

has been held in different ages and countries. The latest English researches are incorporated; but the point of view is cosmopolitan. There could not be a better preparation for a discussion of the nationalisation of the land. This cannot be defended, Mr. Pierson holds, as tending to increased production, but only as a means to more equal division of the produce. Mill's arguments on that head may quite possibly become applicable, if there should be a general rise of rents before another century passes. Still, the financial difficulties of the operation would be formidable. Then the salaries of the public stewards or bailiffs who would be required would be burdensome. These expenses might indeed be reduced by leases of the nationalised lands for 100 years. But who would benefit by the arrangement? Our great-grandchildren. But if we want to benefit posterity, there are more certain ways: *e.g.* paying off the National Debt. In these reasonings it is taken for granted that compensation is to be given to the owners who are to be expropriated. Henry George's plan is rejected as one of the most wasteful forms of confiscation.

It will be understood that we are presenting only samples of Mr. Pierson's reasonings. We still cherish the hope that the work in its entirety will be made accessible to the English reader by being translated into some language more generally familiar than the Dutch. There is a quality in the work which is probably demanded by our public more than much of the home-made article. *The Times* no doubt expressed a national sentiment when, reflecting upon Walker's death, their leader writer said that he resembled the older classical economists who kept in touch with practical life and had no idea of making their science a collection of refinements remote from the business of the world.

That combination of the man of affairs and the professional economist which has become uncommon in England is to be found in the Prime Minister of Holland.

PROFESSOR GRAZIANI ON THE MATHEMATICAL THEORY OF  
MONOPOLY.<sup>1</sup>

ECONOMIC controversy is generally a thankless task. You cannot hope to make any impression on your opponent. Yet he is the only reader on whose interest you can count. However, a

<sup>1</sup> Augusto Graziani, *Sulla ripercussione delle imposte nei casi di monopolio (A proposito di alcune osservazioni del prof. Edgeworth)*, Torino, Fratelli Bocca, 1898, p. 12. [Referring to a review of Professor Graziani's *Istituzioni* by the present writer in the *ECONOMIO JOURNAL*, Vol. VII. p. 403. Above, p. 80.]

discussion with Professor Graziani upon the theory of monopoly may form an exception to this general rule. For Professor Graziani has shown in his courteous reply to my criticisms of his *Istituzioni* that he is amenable to argument. He makes one concession,<sup>1</sup> which is one more than most disputants can be induced to make. Also a more than personal interest may attach to the renewed discussion of the points on which Professor Graziani still holds out. They relate to important problems in the science of taxation; and they raise the general question whether the mathematical can be of any assistance in dealing with such problems. Professor Graziani maintains the negative: that "in the matter of repercussion [shifting and incidence of taxation] Cournot and the other purely mathematical writers have not proved any truths which have not been, or could not easily be, demonstrated without the use of mathematical symbols."<sup>2</sup> I accept this challenge and join issue at all the disputed points.

I. Professor Graziani denies what I, after Cournot, have affirmed, that normally and in general a specific tax (a tax of so much per unit supplied) on a monopolised article will (tend to) raise the price of the article. He argues thus: "In order that in every case an elevation of price should be advantageous to the monopolist, it would be necessary to prove that universally the gain resulting from the diminution of the impost would be greater than the loss which the net profit suffers from the change of price. And this demonstration Edgeworth has not given and could not give. He ought to have proved that if, for example, the price 5 gives the largest net profit to the monopolist before the imposition of the tax, there will be found, after the imposition of the tax, a price greater than 5, which will *necessarily* give a profit greater than any other price. And whereas by hypothesis, abstracting from the tax, any higher price causes the monopolist to obtain a profit smaller than 3,000,<sup>3</sup> which we will call  $3,000 - \alpha$ , if the [total] tax charged on the amount produced at the price 5 is called  $b_1$ , and the tax on the amount produced at a price greater than 5 is called  $b$ , Edgeworth's proposition may be replaced by the formula:  $3,000 - (\alpha + b) > 3,000 - b_1$ ; which reduces to the following:  $\alpha + b < b_1$ . . . . If we represent by  $c$  the excess of  $b_1$  over  $b$ , we may transform the condition above stated into this:  $\alpha + b < b + c$ , which is fulfilled if  $\alpha < c$ ."

But, continues Professor Graziani, as  $\alpha$  depends upon the law of demand, and  $b$  partly upon the law of demand and partly on the

<sup>1</sup> *Op. cit.*, p. 10.

<sup>2</sup> *Op. cit.* p. 4.

<sup>3</sup> Referring to an example given.

amount of the tax per piece, "the law of demand and the rate of taxation may be such that  $a$  is either less or equal or greater than  $c$ ;<sup>1</sup> and it may accordingly be the interest of the monopolist either to raise the price or to maintain the original price."

In this passage Professor Graziani has well stated the problem; but he has not fully utilised one of the data: namely, that the net profit of the monopolist was a *maximum* at the original price. When that circumstance is attended to, the relation of  $a$  to  $c$  is found not to be so indeterminate as Professor Graziani supposes. When that change of price is small,  $a$ , the diminution of the net profit (irrespective of the tax), must be positive ( $-a$  must be negative) whether the small change is in the positive or negative direction. Thus if we designate by  $\pm \Delta p$  the small quantity added to, or subtracted from, the original price, it may be expected that  $a$  will be of the form  $\pm (\Delta p)^2 \times w$ , where  $w$  is positive.<sup>2</sup> Also  $b_1$  the amount of tax paid at the original price, is of the form  $x \times t$ , where  $x$  is the amount of the commodity supplied, and  $t$  the rate of taxation per piece. Accordingly  $b$ , the corresponding amount for the price  $p + \Delta p$  is approximately of the form  $t \times (x - \Delta p \times v)$ ;<sup>3</sup> assuming as usual that for small variations the change in the dependent variable ( $x$ ) is proportional to the change in the independent variable ( $p$ ). Whence  $c (= b_1 - b) = \Delta p \times vt$ . This is a quantity of the first order, proportioned to  $\Delta p$ , while  $a$  is a quantity of the second order, proportioned to  $(\Delta p)^2$ . Accordingly, by taking  $\Delta p$  sufficiently small, the condition which Professor Graziani rightly postulates, viz. that  $a$  should be less than  $c$ , will be fulfilled.

A geometrical illustration may put the matter in a clearer light. In the annexed figure let  $OX$  be a horizontal line, and  $ABC$  a curve which is continuous and concave towards  $OX$  in the neighbourhood of  $B$ , the point at which the vertical height of the curve above  $OX$  is a maximum. We might suppose the line to represent the floor, and the curve the vaulted roof, of a cell, so narrow that a prisoner confined therein could only move backwards or forwards in one line, viz.  $OX$ , so low that he could

<sup>1</sup> "Quindi a norma della ragione dell'imposta e della legge della domanda può avvenire  $a$  o minore od uguale o maggiore di  $c$ ." *Op. cit.*, p. 6.

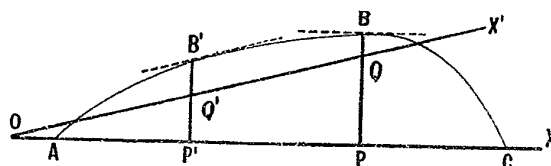
<sup>2</sup> Here  $w$  is put for *minus* half the second differential of the monopolist's net profit with respect to the price, at the original price (abstraction being made of the tax). This net profit being a maximum, that second differential must be negative, and therefore  $w$  positive, in general, and excepting the peculiar cases in which the second differential (as well as the first) vanishes at the point of maximum or becomes infinite.

<sup>3</sup> Here  $v$  is the rate at which the quantity consumed diminishes with the increase of price; what in Cournot's notation is represented by  $-F'(p)$ .

not anywhere raise himself to his full height. The position at which he could most nearly stand erect, which we will suppose to be the position of greatest comfort, is given by the point  $B$ , at which a tangent to the curve is parallel to  $OX$ .

Now suppose that Inquisitorial ingenuity makes the floor of the cell an inclined plane of which  $OX'$  is a section. The new position of greatest comfort is given by the point  $B'$  on the curve which is at the greatest vertical distance from the straight line  $OX'$ , that is the point on the curve at which the tangent to the curve is parallel to  $OX'$ . It is evident that this point is to the left of (the vertical through)  $B$  the original point, the curve being continuously concave. Accordingly our prisoner seeking the position of greatest comfort will move from his original position towards  $O$ .

Is it necessary to interpret the parable? The amount of commodity supplied is represented by a distance measured from  $O$  along the abscissa  $OX$ . The corresponding ordinate to the



curve represents the net profit of the monopolist irrespective of the tax. This profit is a maximum for the amount  $OP$ . The angle  $XOX'$  has for its tangent  $t$ , the tax per unit of commodity; so that the total tax paid on any amount of commodity supplied, e. g.  $OP'$ , is represented by  $P'Q'$ , the vertical distance of  $P'$  from the line  $OX'$ . Accordingly the net profit of the monopolist after the tax is represented by the length of the vertical intercept between  $OX'$  and the curve. It is evident that  $OP'$ , the abscissa for which this intercept is a maximum, is less than  $OP$ . That is *after the tax* the amount supplied is less, and accordingly *the price is higher*.

II. and III. The fate of Professor Graziani's first proposition involves that of two others, which he thus restates: "As to whether the price will be raised or the original price will be maintained, I laid down these principles: that the first alternative will be so much the more probable if the demand approaches the conditions of relative stability," i.e., as Marshall would say, is inelastic, or if the production "obeys the law of diminishing

returns," or that of constant returns.<sup>1</sup> But it has been shown that a tax is attended with a rise of price, not sometimes, but generally; and except in the peculiar case of a certain discontinuity in the curve representing the monopolist's profit—a peculiarity which has nothing to do with the elasticity of the law of demand, or the increasingness of the returns. No doubt it would be an allowable way of speaking to describe a tendency as null when it is insensible, or so small as to be probably counteracted by friction. It might be legitimate to regard the force of gravitation which, say, a satellite of Saturn exercises on a terrestrial object, as *nil*. But Professor Graziani has expressly cut himself off from this sort of interpretation.<sup>2</sup> It is as if he maintained that heavenly bodies sometimes do, and sometimes do not, attract the earth; the former alternative—the case of attraction—being the more probable, the smaller and more distant the bodies are! The cases in which he says the rise of price is most probable, the cases of inelastic demand and diminishing returns, are precisely the cases in which the rise of price is not indeed null, but particularly small, as I pointed out in my review.<sup>3</sup>

Here is his reasoning as to the first case :—"It seems to me that when the demand approaches conditions of stability, that is when it is not elastic, and to a large increase of price there is apt to correspond only a slight diminution of consumption, the net profit attainable by adopting prices superior to that which realises the maximum gain ought to differ little from that maximum. . . . It is true that also the difference between the amount of tax paid [before and after the tax] will not be great, since the contribution

<sup>1</sup> *Op. cit.*, p. 8.

<sup>2</sup> Non è, come sembra sostenere l'Edgeworth, che se l'imposta è molto tenue, il prezzo non si elevi perché il guadagno del monopolista o la minore perdita di lui sarebbe così piccola da sconsigliare una modificazione, ma sta invece che quando l'imposta è tenuissima vi ha più forte probabilità che  $a < c$  . . . che la conservazione del prezzo primitivo giovi più di qualsiasi modificazione. *Op. cit.*, p. 6.

<sup>3</sup> *Economic Journal*, Vol. VII. p. 406. The following restatement may be acceptable. The addition to the price caused by a tax of  $t$  per unit of a monopolised commodity  $= tv \div (2v + u + R)$ , where  $v$  has the same meaning as in the text (the  $t'(p)$  of Cournot with its sign changed);  $u$  is the rate at which the cost of production increases with the increase of the amount produced (the second differential of Cournot's  $\phi(D)$ );  $R$  is a quantity involving the second differential of the quantity consumed with respect to the price, about which coefficient in general we know nothing. This *addendum* is greater (1) the greater  $v$  is,  $u$  being positive, and other things, in particular  $R$ , being the same; and (2), the less  $u$  is, other things and in particular  $v$  and  $R$ , being the same. But  $v$  is greater the greater the elasticity of demand, and  $u$  is smaller the further the case is removed from that of diminishing returns, becoming indeed less than nothing in case of increasing returns. Thus the rise of price is particularly great in cases exactly opposite to those which Professor Graziani distinguishes as favourable to a rise of price.

depends upon the number of units of the product, and the consumption is supposed to be but slightly [*scarsamente*] diminished; yet, except where the tax is exceedingly light, it is evident that its diminution will exceed the difference between the net profits attainable at the two profits; that is, that  $c$  will be greater than  $a$ , and therefore the elevation of price probable." <sup>1</sup>

The correct statement is that, as shown above,  $c$  will *generally* be greater than  $a$  for sufficiently small values of  $\Delta p$ . The question is: Will the value of  $\Delta p$ , at which  $a$  just becomes equal to  $c$ , be particularly large or small when, *ceteris paribus*, the elasticity of demand is small? The answer is that the disturbance of price will then be particularly *small* in the common case of diminishing returns. But it is not allowable to suppose the elasticity of demand to be very slight, indefinitely small. For equilibrium would not have been reached in that case before the tax; the monopolist would have gone on raising the price until checked by a sensible elasticity. <sup>2</sup>

IV. So clear an issue is not presented by Professor Graziani's rejoinder to my remarks on his theory that the monopolist can never shift the whole burden of the tax. The question here is largely one of words, turning on the definition of the term "*shifting*" [*ripercussione*]. I admitted that upon a certain definition of the term, Professor Graziani's statement was not indefensible. <sup>3</sup> I must now admit that he seems to have entertained such a signification.

At the same time I may doubt whether the subtle distinctions of things upon which the selection of a definition turns can be adequately effected by his method. Consider the following contrasts between the regime of monopoly and that of competition.

(i) Whereas in a regime of competition the rise of price consequent on a specific tax cannot in general be greater than the tax, except in the case of increasing returns, or as an indirect result, to afford compensation to the producer for delay and impediment; in a regime of monopoly a rise of price to a greater extent than the tax may occur without these reservations. <sup>4</sup>

(ii) Whereas in a regime of competition, if the demand is perfectly inelastic, it is possible that the producer should not be damnified at all, the price being just raised to the full extent of the

<sup>1</sup> *Op. cit.* p. 9.

<sup>2</sup> If the  $F'(p)$  of Cournot is zero, the first differential of the monopolist's net profit is positive; it will be the interest of the monopolist to go on raising the price until the elasticity is just sufficiently great to reduce the first differential to zero.

<sup>3</sup> *ECONOMIC JOURNAL*, Vol. VII. p. 405, note.

<sup>4</sup> See Cournot, *Recherches*, Art. 33, p. 69, and contrast Art. 51, p. 104.

tax, and the consumption not being diminished; in a regime of monopoly this is not possible, since, as above pointed out, the demand cannot have been before the tax perfectly inelastic.

Professor Graziani seems to acquiesce in the first proposition,<sup>1</sup> as indicated in my review. Nor does he deny the second proposition. But it is hard to see how either could be discovered and proved by his methods.

V. A rejoinder is hardly called for by Professor Graziani's concluding remarks on the dangers of mathematical reasoning. The withers of the mathematician are not wrung by these common-places. The use of the method is not necessarily attended with an exaggeration of its importance. The inability to use it is not a qualification for appreciating its usefulness.

*Letters of David Ricardo to Hutches Trower and others, 1811—1823.*

Edited by JAMES BONAR and J. H. HOLLANDER. (Oxford : Clarendon Press. 1899.)

THE editors have brought to their joint task minds separately trained in Ricardian exegesis. Each of them had independently by his own methods of analysis and commentary illustrated a set of Ricardo's letters : Dr. Bonar those addressed to Malthus, Dr. Hollander those addressed to M'Culloch. Now they apply their united power of critical apparatus to a third set of Ricardo's letters, throwing additional light upon his life and doctrine.

All that we have learnt of the personality of Ricardo, whether from the edited letters or from unpublished memoirs and literary tradition, shows him as more human than unsympathetic critics of his abstract theories have imagined. The trait of character which the letters now before us bring out most conspicuously is public spirit. It is not irrelevant in an *Economic Journal* to allude to some passages of a political complexion which, read with their contexts, may illustrate how broad and liberal were the interests of the great theorist.

His zeal for Parliamentary Reform was ardent :—

“ There is no class in the community whose interests are so clearly on the side of good government as the people. The suffrage must be extensive . . . and the voting must be by ballot” (p. 52, *cp.* pp. 61, 69).

With reference to the Peterloo massacre :—

“ These large assemblages of the people may be regretted, they

<sup>1</sup> *Op. cit.* p. 6, last par., and p. 7