

European, of men. Thus in Japan there is a considerable excess of men; and in the East of Europe the excess of women is less than in the West. Greece perhaps owes her exceptional excess of men to the occupation of the Turks. If so, that occupation left behind it one good result, in the judgment of our author, which deserves to be quoted in full. "In order that women should marry easily men should be in excess. It is desirable that unmarried women should not become numerous, as in politics and social life they are far from being elements of concord and order. We Greeks have an unpleasant experience of this, in spite of the fact that our women are in the minority. The English have an even bitterer experience. The unmarried women of England, the number of whom amounts to about a million, form a peculiar class which has been called 'the third sex.' Lowering wages by their competition with the men, claiming political rights for women, and introducing a freedom of action which is far from favourable to domestic life, this class has become an element of which the disturbing effect is very marked." The views of Pericles respecting the sphere of woman seem to have been inherited by the distinguished modern Athenian. Going on to another head of the census, the "Civil State," as our statisticians say, Prof. Andréadès points out that inferences as to the character of a people from the number of marriages must be made with caution. In Greece, account must be taken of the excellent custom—literally "sacred tradition"—that brothers must see their sisters settled in life before they themselves marry. Prof. Andréadès concludes his striking and instructive address by combating the prejudices against the census, which seem to be as strong in Greece to-day as they once were in England. He reminds the devout of the memorable journey to Bethlehem that was undertaken in obedience to the requirements of the census; he warns the patriotic that deficiency in this branch of statistics is generally regarded as a mark of an imperfect civilisation.

Gold Prices and Wages under the Greenback Standard. By WESLEY C. MITCHELL. (University of California Publications in Economics.) (Berkeley: University Press, 1908. Pp. 627.)

THIS is a continuation of the author's *History of the Greenbacks*; or rather the materials for such a continuation, "the statistical apparatus of a book still to be written."

The statistical apparatus would be well worth studying for its own sake, even if the author had not pointed out its economic significance. In dealing with prices and wages he is not content with the comparison of arithmetic means; he employs largely *medians* and *deciles*. For example, for the prices of October, 1873, compared with 1860, there are *ninety-one* percentages: as thus, in the order of magnitude,

58, 76, 78 *bis*, 85, 86, 89, 91 *bis*, 92, . . . 241, 250, 367.

The deciles are points on this scale which divide the whole group of percentages into ten sub-groups, each comprising a number of percentages which is the tenth part of the total number, that is, 9, or rather 9.1. Thus the first decile is 92, since up to and exclusive of 92 there are nine entries—78 and 91 each occurring twice. Likewise the ninth decile is 196, since up to and exclusive of that entry there occur *eighty-one* percentages. The median is identical with the fifth decile. The graphical representation of this system consists of ten more or less parallel curves.

There can be no question as to the value of this statistical method. But before it is generally adopted its cost in labour should be counted. Is it worth all the trouble that it involves? Could nearly as valuable results be obtained with considerably less trouble? If for summary purposes an abridgment is desiderated, I suggest as a rough and ready rule the following. Find the second, fifth, and eighth decile; these determinations (with some attention to the highest and lowest figures that occur) will adequately characterise the group.¹ Thus for the prices of October, 1873, the data would be as follows:—

Lowest.	2nd decile.	Median.	8th decile.	Highest.
58	104	131	167	367

This presentation brings out the interesting circumstance that the distribution of the inflated prices is not symmetrical; the higher prices exceed the average much more than the lower prices

¹ The constants *c* and *j* pertaining to an asymmetrical curve of error (of the kind described by Mr. Bowley in the Appendix to the second edition of his *Elements of Statistics*) may be roughly calculated from the median together with two percentiles, as shown in the paper on the methods of representing statistics contributed by Mr. Bowley and the present writer to the *Journal of the Royal Statistical Society*, June 1902. A particularly simple construction is afforded by the 16th and 84th percentile (corresponding to the points *R*₁, *R*₂ mentioned by Mr. Bowley in the paper referred to) taken in connection with the median. But Mr. Mitchell's statistics are perhaps too violently asymmetrical to justify the use (for this purpose) of percentiles at such a distance from the central region.

fall short of it. The incident had been observed before, but not, I think, evidenced so fully as by Mr. Mitchell's statistics. For the period 1860 to 1880 the average difference between the second decile and the median is 25.6 (as appears from Mr. Mitchell's Table 5), while the average difference between the median and the eighth decile is 33. The differences between the higher and the lower parts of the scale are much greater.

There is a propriety in fixing attention upon the central portion of the statistical group. For that is the portion most amenable to general law. It is there that we may look for the fulfilment of the law of error—the asymmetrical law, which is a generalisation of the simpler law expounded by Mr. Galton. We may expect to find the law when we have reason to believe that there exist the conditions of its genesis—the sporadic action of independent agencies, a chaos of elements

"Confusedly in their pregnant causes mixed."

Now with regard to price-variations, Mr. Mitchell's statistics afford evidence of this fortuitous distribution. His Table 14, showing the numerical order of the percentages representing price-variations for some ninety commodities at the epoch of 1865 and 1879 respectively, shows in regard to the character of the commodities thus arranged the sort of disorder which is favourable to the application of Probabilities. "Unlike commodities are often side by side, and related commodities are often widely separated." In the period under observation forty-two commodities moved from the lower half of the table to the upper half, or the other way.

The statistics of wages do not, I think, afford such perfect data for the application of the more refined methods; principally for the reason which is thus indicated by Mr. Bowley. "In . . . wage groups we are not dealing with unconnected units; there is a tendency towards a standard wage in many occupations."¹ It is disconcerting to find in Mr. Mitchell's tables of wages coincident deciles; as to which Mr. Mitchell remarks :—"The fact that two or more of the deciles for an industry are frequently identical is due mainly to the fact that the series for an important occupation in a large establishment often represents more than a tenth of the whole number of employees in the industry as shown by the data" (p. 104). This lumping together of numerous statistics at one point appears to me to impair the method of percentiles, as applied to wages, more seriously than the opposite

¹ *Journal of the Statistical Society*, loc. cit.

characteristic—the absence of density—by which Mr. Mitchell is disquieted, even with reference to statistics of prices. “The median,” he says, “is rather erratic within limits of several points, because its precise position is often dependent on the relative price of a single commodity, which stands in the middle of the scale of relative prices” (p. 58). The imperfection is due, as the writer intimates (p. 33, note), to the smallness of the group. He might have added that it is not entirely removed by the use of the arithmetic mean which he proposes to employ collaterally. The advantage of the arithmetic mean in respect of definiteness over the median is to a large extent only in appearance.¹

I am looking at the matter from the point of view of Probabilities, each observation being regarded as representative of some coefficient that would be applicable to an ideally complete set of statistics. There are other points of view; there are diversities of problems, as Mr. Mitchell has pointed out, with respect to prices at least. With reference to one kind of problem it is proper to use price-variations not weighted according to the amounts, but rather selected according to the independence of each commodity, and not necessarily combined by way of the arithmetic mean. For another problem it is proper to weight the prices with corresponding quantities of commodity, and to combine the figures on the principle of the arithmetic mean, simple addition.

Of the latter description is the problem to determine the variation in the cost of living to the wage-earner, the change in the money-price of the set of articles (considered as constant) which he consumes. Mr. Mitchell's statistics throw much light upon this problem. Some of his results may be subsumed in the following generalisation. Let us define as “responsiveness” (to changes in monetary policy) a couple of attributes which are observed to go together, namely, the rise of index-numbers representing prices in paper-money from 1861 to 1864, and the fall thereof from 1864 to 1879. Then in the scale of responsiveness the following classes of prices prove to be in a descending order: (1) Prices of gold (in greenbacks); (2) wholesale prices of commodities; (3) retail prices; (4) cost of living; (5) wages. The paper price of gold rises higher and falls further than the wholesale prices of commodities, and much more than retail prices; the cost of the articles consumed by workmen fluctuates more violently than their wages (pp. 237, 273, 278, *et passim*).

This rough generalisation masks specific differences of great

¹ See Index, *Median*.

interest. For instance, it is found by Mr. Mitchell—contrary to a general belief, I think—that the responsiveness of the lower grades of wages is greater than of the higher grades. The paradox is explained by “the more severe pressure which the increased cost of living puts upon wage-earners with small incomes” (pp. 165, 186). The supply of common labour seems to show an elasticity now commonly supposed to exist only in the pages of Ricardo.

Mr. Mitchell points to other results of economic significance which it may be hoped he will develop in the future continuation of his *History of the Greenbacks*. He will thus obtain as high a rank in Economics as that which he has now attained in the twin science of Statistics.

The Meaning of Money. By HARTLEY WITHERS. (London : Smith and Elder. 1909. Pp. 307.)

EXPERTS seldom avoid the sort of mistake which was committed by a distinguished astronomer when, lecturing to a popular audience about the distance of the stars, he continually employed without explanation the term “parallax.” The distinguished astronomer could not put himself in the position of persons to whom that technical term was unintelligible. Unlike so many experts, Mr. Withers begins at the beginning. He can put himself in the position of those who have not yet lost through familiarity the sense of wonder which the modern monetary system is calculated to excite in the ingenuous mind. “At first sight there is something whimsical in the process of stimulating production and expanding trade by an agreement between two parties to owe one another something.” The conditions under which this “magical business of providing currency and credit on a basis of mutual indebtedness” becomes possible is happily illustrated by the following metaphor. “Just as a man cycling through a crowded street depends for his life, not only on his own skill, but also on the care with which the rest of the traffic is driven, so the English banking system is dependent on the sanity and sense of the public as much as on its own soundness.” Illustrations drawn from literature, as well as metaphors from common life, are employed with effect. The fictitious bill of exchange is illustrated by the “bill of asscolts” which Don Quixote, while wandering in the Sierra Morena, drew on his niece for three colts to be delivered to Sancho Panza “for the like number received of him here in tale.” Still more remote