of utility as a quantity" which we may dispense with when we "seek only the causation of the *objective facts of prices and commodity distribution*": namely,

¹ See above, p. 39.

that one man's utility can be compared to another's. Dr. Fisher, who has a just conception of the great gulf which separates economics from moral philosophy, regards comparisons between the pleasures of different individuals as "mysterious" (p. 99) which "do not belong here" (p. 87). At the same time he throws out some hints which will be valuable to the utilitarian.

"The statistician might begin with those utilities in which men are most alike—food utilities—and those disutilities in which they are most alike as the disutilities of definite sorts of manual labour. By these standards he could measure and correct the money-standard, and if the utility curves for various classes of articles were constructed, he could make rough statistics of total utility, total disutility, gain, and utility-value which would have considerable meaning. Men are much alike in their digestion and fatigue. If a food or a labour standard is established, it can be easily applied to the utilities in regard to which men are unlike, as of clothes, houses, furniture, books, works of art, etc." (p. 87).

There are those who think that the principle of final utility is destined to have a more important use in socialistic politics than even in abstract Economics; that, when the regime of competition shall have passed away, the laws of utility will still be employed to regulate utilitarian distribution. Without forecasting a future so remote, we may at least predict to Dr. Fisher the degree of immortality which belongs to one who has deepened the foundations of the pure theory of Economics.

Public Finance. By Professor C. F. BASTABLE. Third edition. (London: Macmillan & Co. 1903. Pp. 780.)

REFERRING for a general estimate of this now classical work to Mr. Price's appreciative review of the first edition in the first volume of the *ECONOMIC JOURNAL*, we shall here confine ourselves to some of the passages which have been added in the third edition. They relate mostly to recent developments of financial theory and fiscal policy. Under the first head, "the theory of minimum sacrifice as the principle for distribution of the public burdens," the principles of local taxation, "the ever-recurring question of incidence," and other controversial topics are treated. Professor Bastable, referring to diversities of doctrine comprehended under the generic principles of sacrifice, concludes that "these complications in the employment of the sacrifice principle seem to justify adherence to the objective standard of ability.